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Sums of generalized convergent harmonic series with eight periodically repeated numerators

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Abstract

This contribution deals with the generalized convergent harmonic series with eight periodically repeated numerators and it is a follow-up to author's papers dealing with the generalized alternating harmonic series with two up to seven periodically repeated numerators. It is derived the only expression of the last numerator depending on preceding numerators for which this series converges. Then the formula for the sum of this series is analytically derived. This analytical result is numerically verified by using the CAS Maple 16.

Keywords: alternating harmonic series, geometric series, sum of the series, CAS Maple.

2000 AMS subject classifications: 40A05, 65B10.

1 Introduction and basic notions

Let us recall the basic terms and notions. The *harmonic series* is the sum of reciprocals of all natural numbers (except zero), so this is the series

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} + \cdots .$$

The divergence of this series can be proved e.g. by using the integral test or the comparison test of convergence. The series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \cdots$$

is known as the *alternating harmonic series*. This series converges by the alternating series test. In particular, the sum is equal to the natural logarithm of 2:

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \cdots = \ln 2 .$$

2 Sum of generalized alternating harmonic series with two up to seven periodically repeated numerators

This paper is a continuation of the author's contributions [1], [2], [3], [4], [5] and [6]. The paper [1] deals, among others, with the generalized alternating harmonic series with two periodically repeated numerators $(1, a)$, i.e with the series of the form

$$\sum_{n=1}^{\infty} \left(\frac{1}{2n-1} + \frac{a}{2n} \right) = \frac{1}{1} + \frac{a}{2} + \frac{1}{3} + \frac{a}{4} + \frac{1}{5} + \frac{a}{6} + \frac{1}{7} + \frac{a}{8} + \frac{1}{9} + \frac{a}{10} + \cdots ,$$

where $a \in \mathbb{R}$. In entire agreement with the well-known fact it was derived that the only one value of the coefficient a, for which this series converges, is $a = -1$ and that the sum of this series is $s = \ln 2$.

The paper [2] deals with the generalized alternating harmonic series with three periodically repeated numerators $(1, a, b)$, i.e. with the series

$$\sum_{n=1}^{\infty} \left(\frac{1}{3n-2} + \frac{a}{3n-1} + \frac{b}{3n} \right) = \frac{1}{1} + \frac{a}{2} + \frac{b}{3} + \frac{1}{4} + \frac{a}{5} + \frac{b}{6} + \frac{1}{7} + \frac{a}{8} + \frac{b}{9} + \cdots .$$

It was derived that the only value of the coefficient $b \in \mathbb{R}$, for which this series converges, is $b = -a - 1$, and that the sum of this series is given by the formula

$$s(a) = \frac{a+1}{2} \ln 3 - \frac{a-1}{6\sqrt{3}} \pi .$$

The contribution [3] deals with the generalized alternating harmonic series with four periodically repeated numerators $(1, a, b, c)$, i.e. with the series

$$\sum_{n=1}^{\infty} \left(\frac{1}{4n-3} + \frac{a}{4n-2} + \frac{b}{4n-1} + \frac{c}{4n} \right) = \frac{1}{1} + \frac{a}{2} + \frac{b}{3} + \frac{c}{4} + \frac{1}{5} + \frac{a}{6} + \frac{b}{7} + \frac{c}{8} + \cdots .$$

It was derived that the only value of the coefficient $c \in \mathbb{R}$, for which this series converges, is $c = -a - b - 1$ and it was also derived that the sum of this series is

$$s(a,b) = \frac{2a+3b+3}{4}\ln 2 - \frac{b-1}{8}\,\pi\,.$$

The paper [4] is about the generalized alternating harmonic series with five periodically repeated numerators $(1,a,b,c,d)$, i.e. with the series

$$\sum_{n=1}^{\infty}\left(\frac{1}{5n-4}+\frac{a}{5n-3}+\frac{b}{5n-2}+\frac{c}{5n-1}+\frac{d}{5n}\right) = \\ = \frac{1}{1}+\frac{a}{2}+\frac{b}{3}+\frac{c}{4}+\frac{d}{5}+\frac{1}{6}+\frac{a}{7}+\frac{b}{8}+\frac{c}{9}+\frac{d}{10}+\cdots\,.$$

It was derived that the only value of the coefficient $d \in \mathbb{R}$, for which this series converges, is $d = -a-b-c-1$. It was also derived that the sum of this series is

$$s(a,b,c) = \frac{1+a+b+c}{4}\ln 5 + \frac{\sqrt{5}(1-a-b+c)}{20}\ln\frac{3+\sqrt{5}}{2} + \\ + \frac{\sqrt{5}(1-c)+1+2a-2b-c}{\sqrt{10}\sqrt{5+\sqrt{5}}}\arctan\frac{\sqrt{2}\sqrt{5+\sqrt{5}}}{5-\sqrt{5}} + \\ + \frac{\sqrt{5}(1-c)-(1+2a-2b-c)}{\sqrt{10}\sqrt{5-\sqrt{5}}}\arctan\frac{\sqrt{2}\sqrt{5-\sqrt{5}}}{5+\sqrt{5}}\,.$$

The contribution [5] is about the generalized alternating harmonic series with six periodically repeated numerators $(1,a,b,c,d,e)$, i.e. with the series

$$\sum_{n=1}^{\infty}\left(\frac{1}{6n-5}+\frac{a}{6n-4}+\frac{b}{6n-3}+\frac{c}{6n-2}+\frac{d}{6n-1}+\frac{e}{6n}\right) = \\ = \frac{1}{1}+\frac{a}{2}+\frac{b}{3}+\frac{c}{4}+\frac{d}{5}+\frac{e}{6}+\cdots\,.$$

It was derived that the only value of the coefficient $e \in \mathbb{R}$, for which this series converges, is $e = -a-b-c-d-1$. It was also derived that the sum of this series is

$$s(a,b,c,d) = \frac{1+b+d}{3}\ln 2 + \frac{1+a+c+d}{4}\ln 3 + \frac{3+a-c-3d}{12\sqrt{3}}\,\pi\,.$$

The paper [6] deals with the generalized alternating harmonic series with seven periodically repeated numerators $(1,a,b,c,d,e,f)$, i.e. with the series

$$\sum_{n=1}^{\infty}\left(\frac{1}{7n-6}+\frac{a}{7n-5}+\frac{b}{7n-4}+\frac{c}{7n-3}+\frac{d}{7n-2}+\frac{e}{7n-1}+\frac{f}{7n}\right).$$

It was derived that the only value of the coefficient $f \in \mathbb{R}$, for which this series converges, is $f = -a-b-c-d-e-1$. It was also derived that the sum of this series is

$$s(a,b,c,d,e) = 0.4440431881a + 0.2555147273b + 0.1530792957c + \\ + 0.0861379681d + 0.0375971731e + 0.9695377967.$$

3 Sum of generalized alternating harmonic series with eight periodically repeated numerators

Now, we deal with the numerical series of the form

$$\sum_{n=1}^{\infty}\left(\frac{1}{8n-7}+\frac{a}{8n-6}+\frac{b}{8n-5}+\frac{c}{8n-4}+\frac{d}{8n-3}+\frac{e}{8n-2}+ \right. \\ \left. +\frac{f}{8n-1}+\frac{g}{8n}\right) = \frac{1}{1}+\frac{a}{2}+\frac{b}{3}+\frac{c}{4}+\frac{d}{5}+\frac{e}{6}+\frac{f}{7}+\frac{g}{8}+ \\ +\frac{1}{9}+\frac{a}{10}+\frac{b}{11}+\frac{c}{12}+\frac{d}{13}+\frac{e}{14}+\frac{f}{15}+\frac{g}{16}+\cdots, \quad (1)$$

where $a,b,c,d,e,f,g \in \mathbb{R}$. This series we shall call *generalized alternating harmonic series with eight periodically repeated numerators* $(1,a,b,c,d,e,f,g)$. We express the numerator g, for which the series (1) converges, as a function of the numerators a,b,c,d,e,f, and determine the sum of this series.

