

A RECURSIVE ALGORITHM SUITABLE FOR REAL-TIME MEASUREMENT

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ABSTRACT - This paper deals with a recursive algorithm suitable for real-time measurement applications, based on an indirect technique, useful in those applications where the required quantities cannot be measured in a straightforward way. To cope with time constraints a parallel formulation of it, suitable to be implemented on multiprocessor systems, is presented. The adopted concurrent implementation is based on factorization techniques. Some experimental results related to the application of the system for carrying out measurements on synchronous motors are included.

INTRODUCTION

The application of high performance intelligent instruments helps implement complex measurement algorithms, such as those allowing for the determination of the structure of a mathematical model (identification) of the system or for the measurement of the corresponding unknown parameters (estimation), starting from the input and output data.

These new instruments can be advantageously applied for the parameter estimation, as some recent tendencies have shown, which can be practically considered as an indirect measurement of quantities also non-accessible or some what ill-favoured in their accessibility (SYDENHAM-THORN [1]). The indirect measurement of some quantities, such as rotor currents in squirrel-cage induction motors, shaft speed in asynchronous motors, thrust in linear motors, etc., can be mentioned as examples of applications (SZTIPANOVITS [2]). Besides, it is well known that control and compensating strategies generally need a careful on-line estimation of electrical quantities, which cannot be directly measured (VAN DEN BOS-EYKHOFF [3], JAWORSKA-SZULC [4], BACCIGALUPI-LANDI-SCARANO [5], BUCCI-CECATI-ZHANG [6]).

The performance of the instrument is strictly related to that of the adopted algorithm. In the past the Kalman Filter theory has been successfully applied for signal processing and model identification in many fields (SORENSEN-

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ET ALT.[7]); the advantages of its application to the electrical drive field have been recently analysed by some authors both for the estimation of non measurable quantities (AMMAR-PIERTRZAK-DE FORNEL-MIRZAIANI [8], ATKINSON-ACARNLEY-FINCH [9]) and for the signal filtering in presence of harmonic distortion and noise (ANDRIA-SALVATORE [10]).

The applicability of such techniques to real-time signal processing problems is generally limited by the complexity of mathematical operations necessary in computing the estimation algorithm. The real-time requirements are often the most critical constraints, which impose to acquire the input signals and to execute the measurement algorithm with sufficient speed to ensure an appropriate response within the required time.

To cope with these constraints, the solution of increasing the computational power by replicating the processors and by running some tasks concurrently on different processors appears to be very promising. However the use of parallel systems allows one to gain advantages when the parallelism is obtained by decomposing the algorithm into a number of blocks which can be executed concurrently. The sequential structure of standard algorithms generally does not allow for an efficient implementation; a reformulation of them, implementing concurrent programming concepts, is then required.

In the paper a parallel architecture for the Kalman Filter, based on factorization techniques, which allows for fast computation and permits the numerical instability problems to be avoided, has been reported. The algorithm has been implemented on a multiprocessor measurement apparatus, based on the INMOS T414/T800 transputers, and has been applied for carrying out measurements on a synchronous motor. The obtained results are reported, showing the measurement algorithm performances in terms of speed and accuracy of the estimation.

THE PARALLEL EXTENDED KALMAN FILTER

The Kalman Filter has been shown to be the optimal linear estimator, in the least square sense, for dynamic linear systems (KALMAN [11]).

Consider a discrete-time linear stochastic system where a n-dimensional state process $\{X_k, k \geq 0\}$ and a p-dimensional measurement process $\{Y_k, k \geq 0\}$ are given by the stochastic difference equations (KALMAN [11]):

$$X_{k+1} = F_k X_k + G_k W_k \quad (1)$$

$$Y_k = H_k X_k + V_k \quad (2)$$

where: $\{W_k, k \geq 0\}$ and $\{V_k, k \geq 0\}$ are assumed to be independent white noise processes with zero mean and time-varying covariance matrices Q_k and R_k respectively;

F_k, G_k and H_k are known time-varying matrices of suitable dimensions.

