Application of mathematical software in solving the problems of electricity

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Abstract

At the present time great emphasis is put on making accessible new knowledge to students through information and communication technologies in effort to facilitate and introduce objects, phenomena and reality. Information and communication technologies complement and develop traditional methods such as direct observation, manipulation with objects, experiment. It is justified mainly at teaching natural sciences. The possibilities of solving physical problem with the use of software tools are presented in the paper.

Keywords: information and communication technologies, electrical circuit, Kirchhoffov´s lows, MS Excel, Matlab.

1 Introduction

Information and communication technologies currently present a set of modern means that are used for preparation, processing and distribution of data and information, but also process control with the aim of achieving more effective results and searching for optimal problem solutions at various fields and areas of human activities [1], [2]. Information and communication technologies significantly influence even university education. Information and communication technologies provide incomparably bigger information basis as it was several years ago. This gradually changes the style of teaching and makes teachers implement new technologies not only in direct pedagogical activity, but

also at its preparation. Implementation of information and communication technologies into education enables new forms of university studies. We can stimulate the interest of students in studies of natural science subjects as mathematics, physics, chemistry, create conditions for educational individualization and improve conditions to raise the quality of education by a suitable combination of traditional and modern teaching methods [3].

In teaching physics there exist possibilities for effective and suitable integration of information and communication technologies into schooling system. One of them is physical problem solution with the support of computer. This paper concretely presents the solution of physical problem from the part Physics – Power and Magnetism by the use of mathematical software MS Excel a Matlab.

2   Physical analysis of the problem

Problem: Figure out the currents in individual circuit branches in Fig. 1, if source voltage and resistance are: U_{01}=10 V, U_{02}=20 V, U_{03}=15 V, U_{04}=10 V, R_1=10 Ω, R_2=15 Ω, R_3=30 Ω, R_4=20 Ω, R_5=10 Ω, R_6=15 Ω, R_7=10 Ω.

Solution: Kirchhoff´s rules are used to figure out the currents in the circuit [4]. The first of Kirchhoff´s rules describes the law of electric charge preservation: *The sum of all the currents flowing into the junction point must equal the sum of all the currents leaving the point*, i.e. $\sum_{k=1}^{n} I_k = 0$. The second of Kirchhoff´s rules forms the law of electric energy preservation for electric circuits: *Algebraic sum of electromotive voltages in any closed part of the electrical network is equal to the sum of ohmic voltages at individual branches of this closed part*, i.e. $\sum_{i=1}^{m} U_{ei} = \sum_{k=1}^{n} R_k I_k$.
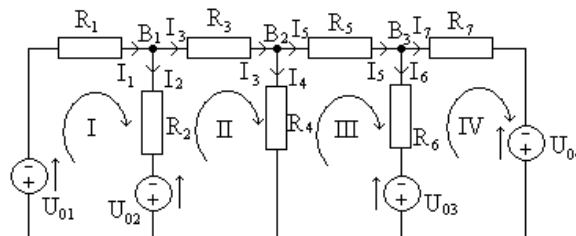

Fig. 1. Circuit

Based on the first and the second of Kirchhoff´s rules (Fig. 1) for the currents and electromotive voltages, it is valid

$$B_1 : \qquad I_1 - I_2 - I_3 = 0$$
$$B_2 : \qquad I_3 - I_4 - I_5 = 0$$
$$B_3 : \qquad I_5 - I_6 - I_7 = 0$$
$$I : \qquad R_1 I_1 + R_2 I_2 = U_{01} - U_{02}$$
$$II : \qquad -R_2 I_2 + R_3 I_3 + R_4 I_4 = U_{02}$$
$$III : \qquad -R_4 I_4 + R_5 I_5 + R_6 I_6 = -U_{03}$$
$$IV : \qquad -R_6 I_6 + R_7 I_7 = U_{03} - U_{04}$$

The set of 7 equations on 7 unknown quantities I_1, I_2, ..., I_7 was obtained. Numeric values are inducted for the known quantities and we have

$$I_1 - I_2 - I_3 = 0$$
$$I_3 - I_4 - I_5 = 0$$
$$I_5 - I_6 - I_7 = 0$$
$$10 I_1 + 15 I_2 = -10$$
$$-15 I_2 + 30 I_3 + 20 I_4 = 20$$
$$-20 I_4 + 10 I_5 + 15 I_6 = -15$$
$$-15 I_6 + 10 I_7 = 5$$

3 Analytic solution of the problem

Based on analysis of the problem and use of electrical laws the system of 7 equations in 7 variables was obtained, where analytic solution is not simple. In general it is possible to solve the system of n equations in n variables in three ways:
1. solving the system of linear equations by means of Cramer´s Rule,
2. solving the system of linear equations by means of inversion matrix,
3. solving the system of linear equations by Gauss elimination method.