The power series corresponding to the series (1) has evidently the form

$$\sum_{n=1}^{\infty}\left(\frac{x^{8n-7}}{8n-7}+\frac{ax^{8n-6}}{8n-6}+\frac{bx^{8n-5}}{8n-5}+\frac{cx^{8n-4}}{8-4}+\frac{dx^{8n-3}}{8n-3}+\frac{ex^{8n-2}}{8n-2}+ \right. \\ \left. +\frac{fx^{8n-1}}{8n-1}+\frac{gx^{8n}}{8n}\right). \quad (2)$$

We denote its sum by $s(x)$. The series (2) is for $x \in (-1,1)$ absolutely convergent, so we can rearrange it and rewrite it in the form

$$s(x) = \sum_{n=1}^{\infty}\frac{x^{8n-7}}{8n-7}+a\sum_{n=1}^{\infty}\frac{x^{8n-6}}{8n-6}+b\sum_{n=1}^{\infty}\frac{x^{8n-5}}{8n-5}+c\sum_{n=1}^{\infty}\frac{x^{8n-4}}{8n-4}+ \\ +d\sum_{n=1}^{\infty}\frac{x^{8n-3}}{8n-3}+e\sum_{n=1}^{\infty}\frac{x^{8n-2}}{8n-2}+f\sum_{n=1}^{\infty}\frac{x^{8n-1}}{8n-1}+g\sum_{n=1}^{\infty}\frac{x^{8n}}{8n}. \quad (3)$$

If we differentiate the series (3) term-by-term, where $x \in (-1, 1)$, we get

$$\begin{aligned} s'(x) = {} & \sum_{n=1}^{\infty} x^{8n-8} + a \sum_{n=1}^{\infty} x^{8n-7} + b \sum_{n=1}^{\infty} x^{8n-6} + c \sum_{n=1}^{\infty} x^{8n-5} + \\ & + d \sum_{n=1}^{\infty} x^{8n-4} + e \sum_{n=1}^{\infty} x^{8n-3} + f \sum_{n=1}^{\infty} x^{8n-2} + g \sum_{n=1}^{\infty} x^{8n-1}. \end{aligned} \tag{4}$$

After reindexing and fine arrangement the series (4) we obtain

$$\begin{aligned} s'(x) = {} & \sum_{n=0}^{\infty} x^{8n} + ax \sum_{n=0}^{\infty} x^{8n} + bx^2 \sum_{n=0}^{\infty} x^{8n} + cx^3 \sum_{n=0}^{\infty} x^{8n} + \\ & + dx^4 \sum_{n=0}^{\infty} x^{8n} + ex^5 \sum_{n=0}^{\infty} x^{8n} + fx^6 \sum_{n=0}^{\infty} x^{8n} + gx^7 \sum_{n=0}^{\infty} x^{8n}, \end{aligned}$$

that is

$$s'(x) = \left(1 + ax + bx^2 + cx^3 + dx^4 + ex^5 + fx^6 + gx^7\right) \sum_{n=0}^{\infty} \left(x^8\right)^n. \tag{5}$$

When we summate the convergent geometric series on the right-hand side of (5) with the first term 1 and the ratio x^8, where $\left|x^8\right| < 1$, i.e. for $x \in (-1, 1)$, we get

$$s'(x) = \frac{1 + ax + bx^2 + cx^3 + dx^4 + ex^5 + fx^6 + gx^7}{1 - x^8}.$$

We convert this fraction using the CAS Maple 16 to partial fractions and get

$$s'(x) = \frac{Ax + B}{x^2 + 1} + \frac{Cx + D}{x^2 - \sqrt{2}x + 1} + \frac{Ex + F}{x^2 + \sqrt{2}x + 1} + \frac{G}{x + 1} + \frac{H}{x - 1},$$

where $x \in (-1, 1)$ and

$$\begin{aligned} A &= \frac{a - c + e - g}{4}, & B &= \frac{1 - b + d - f}{4}, \\ C &= \frac{-1 + b + \sqrt{2}c + d - f - \sqrt{2}g}{4\sqrt{2}}, & D &= \frac{\sqrt{2} + a - c - \sqrt{2}d - e + g}{4\sqrt{2}}, \\ E &= \frac{1 - b + \sqrt{2}c - d + f - \sqrt{2}g}{4\sqrt{2}}, & F &= \frac{\sqrt{2} - a + c - \sqrt{2}d + e - g}{4\sqrt{2}}, \\ G &= \frac{1 - a + b - c + d - e + f - g}{8}, & H &= \frac{-1 - a - b - c - d - e - f - g}{8}. \end{aligned} \tag{6}$$

The sum $s(x)$ of the series (2) we obtain by integration in the form

$$s(x) = \int\left(\frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2-\sqrt{2}x+1} + \frac{Ex+F}{x^2+\sqrt{2}x+1} + \frac{G}{x+1} + \frac{H}{x-1}\right)\mathrm{d}x =$$

$$= \frac{A}{2}\int\frac{2x}{x^2+1}\,\mathrm{d}x + B\int\frac{1}{x^2+1}\,\mathrm{d}x + \int\frac{C(2x-\sqrt{2})/2+D+C\sqrt{2}/2}{x^2-\sqrt{2}x+1}\,\mathrm{d}x +$$

$$+ \int\frac{E(2x+\sqrt{2})/2+F-E\sqrt{2}/2}{x^2+\sqrt{2}x+1}\,\mathrm{d}x + G\ln|x+1| + H\ln|x-1| + K,$$

so

$$s(x) = \frac{A}{2}\ln(x^2+1) + B\arctan x + \frac{C}{2}\ln(x^2-\sqrt{2}x+1) +$$

$$+ \frac{2D+C\sqrt{2}}{2}\int\frac{\mathrm{d}x}{(x-\sqrt{2}/2)^2+(\sqrt{2}/2)^2} + \frac{E}{2}\ln(x^2+\sqrt{2}x+1) +$$

$$+ \frac{2F-E\sqrt{2}}{2}\int\frac{\mathrm{d}x}{(x+\sqrt{2}/2)^2+(\sqrt{2}/2)^2} + G\ln|x+1| + H\ln|x-1| + K =$$

$$= \frac{A}{2}\ln(x^2+1) + B\arctan x + \frac{C}{2}\ln(x^2-\sqrt{2}x+1) + \frac{2D+C\sqrt{2}}{\sqrt{2}}\times$$

$$\times\arctan\frac{2x-\sqrt{2}}{\sqrt{2}} + \frac{E}{2}\ln(x^2+\sqrt{2}x+1) + \frac{2F-E\sqrt{2}}{\sqrt{2}}\arctan\frac{2x+\sqrt{2}}{\sqrt{2}} +$$

$$+ G\ln|x+1| + H\ln|x-1| + K,$$

where K is the constant of integration and where we used the formulas

$$\int\frac{f'(t)}{f(t)}\,\mathrm{d}t = \ln|f(t)| + K \qquad \text{and} \qquad \int\frac{\mathrm{d}t}{t^2+\alpha^2} = \frac{1}{\alpha}\arctan\frac{t}{\alpha} + K\,.$$

From the condition $s(0) = 0$, and because we have $\ln 1 = 0$, $\arctan 0 = 0$, $\arctan(\pm 1) = \pm\frac{\pi}{4}$, we obtain

$$\frac{2D+C\sqrt{2}}{\sqrt{2}}\cdot\frac{-\pi}{4} + \frac{2F-E\sqrt{2}}{\sqrt{2}}\cdot\frac{\pi}{4} + K = 0\,,$$

hence

$$K = \frac{\pi}{4\sqrt{2}}\big(2D + C\sqrt{2} - 2F + E\sqrt{2}\big)\,.$$

Because $2(D-F)+(C+E)\sqrt{2} = \dfrac{a-e}{\sqrt{2}}$, we get $K = \dfrac{\pi}{4\sqrt{2}}\cdot\dfrac{a-e}{\sqrt{2}} = \dfrac{(a-e)\pi}{8}$.