Kalman Filter theory provides recursive formulae for the best linear last-square estimate $\hat{X}_{k|k}$ of the state variable X_k , given observations $[Y_0, Y_1, \dots, Y_k]$, and for error covariance matrices. As it is well known, the Kalman Filter equations are the following:

TIME UPDATE EQUATION:

$$\hat{X}_{k+1|k} = F_k \hat{X}_{k|k} \quad (3)$$

$$P_{k+1|k} = F_k P_{k|k} F_k^T + G_k Q_k G_k^T \quad (4)$$

MEASUREMENT UPDATE EQUATIONS:

$$\hat{X}_{k+1|k+1} = \hat{X}_{k+1|k} + K_{k+1} e_{k+1} \quad (5)$$

$$P_{k+1|k+1} = P_{k+1|k} - P_{k+1|k} H_k^T R_{e,k}^{-1} H_k P_{k+1|k} \quad (6)$$

$$e_{k+1} = Y_{k+1} - H_{k+1} \hat{X}_{k+1|k} \quad (7)$$

$$R_{e,k} = H_k P_{k+1|k} H_k^T + R_k \quad (8)$$

$$K_{k+1} = P_{k+1|k} H_k^T R_{e,k}^{-1} \quad (9)$$

where: $P_{k|k} = E[X_k - \hat{X}_{k|k}][X_k - \hat{X}_{k|k}]^T$ is the filter error covariance matrix;

$P_{k+1|k} = E[X_{k+1} - \hat{X}_{k+1|k}][X_{k+1} - \hat{X}_{k+1|k}]^T$ is the prediction error covariance matrix;

$\{e_k, k > 0\}$ and $\{K_k, k > 0\}$ are the innovation and Kalman gain sequences respectively.

Nonlinear models can be solved by using the Extended Kalman Filter (EKF), which gives an approximate filter. It is based on the linearization, at each time instant, of the model equations around the best estimation of the system state currently available. The EKF equations system is identical to the Kalman Filter one, with F and H Jacobian matrices (state dependent) calculated as follows:

$$(F_k)_{i,j} = \frac{\partial f_i}{\partial x_j}(\hat{x}_{k+1|k}) \quad i,j = 1, \dots, n \quad (10)$$

$$(H_k)_{i,j} = \frac{\partial h_i}{\partial x_j}(\hat{x}_{k+1|k}) \quad i,j = 1, \dots, n \quad (11)$$

where n is the number of state variables.

In order to make the algorithm suitable to be implemented on a multiprocessor system, it is necessary to introduce a different formulation, which also allows numerical instability problems of the filter to be avoided.

Algorithms based on matrices block partitioning are particularly suited to parallel implementation and, in particular, methods involving square root matrix factorization have superior numerical properties (BIERMAN [12]). By adopting this solution, it is possible to reformulate the EKF factors update as a matrix triangularization problem (JOVER-KAILATH [13]).

In our case, the use of this technique appears particularly advantageous in calculating the equations (4) and (6), allowing the algorithm structure to be modified in order to implement it on a multiprocessor computing system. The time update equation (4) may be computed by using a suitable orthogonal transformation to triangularize the augmented matrix $[F_k P_{k|k}^{1/2} \quad G_k Q_k^{1/2}]$.

To avoid the computation of square root of matrices and minimize the execution time, we have used a slight modification (U replaced by L matrix) of the Modified Weighted Gram-Schmidt orthogonalization method (MWG-S) (THORNTON-BIERMAN [14]). This procedure consists in the orthogonalization of n linearly independent vectors w_1, \dots, w_n with respect to the weighted norm $\|w_j\|^2 = \sum_{k=1}^N D_k w_j(k)^2$ where N is vectors dimension and $j=1, \dots, n$.

$$\text{If we define: } \begin{bmatrix} w_1^T \\ \vdots \\ w_n^T \end{bmatrix} = [FL_+ \quad G] \quad (12)$$

$$D = \text{diag}(D_+ \quad Q) = \text{diag}(D_1 \dots D_N) \quad (13)$$

by using MWG-S algorithm and starting from the L-D factors of $P_{k|k}$, we can directly obtain the L-D factors of $P_{k+1|k}$, which in turn are used to calculate the measurement update equations.