Gauss elimination method appears to be a suitable method of solving the system of n equations in n variables, if $n > 3$ [5]. By means of equivalent line adjustment the matrix of the system of equations, which is augmented by the second column (so called augmented matrix of the system) to a triangle shape, is modified. We write to such an augmented matrix an appropriate system, which is equivalent with the original system, i.e. it has the same family of solutions. Frobenius norm and its consequences can be used to solve such a modified system.

The system of heterogeneous equations can be solved only if the rank of a matrix is equal to the rank of an augmented matrix of the system.

Consequence 1: If $h(A) = h(A') = n$ (n is the number of unknowns), then the system has only one solution.

Consequence 2: If $h(A) = h(A') < n$ (n is the number of unknowns), then the system has infinite number of solutions and $n - h$ unknowns can be arbitrarily selected.

Consequence 3: If $h(A) \neq h(A')$, then the system has no solution.

We get the values of unknowns by gradual substitution into previous equations.

The system of equations is written into the form of an augmented matrix and we get by means of equivalent line adjustment

$$
\left(
\begin{array}{ccccccc|c}
1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & -1 & 0 \\
10 & 15 & 0 & 0 & 0 & 0 & 0 & -10 \\
0 & -15 & 30 & 20 & 0 & 0 & 0 & 20 \\
0 & 0 & 0 & -20 & 10 & 15 & 0 & -15 \\
0 & 0 & 0 & 0 & 0 & -15 & 10 & 5
\end{array}
\right) \approx
$$

$$
\approx
\left(
\begin{array}{ccccccc|c}
1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & -15 & 30 & 20 & 0 & 0 & 0 & 20 \\
0 & 0 & 1 & -1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & -20 & 10 & 15 & 0 & -15 \\
0 & 0 & 0 & 0 & 1 & -1 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & -15 & 10 & 5 \\
10 & 15 & 0 & 0 & 0 & 0 & 0 & -10
\end{array}
\right) \begin{array}{c} \\ \\ \\ \\ \\ \\ -10R_1 \end{array} \approx \ldots
$$

$$
\approx
\left(
\begin{array}{ccccccc|c}
1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & -15 & 30 & 20 & 0 & 0 & 0 & 20 \\
0 & 0 & 1 & -1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & -20 & 10 & 15 & 0 & -15 \\
0 & 0 & 0 & 0 & 1 & -1 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & +1590 & -1060 & -530 \\
0 & 0 & 0 & 0 & 0 & -1590 & -960 & 420
\end{array}
\right) \begin{array}{c} \\ \\ \\ \\ \\ \\ +R_6 \end{array} \approx
$$

$$
\approx
\left(
\begin{array}{ccccccc|c}
1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & -15 & 30 & 20 & 0 & 0 & 0 & 20 \\
0 & 0 & 1 & -1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & -20 & 10 & 15 & 0 & -15 \\
0 & 0 & 0 & 0 & 1 & -1 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & +1590 & -1060 & -530 \\
0 & 0 & 0 & 0 & 0 & 0 & -2020 & -110
\end{array}
\right)
$$

The rank of a matrix is equal to the rank of an augmented matrix, i.e. $h(A) = h(A') = n$ (n is the number of unknowns), then the system has one solution that is determined from an appropriate system:

$$-2020I_7 = -110 \quad \Rightarrow \quad I_7 = \frac{11}{202} = 0{,}05446$$

$$1590I_6 - 1060I_7 = -530 \quad \Rightarrow \quad I_6 = \frac{-530 + 1060I_7}{1590}$$

$$I_6 = \frac{-530 + 1060\dfrac{11}{202}}{1590} = -\frac{30}{101} = -0{,}29703$$

$$I_5 - I_6 - I_7 = 0 \quad \Rightarrow \quad I_5 = I_6 + I_7$$

$$I_5 = -\frac{30}{101} + \frac{11}{202} = -\frac{49}{202} = -0{,}24257$$

$$-20I_4 + 10I_5 + 15I_6 = -15 \quad \Rightarrow \quad I_4 = \frac{-15 - 10I_5 - 15I_6}{-20}$$