After application the relations (6), where

$$\sqrt{2}D + C = \frac{1+\sqrt{2}a+b-d-\sqrt{2}e-f}{4\sqrt{2}}\,,$$

$$\sqrt{2}F - E = \frac{1 - \sqrt{2}a + b - d + \sqrt{2}e - f}{4\sqrt{2}},$$

we get

$$\begin{aligned}
s(x) = {} & \frac{a-c+e-g}{8}\ln(x^2+1) + \frac{1-b+d-f}{4}\arctan x + \\
& + \frac{-1+b+\sqrt{2}c+d-f-\sqrt{2}g}{8\sqrt{2}}\ln(x^2-\sqrt{2}x+1) + \\
& + \frac{1+\sqrt{2}a+b-d-\sqrt{2}e-f}{4\sqrt{2}}\arctan(\sqrt{2}x-1) + \\
& + \frac{1-b+\sqrt{2}c-d+f-\sqrt{2}g}{8\sqrt{2}}\ln(x^2+\sqrt{2}x+1) + \\
& + \frac{1-\sqrt{2}a+b-d+\sqrt{2}e-f}{4\sqrt{2}}\arctan(\sqrt{2}x+1) + \\
& + \frac{1-a+b-c+d-e+f-g}{8}\ln|x+1| - \\
& - \frac{1+a+b+c+d+e+f+g}{8}\ln|x-1| + \frac{(a-e)\pi}{8}.
\end{aligned}$$

Now, we will deal with the convergence of the power series (2) in the point $x = 1$. Substituting $x = 1$ to the series (2) – it can be done by the extended version of Abel's theorem (see [7], p. 23) – we get the numerical series (1). By the integral test we can prove that the series (1) converges if and only if $H = 0$, i.e. for $g = -a-b-c-d-e-f-1$. Simplifying the formula for $s(x)$ above, where $g = -a-b-c-d-e-f-1$, and for $x = 1$ we get

$$\begin{aligned}
s(1) = {} & \frac{1+2a+b+d+2e+f}{8}\ln 2 + \frac{1-b+d-f}{4}\arctan 1 + \\
& + \frac{\sqrt{2}-1+\sqrt{2}a+(\sqrt{2}+1)b+2\sqrt{2}c+(\sqrt{2}+1)d+\sqrt{2}e+(\sqrt{2}-1)f}{8\sqrt{2}} \times \\
& \times \ln(2-\sqrt{2}) + \frac{1+\sqrt{2}a+b-d-\sqrt{2}e-f}{4\sqrt{2}}\arctan(\sqrt{2}-1) + \\
& + \frac{\sqrt{2}+1+\sqrt{2}a+(\sqrt{2}-1)b+2\sqrt{2}c+(\sqrt{2}-1)d+\sqrt{2}e+(\sqrt{2}+1)f}{8\sqrt{2}} \times \\
& \times \ln(2+\sqrt{2}) + \frac{1-\sqrt{2}a+b-d+\sqrt{2}e-f}{4\sqrt{2}}\arctan(\sqrt{2}+1) + \\
& + \frac{1+b+d+f}{4}\ln 2 - 0 + \frac{(a-e)\pi}{8}.
\end{aligned}$$

Because $\ln 1 = 0$, $\arctan 1 = \dfrac{\pi}{4}$, $\arctan(\sqrt{2}-1) = \dfrac{\pi}{8}$, $\arctan(\sqrt{2}+1) = \dfrac{3\pi}{8}$, we have

$$s(1) = \frac{1-b+d-f}{16}\pi + \frac{-1+b+d-f+\sqrt{2}(1+a+b+2c+d+e+f)}{8\sqrt{2}} \times$$
$$\times \ln(2-\sqrt{2}) + \frac{1+b-d-f+\sqrt{2}(a-e)}{32\sqrt{2}}\pi +$$
$$+ \frac{1-b-d+f+\sqrt{2}(1+a+b+2c+d+e+f)}{8\sqrt{2}}\ln(2+\sqrt{2}) +$$
$$+ \frac{3(1+b-d-f)-3\sqrt{2}(a-e)}{32\sqrt{2}}\pi +$$
$$+ \frac{3(1+b+d+f)+2(a+e)}{8}\ln 2 + \frac{a-e}{8}\pi\,.$$

After simplification and after re-mark $s(1)$ as $s(a,b,c,d,e,f)$ we obtain

$$s(a,b,c,d,e,f) = \frac{1+2a-b+d-2e-f}{16}\pi +$$
$$+ \frac{\sqrt{2}-a+\sqrt{2}b-\sqrt{2}d+e-\sqrt{2}f}{16}\pi + \frac{1-b-d+f}{8\sqrt{2}}\ln(3+2\sqrt{2}) +$$
$$+ \frac{1+a+b+2c+d+e+f+3+2a+3b+3d+2e+3f}{8}\ln 2\,.$$

Finally, we get the required formula

$$s(a,b,c,d,e,f) = \frac{\sqrt{2}(1+b-d-f)+1+a-b+d-e-f}{16}\pi +$$
$$+ \frac{1-b-d+f}{8\sqrt{2}}\ln(3+2\sqrt{2}) + \frac{4(1+b+d+f)+3a+2c+3e}{8}\ln 2\,. \quad (7)$$

4 Numerical verification

We have solved the problem to determine the sum $s(a,b,c,d,e,f)$ above for several values of a, b, c, d, e, f by using the basic programming language of the computer algebra system Maple 16. It was used the following simple procedure `sumgenhar1abcdefg`. As a sample of the hexads (a,b,c,d,e,f) we took 12 hexads

$$(1,0,0,0,0,0)\,,\ (0,1,0,0,0,0)\,,\ (0,0,1,0,0,0)\,,\ (0,0,0,1,0,0)\,,$$

$$(0,0,0,0,1,0)\,,\ (0,0,0,0,0,1)\,,\ (0,0,0,0,0,0)\,,\ (-1000,0,0,0,0,0)\,,$$
$$(6,-1,0,1,-6,-1)\,,\ (-1,1,-1,1,-1,1)\,,\ (-2,1,-2,1,-2,1)\,,$$
$$\text{and}\ \ (1/2,1/4,1/8,1/16,1/32,1/64)\,.$$

It was chosen $t = 10^6$ summands with 8 terms

$$\frac{1}{8n-7} + \frac{a}{8n-6} + \frac{b}{8n-5} + \frac{c}{8n-4} + \frac{d}{8n-3} + \\ + \frac{e}{8n-2} + \frac{f}{8n-1} - \frac{a+b+c+d+e+f+1}{8n}$$

for the computations whose results will be compared with the results obtained by the formula (7). The procedure `sumgenhar1abcdefg` consists of the following commands:

```
sumgenhar1abcdefg:=proc(t,a,b,c,d,e,f)
   local g,r,k,s,w;
   s:=0; r:=0;
   g:=-a-b-c-d-e-f-1;
   for k from 1 to t do
       r:=1/(8*k-7)+a/(8*k-6)+b/(8*k-5)+c/(8*k-4)+d/(8*k-3)
         +e/(8*k-2)+f/(8*k-1)+g/(8*k);
       s:=s+r;
   end do;
   print("t=",k-1,"s(",a,b,c,d,e,f,")=",evalf[20](s));
   w:=Pi*(sqrt(2)*(1+b-d-f)+1+a-b+d-e-f)/16
        +ln(3+2*sqrt(2))*(1-b-d+f)/(8*sqrt(2))
        +ln(2)*(4*(1+b+d+f)+3*a+2*c+3*e)/8
   print("s(",a,b,c,d,e,f")=",evalf[20](w));
end proc:
```

Computation of the twelve sums $s(10^6, a, b, c, d, e, f)$ took about 47 hours and 39 minutes. The relative quantification accuracies of the twelve sums $s(10^6, a, b, c, d, e, f)$, that is the ratio

$$\left| \frac{s(10^6, a, b, c, d, e, f) - s(a, b, c, d, e, f)}{s(10^6, a, b, c, d, e, f)} \right| ,$$

have here place value about 10^{-7}.