In particular, to calculate the measurement update equation (6) the Modified Givens transformation introduced by GENTLEMAN [15] appears to be a good solution in terms of numerical stability and execution speed. The method consists in defining elementary orthogonal transformations Θ_D which allow any given vector $[p_1 \quad p_2]$ to be rotated to lie along $[1 \quad 0]$, keeping equality of weighted norms, that is:

$$[p_1 \quad p_2] \begin{bmatrix} d_{p1} & 0 \\ 0 & d_{p2} \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = [1 \quad 0] \begin{bmatrix} d_{q1} & 0 \\ 0 & d_{q2} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad (14)$$

In most problems d_{p1} and d_{p2} are known but d_{q1} and d_{q2} have to be determined after the matrix Θ_D has been defined. If we choose:

$$\Theta_D = \begin{bmatrix} \frac{p_1 d_{p1}}{d_{q1}} & \frac{p_2 d_{p2}}{d_{q1}} \\ -p_2 & p_1 \end{bmatrix}^T \quad (15)$$

after some algebraic manipulations, we obtain:

$$d_{q1} = p_1^2 d_{p1} + p_2^2 d_{p2} \tag{16}$$

$$d_{q2} = \frac{d_{p1} d_{p2}}{d_{q1}} \tag{17}$$

The above transformation Θ_D , applied to an arbitrary vector $[p'_1 \ p'_2]$, allows one to obtain a vector $[q'_1 \ q'_2]$ where:

$$q'_2 = -p_2 p'_1 + p_1 p'_2 \tag{18}$$

$$q'_1 = \frac{d_{p1}}{d_{q1}} p_1 p'_1 + \frac{d_{p2}}{d_{q1}} p_2 p'_2 \tag{19}$$

By systematically applying a sequence of such elementary transformations it is possible to triangularize any given matrix. In our case the use of the above method allows the factors L-D (unit lower triangular and diagonal) of the matrix $P_{k+1|k+1}$ to be obtained from the ones of the matrix $P_{k+1|k}$, according to the following transformation:

$$\begin{bmatrix} L_R & HL \\ 0 & L \end{bmatrix} \Theta = \begin{bmatrix} L_{Re} & 0 \\ KL_{Re} & L_+ \end{bmatrix} \tag{20}$$

where:

Θ is the product of the elementary matrices Θ_D ;

L_+ , L , L_R and L_{Re} are lower triangular unit diagonal matrices such that

$P_{k+1|k+1} = L_+ D_+ L_+^T$, $P_{k+1|k} = L D L^T$, $R_{kk} = L_R D_R L_R^T$ and $R_{e,k} = L_{Re} D_{Re} L_{Re}^T$.

The method above, using Gentleman's square root free Givens rotation, may be seen to be equivalent to Bierman's U-D method for the case of scalar ents (BIERMAN [16]).

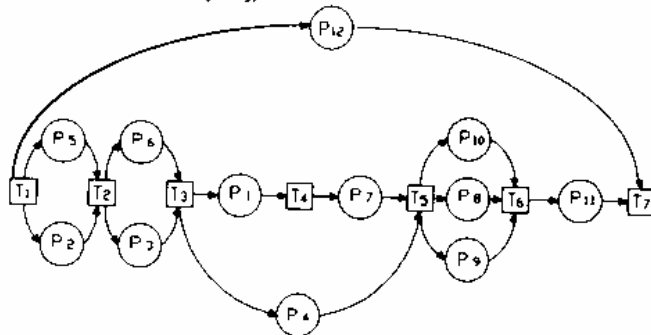


Fig.1 - Synchronization graph of the PEKF

The Parallel Extended Kalman Filter (PEKF) has been structured defining some elementary processes, (P_1, \dots, P_{12}) , whose priority levels have been analyzed in order to synchronize their activities (BUCCI-GERMANO-TOFONI [17]). The synchronization graph, reported in Fig.1, has been used to represent processes and their time constraints; the circles, labelled as P_i , represent processes in execution and boxes, labelled as T_i , represent time constraints. Each process, starting at time T_i , must be executed before T_{i+1} ; concurrent processes generally require different execution times but they are synchronized by the final time constraint T_{i+1} . Processes which start from a synchronization point T_i generally use the results of processes which terminate before T_i .