$$I_4 = \frac{-15 - 10\left(-\dfrac{49}{202}\right) - 15\left(-\dfrac{30}{101}\right)}{-20} = \frac{41}{101} = 0{,}40594$$

$$I_3 - I_4 - I_5 = 0 \quad \Rightarrow \quad I_3 = I_4 + I_5$$

$$I_3 = \frac{41}{101} + \left(-\frac{49}{202}\right) = \frac{33}{202} = 0{,}16337$$

$$-15I_2 + 30I_3 + 20I_4 = 20 \quad \Rightarrow \quad I_2 = \frac{20 - 30I_3 - 20I_4}{-15}$$

$$I_2 = \frac{20 - 30\left(\dfrac{33}{202}\right) - 20\left(\dfrac{41}{101}\right)}{-15} = -\frac{47}{101} = -0{,}46534$$

$$I_1 - I_2 - I_3 = 0 \quad \Rightarrow \quad I_1 = I_2 + I_3$$

$$I_1 = \left(-\frac{47}{101}\right) + \frac{33}{202} = -\frac{61}{202} = -0{,}30198$$

We get the following current values in the circuit:

$$I_1 = -0{,}30198 \ \ A$$
$$I_2 = -0{,}46534 \ \ A$$
$$I_3 = \ \ 0{,}16337 \ \ A$$
$$I_4 = \ \ 0{,}40594 \ \ A$$
$$I_5 = -0{,}24257 \ \ A$$
$$I_6 = -0{,}29703 \ \ A$$
$$I_7 = \ \ 0{,}05446 \ \ A$$

It results from the negative current values that currents in the circuit are in the opposite direction as we selected.

Analytic solution of the system of 7 equations in 7 variables by Gauss elimination method requires not only knowledge of linear algebra (matrix

algebra), but also good mathematical skills and time. Numerical solution of the system of equations by means of various mathematical software tools such as MS Excel, Mathematica or MATLAB is much more easier.

4 Using software tools at the problem solution

4.1 The problem solution by means of MS Excel

In current computing technique it is possible to use standard programs for the matrix inversion up to relatively big number of equations (hundreds of variables). One of the possibilities is the solution in MS Excel [6]. We write the system of equations in matrix form

$$\begin{pmatrix} a_{11} & a_{21} & \cdots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ & & \vdots & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix} = \mathbf{A}.x = \mathbf{b}$$

where $\mathbf{A}$ is the matrix of coefficients, x is the vector of unknowns and $\mathbf{b}$ is the vector of the second members. We get by multiplying $\mathbf{A^{-1}}$ from the left $\mathbf{A^{-1}} \cdot \mathbf{A} \cdot x = x = \mathbf{A^{-1}} \cdot \mathbf{b}$. If we calculate the inversion matrix, x_k unknowns can be obtained by multiplication of the matrix and vector, which is procedure that is optimized very well and is the part of standard libraries of subprograms. To calculate the inversion matrix MINVERSE functions from the offer of MS Excel More Functions is used. To calculate the roots of the system of equations ($\mathbf{A^{-1}} \mathbf{b}$) MMULT function is used. The result of the solution can be found in Fig. 2.
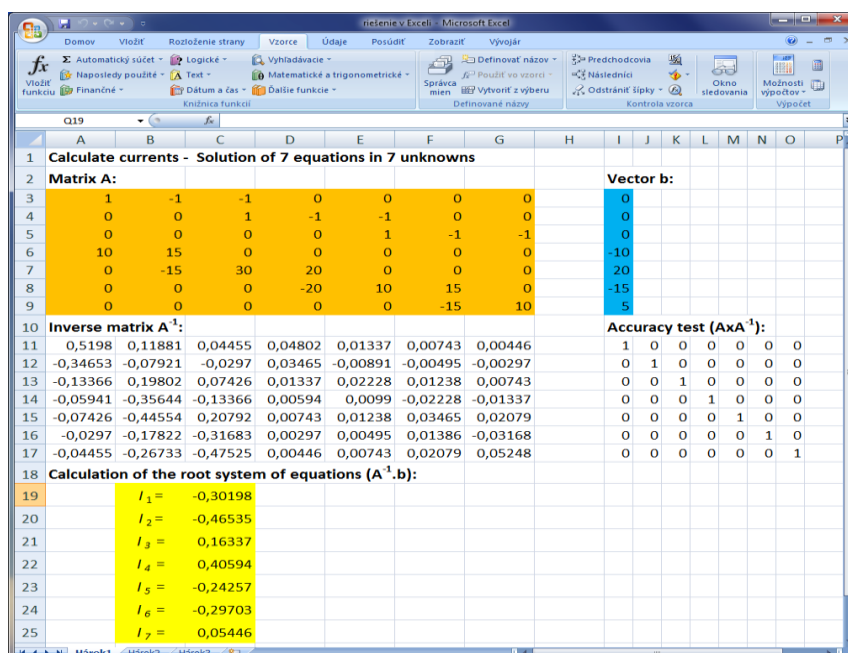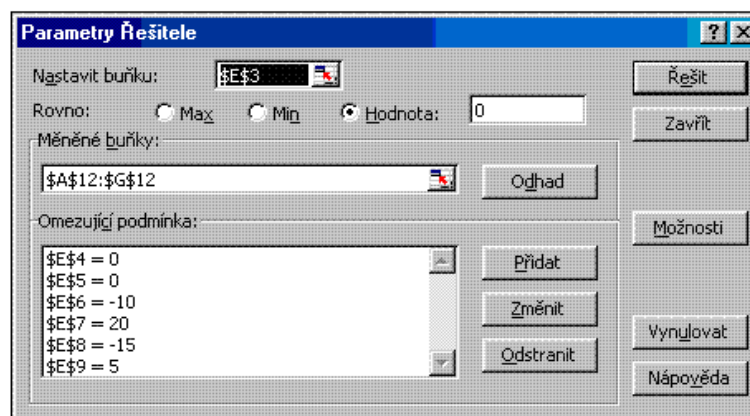
Fig. 2. Numerical solution of the set of equations in MS Excel