The results of the procedure above are presented in the Table 1, where the computed sums are denoted briefly $s(10^6)$ instead of $s(10^6, a, b, c, d, e, f)$ and the sums $s(a, b, c, d, e, f)$ are denoted as $s(abcdef)$ and are evaluated by means of the formula (7):

a	b	c	d	e	f	$s(10^6)$	$s(abcdef)$
1	0	0	0	0	0	0.9591518	0.9591520
0	1	0	0	0	0	1.4326892	1.4326894
0	0	1	0	0	0	1.1496962	1.1496964
0	0	0	1	0	0	1.0858461	1.0858463
0	0	0	0	1	0	1.0399901	1.0399903
0	0	0	0	0	1	1.0047597	1.0047598
0	0	0	0	0	0	0.9764095	0.9764096
-10^3	0	0	0	0	0	-455.30323	-455.30332
6	-1	0	1	-6	-1	3.1415922	3.1415927
-1	1	-1	1	-1	1	0.6931471	0.6931472
-2	1	-2	1	-2	1	0.0000001	0
1/2	1/4	1/8	1/16	1/32	1/64	1.3035043	1.3035045

Table 1: The approximate values of the sums of the generalized harmonic series with periodically repeating numerators $(1, a, b, c, d, e, f, -a-b-c-d-e-f-1)$ for 12 hexads (a, b, c, d, e, f)

5 Conclusion

We dealt with the generalized convergent harmonic series with eight periodically repeated numerators $(1, a, b, c, d, e, f, g)$, where $a, b, c, d, e, f, g \in \mathbb{R}$, i.e. with the series

$$\sum_{n=1}^{\infty}\left(\frac{1}{8n-7}+\frac{a}{8n-6}+\frac{b}{8n-5}+\frac{c}{8n-4}+\frac{d}{8n-3}+\frac{e}{8n-2}+\frac{f}{8n-1}+\frac{g}{8n}\right).$$

We derived that the only value of the numerator g, for which this series converges, is $g = -a-b-c-d-e-f-1$, and we derived that the sum of this series is given by the formula

$$s(a, b, c, d, e, f) = \frac{\sqrt{2}(1+b-d-f)+1+a-b+d-e-f}{16}\pi + \\ + \frac{1-b-d+f}{8\sqrt{2}}\ln(3+2\sqrt{2}) + \frac{4(1+b+d+f)+3a+2c+3e}{8}\ln 2\,.$$

This formula allows to determine another sums whose periodically repeated numerators need not be $(1, a, b, c, d, e, f, -a-b-c-d-e-f-1)$, but also $(k, \ell, m, n, p, q, r, -k-\ell-m-n-p-q-r)$, for $k, \ell, m, n, p, q, r \in \mathbb{R}$, at least one nonzero. For example, the series

$$\sum_{n=1}^{\infty}\left(\frac{64}{8n-7}+\frac{32}{8n-6}+\frac{16}{8n-5}+\frac{8}{8n-4}+\frac{4}{8n-3}+\frac{2}{8n-2}+\frac{1}{8n-1}-\frac{127}{8n}\right)$$

has the sum $S(64,32,16,8,4,2,1,-127) =$

$$= 64\cdot s\left(\frac{1}{2},\frac{1}{4},\frac{1}{8},\frac{1}{16},\frac{1}{32},\frac{1}{64}\right) \doteq 64\cdot 1.303\,504 \doteq 83.424\,.$$

There are special series with sums expressed only by one summand or with the special or the null sum. It can be easily derived that

$$s(a,b,0,-b,-a,-1) = \frac{1+a-b+\sqrt{2}(1+b)}{8}\,\pi\,,$$

$$\text{so e.g.} \qquad s(6,-1,0,1,-6,-1) = \pi\,,$$

$$s(a,1,c,1,a,1) = \frac{8+3a+c}{4}\ln 2\,, \qquad \text{so e.g.} \qquad s(-1,1,-1,1,-1,1) = \ln 2\,,$$

$$\text{and} \qquad s(-2,-1,0,1,2,-1) = 0\,, \qquad s(-2,1,-2,1,-2,1) = 0\,.$$

We verified the main result (7) by computing 12 sums by using the CAS Maple 16. The generalized convergent harmonic series with eight periodically repeated numerators so belong to special types of infinite series, such as geometric and telescoping series, which sums are given analytically by means of a relatively simple formula.

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Approach of the value of an annuity when non-central moments of the capitalization factor are known: an R application with interest rates following normal and beta distributions

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Abstract

This paper proposes an expression of the value of an annuity with payments of 1 unit each when the interest rate is random. In order to attain this objective, we proceed on the assumption that the non-central moments of the capitalization factor are known. Specifically, to calculate the value of these annuities, we propose two different expressions. First, we suppose that the random interest rate is normally distributed; then, we assume that it follows the beta distribution. A practical application of these two methodologies is also implemented using the R statistical software.

Keywords: annuity; random interest rate; non-central moments.

2010 AMS subject classification: 91G30; 46N30; 65C60; 91G70; 62P05.

1 Introduction

This study aims to determine an approximate expression for the present, or final, value of an annuity when the interest rate is random. In the context of annuities assessment, the interest rate has a great relevance because even small changes may cause major changes in the total annuity value. Thus, the determination of the value of the interest rate should be carried out as accurately as possible.

The traditional approach treats interest rates deterministically; indeed, in contexts of certainty, the use of a single possible value for each period may be enough [8]. However, for those operations developed in uncertain environments, it is more reasonable the formulation of potential scenarios, which are subsequently reduced to one by statistical treatment [2].

The determination of the interest rate value must be based on the current situation, as well as on its possible future evolution, of both companies and environment. In this way, if prospects are unfavorable, interest rates must be higher, compared to more favorable situations, and hence to reduce the operation value as a consequence of the risk attached to it. However, in most cases, determining the interest rate of a financial operation is subject to the propensity/aversion to risk of the agent to be responsible for the assessment. In this sense, the adopted interest rate would be affected by a degree of subjectivity that may over/undervalue the project [7].

In this paper, we consider the interest rate as a random variable that is represented as *X*. Therefore, the capitalization factor, $1+i$, is also a random variable represented as *U*. Obviously, it is verified that $U=1+X$, thus, the relationship between the mean and standard deviation of both variables is as follows:

$$\mu_U = 1+\mu_X \text{ and } \sigma_U = \sigma_X .$$

As a result, if *X* is defined in an interval $[a,b]$, then *U* will be in the interval $[a+1,b+1]$. Henceforth, when the mean and standard deviation are mentioned we will refer, unless otherwise specified, to the random variable *U*.

In this case, the final value of an *n*-payment annuity, with payments of 1 unit each made at the end of every year (annuity-immediate), valued at the rate *X*, would be the following random variable:

$$s_{\overline{n}|U-1} = 1+U+U^2+\cdots+U^{n-1}. \tag{1}$$

Thus, its expected value is:

$$E(s_{\overline{n}|U-1}) = E(1)+E(U)+E(U^2)+\cdots+E(U^{n-1}) =$$
$$=1+\mu+\mu_2+\cdots+\mu_{n-1}.$$

On the other hand, the final expected value of an *n*-payment annuity, with payments of 1 unit each made at the beginning of every year (annuity-due), valued at the rate *X*, would be:

$$E(\ddot{s}_{\overline{n}|U-1}) = E(U) + E(U^2) + \cdots + E(U^n) = \mu + \mu_2 + \cdots + \mu_n, \tag{2}$$

being $\mu_r = E(U^r)$ the moment of order *r*, with respect to the origin, of the random variable *U*; hence, if the random variable is discrete, it adopts the following expression [1]:

$$\mu_r = E(U^r) = \sum_{i=1}^{k} p_i u_i^r , \tag{3}$$

being p_i the probability that the random variable takes the value u_i. In the continuous case, the expression of the moment of order *r* is:

$$\mu_r = E(U^r) = \int_{u_{\min}}^{u_{\max}} u f(u)\mathrm{d}u , \tag{4}$$

for all values of *r*, being $f(u)$ the density function of the random variable *U*.

As indicated, this paper proposes a mathematical expression of the final value of an annuity, immediate or due; specifically, we compute it using a random interest rate and suppose that the non-central moments of the capitalization factor are known. Section 2 shows the case of interest rates following the normal distribution. Section 3 takes into account the beta distribution, as an example of distribution with finite range. Section 4 shows a practical application using the R statistical software. Lastly, the conclusions are presented.