THE MEASUREMENT APPARATUS

The proposed algorithm is suitable for implementation on a multiprocessor system. For the aim of investigating its performance, an actual measurement apparatus, composed by a signal processing unit and two acquisition sections (BUCCI-GERMANO-LANDI [18]), has been adopted.

The signal processing unit, a transputer network, allows measurement results to be obtained in real-time; it adopts a personal computer PC-AT both as host computer and user interface. The transputer network is composed, in its full extension, by an array of eight IMS T800 transputers. The network topology can be changed dynamically depending on the process requirements.

In order to acquire signals with high and low dynamic two input sections have been implemented: the first one is connected via link adaptors and the second one makes use of the transputer external memory bus (BUCCI-GERMANO-LANDI [18], [19]).

The measurable quantities are caught from the object under test and then converted in numerical form by means of ADC sections. The samples, coming from the acquisition and conversion section, are elaborated on-line by the processing unit. The output quantities are then estimated by solving the equations relative to the model of the system under measurement.

APPLICATION

The knowledge of the instantaneous position of the rotor in a permanent magnet synchronous motor (PMSM) is needed to control the stator phase currents and the torque. It also allows a position or a speed controller to be implemented. An estimation of rotor position, starting from measurements of stator voltages and currents, allows the electro-mechanical sensor (encoder or resolver) to be removed, increasing the reliability and reducing the maintenance costs.

For better clarity in Fig.2 the block diagram of a PMSM drive is reported.

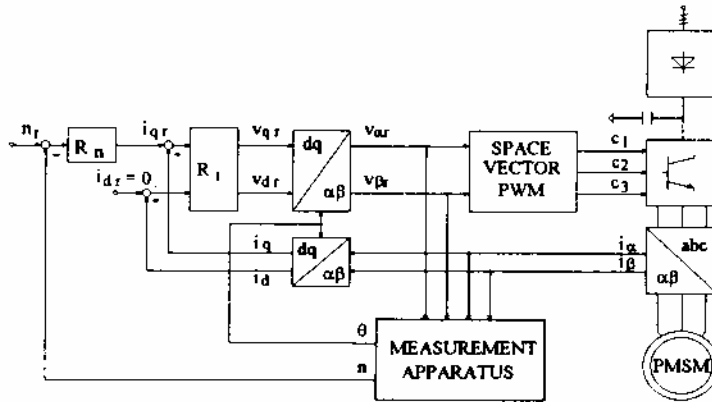


Fig.2 - Block diagram of a magnet permanent synchronous motor drive

The motor is fed by a PWM inverter whose switch command signals (c_1 - c_3) are generated by the controller blocks, adopting a space vector modulation strategy. The estimated rotor speed (n) and position (θ) are used as feedback signals for the controller. The speed controller generates the q-axis current reference i_{qr} (the torque reference), while the d-axis current reference i_{dr} is set to zero. A d-q transformation is applied in order to convert the measured values i_α and i_β into the d-q components i_d and i_q . The motor model is derived by taking the stator-fixed reference frame (α, β), assuming the currents i_α and i_β , the rotor speed n and the rotor position θ as state variables. In this case, denoting by:

- $T_m = J\omega_b / pM_b$ the mechanical time constant;
- J the equivalent inertia;
- ω_b the angular frequency;
- p the pole pairs;
- M_b the reference value torque;
- r_s the stator resistance;
- x_s the stator reactance;
- m_r the load torque;
- v_a, v_b the stator voltage a-b components.

the state space model assumes the following form:

$$X_k = [i_\alpha \ i_\beta \ n \ \theta]^T \tag{21}$$

$$f(X_k) = \begin{bmatrix} \left(1 - \frac{T r_s \omega_b}{x_s}\right) i_\alpha(k) + \frac{T \omega_b}{x_s} n(k) \sin\theta(k) + \frac{T \omega_b}{x_s} v_\alpha(k) \\ \left(1 - \frac{T r_s \omega_b}{x_s}\right) i_\beta(k) - \frac{T \omega_b}{x_s} n(k) \cos\theta(k) + \frac{T \omega_b}{x_s} v_\beta(k) \\ -\frac{T}{T_m} i_\alpha(k) \sin\theta(k) + \frac{T}{T_m} i_\beta(k) \cos\theta(k) + n(k) - \frac{m_r T}{T_m} \\ T \omega_b n(k) + \theta(k) \end{bmatrix} \tag{22}$$

The observation equation is:

$$Y_k = [i_\alpha(k) \quad i_\beta(k)]^T + V_k \quad (23)$$

where the random vector V_k models measurement uncertainties.