Another possibility to solve the system of equations is to use the MS Excel Solutionist [7], [8]. From the task and solution of the problem in the Solutionist we get (Fig. 3)

Fig. 3. Numerical solution of the system of problems in the MS Excel Solutionist

From both solutions in MS Excel we get the following current values in the circuit

$$I_1 = -0{,}30198 \text{ A}, \quad I_2 = -0{,}46535 \text{ A}, \quad I_3 = 0{,}16337 \text{ A}, \quad I_4 = 0{,}40594 \text{ A},$$
$$I_5 = -0{,}24257 \text{ A}, \quad I_6 = -0{,}29703 \text{ A}, \quad I_7 = 0{,}05446 \text{ A}$$

It results from the negative values of the current that currents have reverse directions as it was selected.

4.2 Problem solution by means of Matlab

MATLAB presents highly functional language for technical calculations. It integrates the calculations, visualization and programming into simply usable environment where the problems and solutions are expressed in natural form [9], [10]. The field is the basic data type of this interactive system. This property together with number of built-in functions enables relatively easy solution of many technical problems, mainly those that lead to the vector or matrix formulations, in much shorter time as solution in classic program languages. To calculate the currents I_1, I_2, ..., I_7 the method of node voltage is used. This method comes from the fact that $(u-1)$ equations is written by means of Kirchhoff´s first law applied to suitably selected nodes. In these equations the equations of Kirchhoff´s second law written for appropriate loops are implicitly included. That is why voltages on tree´s branches are selected as unknowns at the method of node voltage. To determine node voltages it is necessary to solve $(u-1)$ equations. After calculation of node voltages the currents of the circuit are determined.

We write for B_1, B_2 and B_3 nodes according to Kirchhoff´s first law

$$B_1 : \qquad I_1 - I_2 - I_3 = 0$$
$$B_2 : \qquad I_3 - I_4 - I_5 = 0$$
$$B_3 : \qquad I_5 - I_6 - I_7 = 0$$

It is possible to express above mentioned currents by means of known node voltages with regard to the selected reference node

$$B_1 : \qquad \frac{U_{01} - U_{B1}}{R_1} - \frac{U_{B1} - U_{02}}{R_2} - \frac{U_{B1} - U_{B2}}{R_3} = 0$$

$$B_2 : \qquad \frac{U_{B1} - U_{B2}}{R_3} - \frac{U_{B2}}{R_4} - \frac{U_{B2} - U_{B3}}{R_5} = 0$$

$$B_3 : \qquad \frac{U_{B2} - U_{B3}}{R_5} - \frac{U_{B3} - U_{03}}{R_6} - \frac{U_{B3} - U_{04}}{R_7} = 0$$