2 The expression of the final value of an annuity when the interest rate follows a normal distribution

The successive non-central moments of order r, with respect to the normal distribution, can be computed according to its mean μ and variance σ^2 [4]:

- $\mu_0 = 1$;
- $\mu_1 = \mu$;
- $\mu_2 = \mu^2 + \sigma^2$;

- $\mu_3 = \mu^3 + 3\mu\sigma^2$;
- $\mu_4 = \mu^4 + 6\mu^2\sigma^2 + 3\sigma^4$;
- $\mu_5 = \mu^5 + 10\mu^3\sigma^2 + 15\mu\sigma^4$;
- $\mu_6 = \mu^6 + 15\mu^4\sigma^2 + 45\mu^2\sigma^4 + 15\sigma^6$;
- $\mu_7 = \mu^7 + 21\mu^5\sigma^2 + 105\mu^3\sigma^4 + 105\mu\sigma^6$;
- $\mu_8 = \mu^8 + 28\mu^6\sigma^2 + 210\mu^4\sigma^4 + 420\mu^2\sigma^6 + 105\sigma^8$.

$$\vdots$$

Therefore, the final value of an n-payment annuity, with payments of 1 unit each made at the end of every year (annuity-immediate) that is the sum of the n first non-central moments $\sum_{r=0}^{n-1} \mu_r$, is composed of the following partial sums:

- $1 + \mu + \mu^2 + \cdots + \mu^{n-1} = \sum_{r=0}^{n-1} \mu^r = \dfrac{1-\mu^n}{1-\mu}$.

This can be written as $\sigma^0 \sum_{r=0}^{n-1} \binom{r}{0} \mu^r$.

- $\sigma^2(1 + 3\mu + 6\mu^2 + 10\mu^3 + 15\mu^4 + 21\mu^5 + 28\mu^6 + \cdots) = \sigma^2 \sum_{r=2}^{n-1} \binom{r}{2} \mu^{r-2}$.

The coefficients of successive powers of μ, in parentheses, are the numbers in red in the following Tartaglia's triangle (Figure 1):

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1

Figure 1: Tartaglia's triangle.

- $3\sigma^4(1 + 5\mu + 15\mu^2 + 35\mu^3 + 70\mu^4 + \cdots) = 3\sigma^4 \sum_{r=4}^{n-1} \binom{r}{4} \mu^{r-4}$.

The coefficients of successive powers of μ, enclosed in the parentheses, are the numbers in green of the previous Tartaglia's triangle.

- $15\sigma^6(1+7\mu+28\mu^2+\cdots)=15\sigma^6\sum_{r=6}^{n-1}\binom{r}{6}\mu^{r-6}$,

whose coefficients are in blue.

- And so forth.

In short, the sum of the n first non-central moments is:

$$\sum_{r=0}^{n-1}\mu_r=\frac{1-\mu^n}{1-\mu}+\sum_{k=1}^{\mathrm{E}((n-1)/2)}1\cdot3\cdot5\cdots(2k-1)\sigma^{2k}\sum_{r=2k}^{n-1}\binom{r}{2k}\mu^{r-2k},$$

which can also be written as follows:

$$\sum_{r=0}^{n-1}\mu_r=\frac{1-\mu^n}{1-\mu}+\sum_{k=1}^{\mathrm{E}((n-1)/2)}\frac{(2k-1)!}{2^{k-1}(k-1)!}\sigma^{2k}\sum_{r=2k}^{n-1}\binom{r}{2k}\mu^{r-2k}. \tag{5}$$

This method is used to calculate the final value of an n-payment annuity, with payments of 1 unit each made at the end of every year (annuity-immediate), with a random interest rate. Whereas, the calculation of the final value of an n-payment annuity, with payments of 1 unit each made at the beginning of every year (annuity-due), has the following expression:

$$\sum_{r=1}^{n}\mu_r=\mu\frac{1-\mu^n}{1-\mu}+\sum_{k=1}^{\mathrm{E}(n/2)}1\cdot3\cdot5\cdots(2k-1)\sigma^{2k}\sum_{r=2k}^{n}\binom{r}{2k}\mu^{r-2k}. \tag{6}$$

In equations (5) and (6), the function $\mathrm{E}(x)$ represents the integer part of x. To carry out the calculations in a comfortable and orderly manner, we propose to refer to Table 1.

3 The expression of the final value of an annuity when the interest rate follows a beta distribution

The best known random variable with a bounded range is the beta distribution. The expression of the non-central moments of the standard beta distribution of parameters α and γ is the following ($r\in N$) [3]:

$$\mu_r = \frac{\Gamma(\alpha+\gamma)\Gamma(\alpha+r)}{\Gamma(\alpha)\Gamma(\alpha+\gamma+r)} = \frac{\alpha(\alpha+1)\cdots(\alpha+r-1)}{(\alpha+\gamma)(\alpha+\gamma+1)\cdots(\alpha+\gamma+r-1)}. \tag{7}$$

In this case, it is not feasible to give a closed expression of the sum of the *n* first non-central moments, but, having in mind that $\Gamma(\alpha+1) = \alpha\Gamma(\alpha)$, we can write the following recurrence relation [5]:

$$\mu_{r+1} = \frac{\alpha+r}{\alpha+\gamma+r}\mu_r. \tag{8}$$

Tartaglia's triangle										**Exponents of σ**				
										0	1	2	3	4
1										μ^0	-	-	-	-
1	1									μ^1	-	-	-	-
1	2	1								μ^2	μ^0	-	-	-
1	3	3	1							μ^3	μ^1	-	-	-
1	4	6	4	1						μ^4	μ^2	μ^0	-	-
1	5	10	10	5	1					μ^5	μ^3	μ^1	-	-
1	6	15	20	15	6	1				μ^6	μ^4	μ^2	μ^0	-
1	7	21	35	35	21	7	1			μ^7	μ^5	μ^3	μ^1	-
1	8	28	56	70	56	28	8	1		μ^8	μ^6	μ^4	μ^2	μ^0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋱	⋮	⋮	⋮	⋮	⋮
										Σ	Σ	Σ	Σ	Σ
										σ^0	σ^2	σ^4	σ^6	σ^8
										1	1	3	15	105
										Π	Π	Π	Π	Π
										Sum of all products				

Table 1. Tabular organization for calculations (the number that occupies the place (r,s) in Tartaglia's triangle is equal to the sum of those in places $(r+1,s-1)$ and $(r+1,s)$.

However, it should be considered that the above mentioned moments refer to the standard beta distribution, *Z*, of parameters, α and γ, that is, with range [0,1]. Furthermore, to obtain the moments μ_r' corresponding to the distribution

U without normalize, that is to say, the beta distribution of parameters α and γ, with range $[a,b]$:

$$U = a + (b-a)Z, \tag{9}$$

it is necessary to consider the relationship between its moment-generating functions [6]:

$$M_U(t) = M_{a+(b-a)Z}(t) = \mathrm{e}^{at} M_Z((b-a)t). \tag{10}$$

Therefore, having in mind the expression of the *n*th derivative of a product of functions, we can write:

$$\frac{\mathrm{d}^r}{\mathrm{d}t^r} M_U(t) = \sum_{k=0}^{r} \binom{r}{k} \frac{\mathrm{d}^k}{\mathrm{d}t^k} \mathrm{e}^{at} \frac{\mathrm{d}^{r-k}}{\mathrm{d}t^{r-k}} M_Z((b-a)t) =$$
$$= \sum_{k=0}^{r} \binom{r}{k} a^k \mathrm{e}^{at} (b-a)^{r-k} M_Z^{r-k)}((b-a)t).$$

Therefore,

$$\mu_r' = \frac{\mathrm{d}^r}{\mathrm{d}t^r} M_U(t)\bigg|_{t=0} = \sum_{k=0}^{r} \binom{r}{k} a^k (b-a)^{r-k} \mu_{r-k} \,. \tag{11}$$

4 Calculation of the value of an annuity, with payments of 1 unit each: an R application

Next, we are going to obtain the final value of an annuity, with payments of 1 unit each for five years through the different expressions developed in this work. In its calculus we consider that the payments are made at the end, or the beginning, of each period. Present value calculation has been omitted provided it can be carried out similarly.