The high speed of execution and the parametric variation sensitivity are critical aspects for the EKF application to the position and speed estimation. The performance of the proposed estimation algorithm has been analysed not only in the standard working conditions, but during critical states of the system too, as when the motor starts up or runs at low speed or when the load changes suddenly.

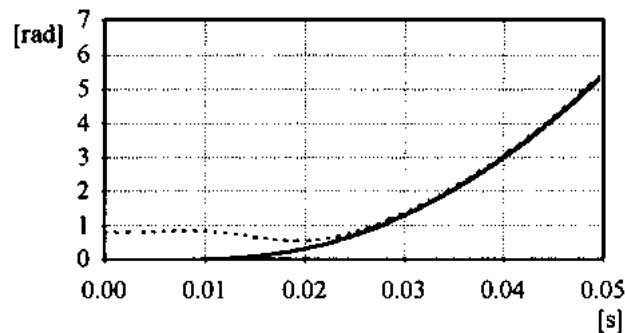


Fig.3 - Rotor position estimation at start-up

As an example, in Fig.3 the rotor position estimation (dashed line), directly compared to the actual value, has been reported during the start-up state of the motor. In order to analyze the effect of the uncertainty relative to the initial rotor position on the control system performance, an error (up to $\pi/4$ rad) has been supposed on the initial value of θ . As can be seen, the filter is still able to track the system state and after a brief transient (about 0.02 s) the estimation converges to the true value. In this case in the steady state the estimation error is lower than 0.1 rad.

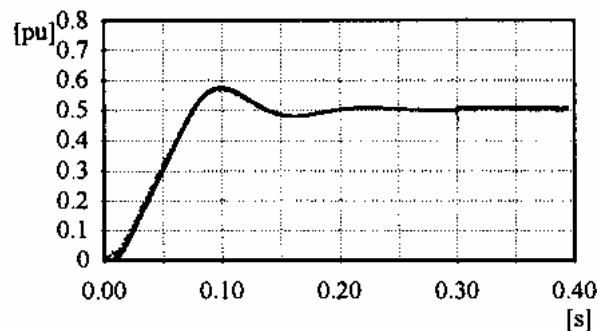


Fig.4 - Rotor speed estimation during a transient

The Fig.4 reports the estimated and actual rotor speed during a load variation ($m_r=0.7$ p.u. and $m_r=0.1$) and at low speed. As can be seen, the filter is insensible to the parametric variations due to load and the estimated

quantities perfectly track the dynamic of the system, following its changes quickly. In this case the estimation error is lower than 0.007 at $m_r=0.1$ p.u. and equal to 0.002 at $m_r=0.7$ p.u. However in this case a lower limit on the speed has been observed: if the reference speed falls to lower values than 0.5 p.u., the relative estimation error becomes unacceptable and, in this case, the filter cannot be applied in the control of the machine. When a speed lower than 0.5 p.u. is required, it is the authors' opinion that a better tuning of the filter allows the estimation error to be significantly reduced, so making it possible to extend the lower bound of applicability range.

CONCLUSIONS

In this paper the authors have presented a recursive algorithm suitable for on-line applications, such as the indirect measurement of quantities which are either non-accessible or ill-favoured in their accessibility. The measuring algorithm, a concurrent implementation of the Extend Kalman Filter, is based on factorization methods, which allow the algorithm structure to be modified in order to adopt a multiprocessor computing system. The PEKF performance has been tested in the estimation of speed and rotor position of a synchronous motor. The obtained results show a high accuracy and a good speed of convergence of estimation and confirm the effectiveness of the proposed algorithm.

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