We write the equations in the matrix form

$$
\begin{bmatrix}
1/R_1 + 1/R_2 + 1/R_3 & -1/R_3 & 0 \\
1/R_3 & -(1/R_3 + 1/R_4 + 1/R_5) & 1/R_5 \\
0 & -1/R_5 & 1/R_5 + 1/R_6 + 1/R_7
\end{bmatrix}
\begin{bmatrix}
U_{B1} \\
U_{B2} \\
U_{B3}
\end{bmatrix} =
$$

$$
=
\begin{bmatrix}
U_{01}/R_1 + U_{02}/R_2 \\
0 \\
U_{03}/R_6 + U_{04}/R_7
\end{bmatrix}
$$

To solve such written equations the matrix solution in Matlab is used. We form the m-file *prudy.m* (Fig. 4)

```
U01=10; U02=20; U03=15; U04=10;
R1=10; R2=15; R3=30; R4=20; R5=10; R6=15; R7=10;

A=[(1/R1+1/R2+1/R3), -1/R3            , 0        ;
     1/R3             , -(1/R3+1/R4+1/R5) , 1/R5 ;
     0                ,-1/R5             , (1/R5+1/R6+1/R7)];

I=[(U01/R1+U02/R2);0;U03/R6+U04/R7];

U=A\I;

I1=(U01-U(1))/R1;
I2=(U(1)-U02)/R2;
I3=(U(1)-U(2))/R3;
I4=U(2)/R4;
I5=(U(2)-U(3))/R5;
I6=(U(3)-U03)/R6;
I7=(U(3)-U04)/R7;

fprintf('\n');
fprintf('I1 = %8.5f A\n', I1);
fprintf('I2 = %8.5f A\n', I2);
fprintf('I3 = %8.5f A\n', I3);
fprintf('I4 = %8.5f A\n', I4);
fprintf('I5 = %8.5f A\n', I5);
fprintf('I6 = %8.5f A\n', I6);
fprintf('I7 = %8.5f A\n', I7);
```

Fig. 4. M-file *prudy.m* for calculation of matrices and currents

After solving the system of equations we get the values of node voltages, which are converted to the currents in branches of the circuit. The result of solution is launching the script of *prudy.m* and print of results.

```
>> prudy
I1 = -0.30198 A
I2 = -0.46535 A
I3 =  0.16337 A
I4 =  0.40594 A
I5 = -0.24257 A
I6 = -0.29703 A
I7 =  0.05446 A
```

Another possibility of the problem solution in Matlab is use of Symbolic Math Toolbox, which provides functions for solution and graphic description of mathematical functions. Tool panel provides libraries of functions in common mathematical areas such as mathematical analysis, linear algebra, algebraic and common differential equations and so on. Symbolic Math Toolbox uses MuPAD language as a part of its calculus core. The language has a extensive set of functions, which are optimized to create and operate symbolic arithmetical

expressions. To solve the system of equations *linsolve* ([*eqs*], [*vars*]) function was used*,* where *eqs* is a list or a set of linear equations or arithmetical expressions, *vars* is a list or a set of unknowns to solve for: typically identifiers or indexed identifiers. The solution of the system can be found in Fig. 5, where $x = I_1$, $y = I_2$, $z = I_3$, $k = I_4$, $l = I_5$, $m = I_6$, $n = I_7$:
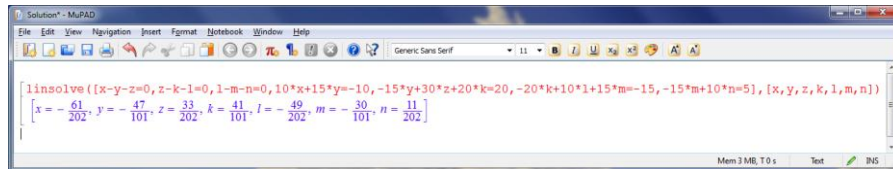

Fig. 5. Numeric solution of the system of equations in MuPad

The same values are obtained from the problem solution in Matlab as in the case of the problem solution in MS Excel.

5   Conclusions

It accrues from the solution results that solution of the system of equations of the physical problem in an analytic way as well as by using mathematical software tools leads to certain numeric values. Analytic solution of the system of *n* equations in *n* variables requires certain mathematical knowledge and skills to solve matrices. Use of modern software tools to solve the system of equations facilitates the problem solution. On the other side it requires certain computing skills. The physical problem being solved points out importance and necessity of using modern information and communication technology means and their utilization in educational process that makes "learning" for pupils and students more interesting and attractive.

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