Given that in this work it has been contemplate that non-central moments of the capitalization factor are known, two possible options have been considered, where discount rate, X, follows:

- a normal distribution;
- a beta distribution.

Discount rate with a normal distribution

To estimate the mean and variance of the normal distribution, we consider Euribor's data containing the estimated annual Euribor of different banks (Table 2), available at http://www.emmi-benchmarks.eu/euribor-org/euribor-rates.html. Specifically, the Euribor at 12 months is considered on 27/07/2016.

Estimates	Euribor in 1 year (%)
BNP-Paribas	0.00
Banca Monte Dei Paschi Di Siena	–0.05
Banco Bilbao Vizcaya Argentaria	–0.05
Banco Santander	–0.06
Banque et Caisse d'Épargne de l'État	–0.06
Barclays Bank	0.02
Belfius	–0.06
CECABANK	–0.05
Caixa Geral De Depósitos	–0.04
CaixaBank S.A.	–0.05
Crédit Agricole s.a.	–0.03
DZ Bank	–0.06
Deutsche Bank	0.04
HSBC France	–0.05
ING Bank	–0.08
Intesa Sanpaolo	–0.05

Table 2: Euribor distribution at 27/07/2016.
Source: http://www.emmi-benchmarks.eu/euribor-org/euribor-rates.html.

To enter the data in the R environment, it is necessary to create a vector as follows:

>data=c(0.00,-0.05,-0.05,-0.06,-0.06,0.02,-0.06,-0.05,-0.04,-0.05,-0.03,-0.06,0.04,-0.05,-0.08,-0.05,-0.12,-0.03,-0.05,-0.04,-0.06)

To check for normality of the data, we need the R package "tseries"; thus, the Jarque-Bera Test is implemented:

> library(tseries)

> jarque.bera.test(data)

We can accept the normality of the data because the p-value is greater than 0.05 (X-squared = 3.559, df = 2, p-value = 0.1687).

Following a preliminary analysis, we obtain $\mu = -0.0442$ and $\sigma = 0.0332$ using the following scripts:

```
> mean(data)
> sd(data)
```

The expressions formulated in Section 2 allow computing the expected final value when the expression of the non-central moments of the capitalization factor is known.

Now, we suppose to compute the final value of an annuity, with payments of 1 unit each for five years using the estimated mean and variance. Thus, if the payments of the annuity are at the end of each period, the final value is:

$$\sum_{r=0}^{4} \mu_r = 4.958541.$$

It is possible to check this result building a function to compute the sum of the non-central moments of the normal distribution for annuities whose payments are at the end of each period, with a duration of k years (however, firstly, we need to load the "moments" library):

```
>library(moments)
>sum_k_moments_post=function(data,k){
 app.moments_post=rep(NA,k)
 for (i in 0:(k-1)) app.moments_post[i+1]=moment(data, central = FALSE,
absolute = FALSE, order =i)
  sum_moments_post=sum(app.moments_post)+(k-1)
  return(sum_moments_post) }
>sum_k_moments_post(data,5)
```

Instead, in case the final value of an annuity, with payments of 1 unit each at the beginning of every year, for five years, we obtain:

$$\sum_{r=1}^{5} \mu_r = 4.958539.$$

Also in this case, we can check the result by creating a function to compute the sum of the non-central moments of the normal distribution for annuities whose payments are at the beginning of each period, with a duration of k years:

```
> sum_k_moments_ant=function(data,k){
```

```
app.moments_ant=rep(NA,k)
for (i in 1:k) app.moments_ant[i]=moment(data, central = FALSE, absolute = FALSE, order =i)
sum_moments_ant=sum(app.moments_ant)+k
return(sum_moments_ant) }
>sum_k_moments_ant(data,5)
```

Using the formulation proposed in equation 5 (annuities with payments at the end of each period) and equation 6 (annuities with payments at the beginning of each period), we reach the same results (replacing the values of the mean and the standard deviation, it is simple to demonstrate this identity).

Discount rate with a beta distribution

Because the beta distribution is suitable to approximate also the data of Table 2, we refer to the same data of Euribor with the aim to compare it with the results obtained in the previous paragraph.

At this purpose we load the data of Table 2 and the R packages "actuar" and "EnvStats" as follows:

```
>library(actuar)
>library(EnvStats)
```

Then, we create a function to normalize data and we apply it to the data of Table 2 as follows:

```
>nor=function(x){(x-min(x))/(max(x)-min(x))}
>data2=nor(data)
```

Afterwards, we estimate the shapes of the beta distribution with the function "ebeta":

```
>ebeta(data2, method = "mle")
```

Following this approach, we get the shapes parameters $a = 2.394501$ and $b = 2.66557$. Then, we build a function to compute the mean and the standard deviation:

```
>mean_beta=function(a,b){a/(a+b)}
>mean_beta(2.394501,2.665577)
0.4732142
```

```
>var_beta=function(a,b){a*b/((a+b)^2*(a+b+1))}
>var_beta(2.394501,2.665577)
0.0411352
```

As known, a generic moment of the standard beta distribution is given by:

$$\mu_r = \frac{(\alpha)_r}{(\alpha+\gamma)_r} \tag{12}$$

where $(\alpha)_r = \alpha(\alpha+1)\cdots(\alpha+r-1)$.

To compute the moments of the standard beta distribution using this method, we need a function to calculate the factorial:

```
>fattoriale_crescente=function(n,f){n*factorial(x=n+f-1)/factorial(x=n)}
```

In this way, we can compute the moments as:

```
>momento_beta=function(a,b,f){fattoriale_crescente(a,f)/fattoriale_cresce
nte((a+b),f)}
```

where *a* and *b* are the shapes parameters and *f* represents the order of the moment.

Using these codes, it is simple to calculate the sum of the moments of the standard beta distribution. However, we need the non-central moments of the original beta distribution (without normalize). At this purpose, we build a function to perform Equation (11) and obtain the non-central moments of the original beta distribution.

We set *a* as the lower bound of our interest rates, *b* as the upper bound of our interest rates, *c* and *d* as the parameters of the standard beta distribution, *n* as the number of years:

```
>c=2.394501
>d=2.665577
>a=-0.12
>b=0.04
>n=5
```

```
>momento_nn_normalizzato=function(n,a,b,c,d){for(k in 0:n) {
m=mbeta(n-k, c, d)
    moment_nn_norm=sum(
    choose(n,k)*(a^k)*((b-a)^(n-k))*m
     )} return(moment_nn_norm)}
```

Afterwards, we present two functions: the first one computes the final value of an *n*-payment annuity, with payments of 1 unit each made at the end of every year (annuity-immediate); the second one calculates the final value of an *n*-payment annuity, with payments of 1 unit each made at the beginning of every year (annuity-due).

The first function is built as follows:

```
>sum_n_moments_non_norm_beta_pag_anticip=function(n,a,b,c,d){
app=rep(NA,n)
 for (i in 1:n) app[i]=(momento_nn_normalizzato(i,a,b,c,d)+1)
  return(sum(app))}
>sum_n_moments_non_norm_beta_pag_anticip(5,a,b,c,d)
```

Thus, the final value of an *n*-payment annuity, with payments of 1 unit each made at the end of every year (annuity-immediate) for five years is:

$$\sum_{r=1}^{5} \mu'_r = 4.892854 .$$

The second function is provided by the following code:

```
>sum_n_moments_non_norm_beta_pag_post=function(n,a,b,c,d){
app2=rep(NA,n)
 for (i in 0:(n-1)) app2[i+1]=momento_nn_normalizzato(i,a,b,c,d)
 app2[2:n]=app2[2:n]+1
 return(sum(app2))}
>sum_n_moments_non_norm_beta_pag_post(5,a,b,c,d)
```

Therefore, with our data, the final value of an *n*-payment annuity, with payments of 1 unit each made at the beginning of every year (annuity-immediate) for five years is:

$$\sum_{r=0}^{4} \mu'_r = 4.892879 .$$

5 Conclusions

In this paper we have presented two methodologies to obtain the value of an annuity whose discount rate is a variable known in terms of random. Once the expected value of the discount rate has been analyzed through the non-central moments of the discount factor, the expression for determining the expected final value of an *n*-payment annuity has been deduced.

Specifically, the theoretical development of this methodology has been carried out in two different ways: by supposing that the interest rate follows a normal distribution, and considering that it follows a beta distribution.

Furthermore, we provided the code to reproduce our results with the R statistical software (available in Appendix 1).

Our results show slight differences between the estimates of the same data, approximating these with different distributions. This shows how the choice of the distribution of the approximation of data is important for the calculation of the value of an annuity when interest rates are represented by random variables.

Appendix 1: Replication material

```
data=c(0.00,-0.05,-0.05,-0.06,-0.06,0.02,-0.06,-0.05,-0.04,-0.05,-0.03,-
0.06,0.04,-0.05,-0.08,-0.05,-0.12,-0.03,-0.05,-0.04,-0.06)

library(tseries)
jarque.bera.test(data)
mean(data)
sd(data)

library(moments)

sum_k_moments_post=function(data,k){
  app.moments_post=rep(NA,k)
  for (i in 0:(k-1)) app.moments_post[i+1]=moment(data, central = FALSE,
absolute = FALSE, order =i)

  sum_moments_post=sum(app.moments_post)+(k-1)
  return(sum_moments_post)}

sum_k_moments_post(data,5)

sum_k_moments_ant=function(data,k){
  app.moments_ant=rep(NA,k)
  for (i in 1:k) app.moments_ant[i]=moment(data, central = FALSE,
absolute = FALSE, order =i)

  sum_moments_ant=sum(app.moments_ant)+k
  return(sum_moments_ant)}

sum_k_moments_ant(data,5)

library(actuar)
library(EnvStats)

nor=function(x){(x-min(x))/(max(x)-min(x))}

data2=nor(data)
ebeta(data2, method = "mle")
mean_beta=function(a,b){a/(a+b)}

mean_beta(2.394501,2.665577)
```

```
var_beta=function(a,b){a*b/((a+b)^2*(a+b+1))}

var_beta(2.394501,2.665577)
fattoriale_crescente=function(n,f){n*factorial(x=n+f-1)/factorial(x=n)}

momento_beta=function(a,b,f){fattoriale_crescente(a,f)/fattoriale_crescente
((a+b),f)}

momento_nn_normalizzato=function(n,a,b,c,d){
  for(k in 0:n) {   m=mbeta(n-k, c, d)
    moment_nn_norm=sum(
      choose(n,k)*(a^k)*((b-a)^(n-k))*m
    ) }
  return(moment_nn_norm)}

sum_n_moments_non_norm_beta_pag_anticip=function(n,a,b,c,d){app=rep
(NA,n)
  for (i in 1:n) app[i]=(momento_nn_normalizzato(i,a,b,c,d)+1)
  return(sum(app))}

sum_n_moments_non_norm_beta_pag_anticip(5,a,b,c,d)
sum_n_moments_non_norm_beta_pag_post=function(n,a,b,c,d){
app2=rep(NA,n)
  for (i in 0:(n-1)) app2[i+1]=momento_nn_normalizzato(i,a,b,c,d)
  app2[2:n]=app2[2:n]+1
  return(sum(app2))}

sum_n_moments_non_norm_beta_pag_post(5,a,b,c,d)
```

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Influence of information on behavioral effects in decision processes

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Abstract

Rational models in decision processes are marked out by many anomalies, caused by behavioral issues. We point out the importance of information in causing inconsistent preferences in a decision process. In a single or multi agent decision process each mental model is influenced by the presence, the absence or false information about the problem or about other members of the decision making group. The difficulty in modeling these effects increases because behavioral biases influence also the modeler. Behavioral Operational Research (BOR) studies these influences to create efficient models to define choices in similar decision processes.

Keywords: Behavioral Operational Research, Intertemporal Choice, Information in Decision Processes.

2000 AMS subject classifications: 91B16, 91B08, 91E10.

1 Introduction

Some failures occur when dealing with traditional Discounted Utility (DU) model from both normative and descriptive setting. Indeed, some studies, especially in psychology and neuroeconomics (a more specialized field of decision neuroscience), point out anomalies that violate some axioms in the traditional model (Sec. 2).

Bechara et al. [1] show positive effects of anomalies in intertemporal choices and the use of hyperbolic delay discounting (declining as the length of the delay increases) to represent inconsistent preferences. On the other hand, the negative effects of emotions mainly stem from impulsivity. To properly describe individual differences in intertemporal choices, derived from impulsivity and inconsistency, behavioral economists have proposed the Q-exponential Delay Discount Function and a Multiple Selves Model (quasi-hyperbolic discount model) (Sec. 3).

Mental models of each person, based on different assumptions and preferences, influence the effects of emotions (positives and negatives) and impulsivity. To control impulsivity Strotz [11] proposed two strategies that might be employed by a person who foresees how his preferences will change over time, and Thaler and Shefrin [12] proposed a model in which the individual is treated as if he contained two distinct psyches denoted as *planner and doer* (Sec. 4).

The information held by the agent plays an important role on the delineation of his mental model. In a multi agent decision context all people involved have their mental model and influence other mental models. Hence, in these strategic decisions information about others and about the problem definitely influence the final choice (Sec. 5). If there is no information about other players, as shown in an experiment of Engelmann and Strobel [4], people weight their own decisions more heavily than that of a randomly selected person from the same population (false consensus effect). This happens in non-cooperative decision problems, not properly modeled in OR. Indeed, because false consensus effect and impulsivity not always lead each agent towards the best strategy according to the theory of games, so obtaining a common decision is only a chance (Sec. 6). On the contrary, when all information is explicit people can consider the choices of others as more informative then their own (excess of consensus or overconfidence). An example is a cooperative decision problem, modeled by OR with cooperative games, in which final decision is based on mental models of the participants and their tendency to overconfidence (Sec. 7).

However, the final decision is influenced not only by intrinsic characteristics of every one, but also by the way in which information is passed: misunderstandings and manipulations (above all for self-interest) change people's reactions (Sec. 8).

At last, also the modeler is influenced by his mental models: creating a model to predict a decision making process is itself a decision making process. A new branch of research (BOR, Behavioral Operational Research) studies human impacts of using OR models in decisional processes (Sec. 9).

2 Effects of behavioral aspects: violations of traditional Discount Utility Model

Operational Research (OR) has modeled human behavior in intertemporal choice in terms of DU model, which assumes an exponential temporal discounting function and a constant discount rate: this represents the individual's pure rate of time preference. An important implication of constant discount rate and exponential discounting function is that a person's intertemporal preference is time-consistent.

However, decision neuroscience, whose goal is to integrate research in neuroscience and behavioral decision-making, highlights that there are a number of behavior patterns that violate rational choice theory. Several empirical studies on individual behavior, when discounting real or hypothetical rewards, stress the existence of violations of the traditional discounting model [2].

Theory and algorithms of OR models are free of behavioral effects but as soon as we use them in real life problem solving behavioral effects will be present. Hence behavioral perspective is essential in decision analysis [6]. Research in psychology has reported many types of cognitive and motivational biases as well as heuristics which relate to human behavior and may significantly distort the decision analysis generating inconsistent preferences in intertemporal choices. Delay effect, magnitude effect and sign effect are among the relevant anomalies in intertemporal choice, we will deal with (see also [13]).

The delay effect. As waiting time increases, the discount rates tend to be higher in short intervals than in longer ones. We can set out this effect as follows:

$$(x, s) \sim (y, s) \text{ but } (x, s+h) < (y, t+h), \text{ for } y > x, s < t \text{ and } h > 0$$

The magnitude effect. Larger outcomes are discounted at a lower rate than smaller outcomes. This effect can be formulated as follows:

$$(x, s) \sim (y, s) \text{ implies } (ax, s) < (ay, t), \text{ for } y > x > 0, s < t$$

and

$$(-x, s) \sim (-y, s) \text{ implies } (-ax, s) > (-ay, t)$$

The sign effect. Gains are discounted at a higher rate than losses of the same magnitude. This anomaly implying that, changing the sign of an amount from gains to losses, the weight of this amount increases:

$$(x, s) \sim (y, s) \text{ implies } (-x, s) > (-y, t) \text{ for } y > x > 0 \text{ and } s < t$$

3 Effects of emotions: violations of traditional Discount Utility model

In a series of studies (see, e. g., ([1], [3], [8])) using a gambling task, it emerges that individuals with emotional dysfunction tend to perform poorly compared with those who are endowed with intact emotional processes. Bechara et al. [1] demonstrated that normal people possess anticipatory SCRs (*Skin Conductance Response*) – indices of somatic states – which represent unconscious biases that are linked to prior experiences with reward or punishment and produce inconsistent preferences. These biases alarm the normal subject about selecting a disadvantageous course of action, even before the subject becomes aware of the goodness or badness of the choice he is about to make. As a consequence there is considerable agreement among psychologists and economists that the notion of exponential discounting should be replaced by some form of hyperbolic discounting, which can point out the delay effect (or present bias), that is the tendency of the individuals to increasingly choose a smaller-sooner reward over a larger-later reward as the delay occurs sooner in time.

Many authors proposed different hyperbolic discount functions, in which temporal discount function increases with the delay to an outcome. One of these proposed functions has the following form:

$$d(t) = \left(\frac{1}{1+\alpha t}\right)^{\beta/\alpha}$$

where $\beta > 0$ is the degree of discounting and $\alpha > 0$ is the departure from exponential discounting. Hyperbolic discounting has been applied to a wide range of phenomena, including consumption-saving behavior. Consistent with hyperbolic discounting, people's investment behavior exhibits patience in the long run and impatience in the short run [13].

A second type of empirical support for hyperbolic discounting comes from experiments on dynamic inconsistency. Studies and empirical evidences show that delay effect can derive in preference reversal between two rewards as the time-distance to these rewards diminishes. A hyperbolic discount model can clarify this; in fact, hyperbolic time-preference curves can cross [11] and consequently the preference for one future reward over another may change with time [13].

However, in some contexts individuals deprived of normal emotional reactions might actually make better decisions than normal individuals, because of the loss of self-control, as Damasio found when studying behavior of people with ventromedial prefrontal damage [3]. Temptations are manifestations of loss of self-control and in many cases induce disadvantageous behavior. Indeed, as far as temptation increases the best long run interest of the problem solver conflicts with his short run desires, moreover impulsive behavior may fail to evaluate the consequences of his behavior appropriately [13].

Other evidences suggest that even relatively mild negative emotions that do not result in a loss of self-control can play a counterproductive role among normal individuals in some situations. When gambles that involve some possible loss are presented one at a time, most people display extreme levels of risk aversion toward the gambles, a condition known as myopic loss aversion. Shiv et al. [9] show that individuals deprived of normal emotional reactions might, in certain situations, make more advantageous decisions than those not deprived of such reactions; so the lack of emotional reactions may lead to more advantageous decisions.

Inconsistent preference is the greatest contradiction of rational theory in intertemporal choice. This behavior can be typically seen in psychiatric disorders (alcoholism, drug abuse), but also in more ordinary phenomena (overeating, credit card debt) [13]. Neuroeconomics has found that addicts are more myopic (have large time-discount rates) in comparison with non-addicted population.

However, the preference for more immediate rewards per se is not always irrational or inconsistent; addicts' behavior is clinically problematic, but economically rational when their choices are time-consistent (if they have large discount rates with an exponential discount function). But addicts also discount delayed outcomes hyperbolically, suggesting the intertemporal choices of addicts are time-inconsistent, resulting in a loss of self-control: they act more impulsively at the moment of the choice, against their own previously intended plan. Moreover if large discount rates are due to habitual drug intake, it is expected that discount rates decreased after long-term abstinence.

Behavioral neuroeconomics and econophysical studies have proposed two discount models, in order to better describe the neural and behavioral correlates of impulsivity and inconsistency in intertemporal choice.

Q-exponential discount model. This function has been proposed and examined for subjective value $V(D)$ of delayed reward:

$$V(D) = \frac{A}{\exp_q(k_q D)} = A/[1 + (1-q)k_q]D^{\frac{1}{1-q}}$$

where D denotes a delay until receipt of a reward, A the value of a reward at $D = 0$, and k_q a parameter of impulsivity at delay $D = 0$ (q-exponential discount rate) and the q-exponential function is defined as:

$$\exp_q(x) = (1 + (1-q))^{\frac{1}{1-q}}$$

The function can distinctly parametrize impulsivity and inconsistency [13].

Quasi-hyperbolic discount model. Behavioral economists have proposed that the inconsistency in intertemporal choice is attributable to an internal conflict between "multiple selves" within a decision maker. As a consequence, there are (at least) two exponential discounting selves (with two exponential discount rates) in a single human individual; and when delayed rewards are at the distant future (> 1 year), the self with a smaller discount rate wins, while delayed rewards approach to the near future (within a year), the self with a larger discount rate wins, resulting in preference reversal over time. This intertemporal choice behavior can be parametrized in a quasi-hyperbolic discount model (also as a $\beta - \delta$ model). For discrete time τ (the unit assumed is one year) it is defined as:

$$F(\tau) = \beta\delta^t \text{ (for } \tau = 1, 2, 3, \ldots) \quad \text{and} \quad F(0) = 1 \quad (0 < \beta < \delta < 1)$$

A discount factor between the present and one-time period later (β) is smaller than that between two future time-periods (δ). In the continuous time, the proposed model is equivalent to the linearly-weighted two-exponential functions (generalized quasi-hyperbolic discounting):

$$V(D) = A[w \exp(-k_1 D) + (1 - w) \exp(-k_2 D)]$$

where w, $0 < w < 1$, is a weighting parameter and k_1 and k_2 are two exponential discount rates $(k_1 < k_2)$. Note that the larger exponential discount rate of the two k_2, corresponds to an impulsive self, while the smaller discount rate k_1 corresponds to a patient self [13].

4 Mental models: self-control against impulsivity

Behavioral issues fit in each phase of the problem solving process, both if it is a single agent decision process or a multi agent one. Every individual choice is influenced by impulsivity and by all positive and negative biases derived from it. The impulsive choices derived from mental models, which are informal models, quickly constructed by problem solvers, which go on constantly during problem solving.

Mental models help us to relate cause and effect, but often in a highly simplified and incomplete way. They are always influenced by our preferences and our personal experiences. So they can be extremely limiting. This explains why emotions do not have always positive or negative effects on decision process and why impulsivity generates sometimes positive and sometimes negative effects.

Strotz proposed two strategies that might be employed by a person who foresees how his preferences will change over time [11]:

1) The "strategy of precommitment": a person can commits to some plan of action;

2) The "strategy of consistent planning": an individual take into account future changes in the utility function and reject any plan that he will not follow through. His problem is then to find the best plan among those she will actually follow.

Hyperbolic discounting predicted a number of mechanisms of self-control. However, the hyperbolic model, as well as the exponential one, is only a special case of interpreting reality. Common sense highlights how people, when are in front of identical short term opportunities, perform only sometimes self-control, independently of the use of one's Strotz strategy. In the setting of Multiple Selves Models, to control impulsivity, Thaler and Shefrin proposed a "planner-doer" model which draws upon principal agent theory [12]. They deal with an individual as if he contained two distinct psyches: one planner, which pursue longer-run results; and multiple doers, that are concerned only with short-term satisfactions, so they care only about their own immediate gratification (and have no affinity for future or past doers). For example, consider an individual with a fixed income stream, where which has to be allocated over the finite interval $(0, T)$. The planner would choose a consumption plan to maximize his utility function

$$V(Z_1, Z_2, \ldots, Z_T) \quad \text{subject to} \quad \sum_{t=1}^{t} c_t \leq Y$$

in which is a utility function of consumption level in t.

On the other hand, an unrestrained doer 1 would borrow $Y - y_1$ on the capital market and therefore choose $c_1 = Y$; the resulting consequence is naturally $c_2 = c_3 = \cdots = c_T = 0$. Such action would suggest a complete absence of psychic integration. The model proposes two instruments that the planner can use to control the behavior of the doers:

(a) he can impose *rules* on the doers' behavior, which operate by altering the constraints imposed on any given doer;

or

(b) he can use *discretion* accompanied by some method of altering the incentives or rewards to the doer without any self-imposed constraints [13].

5 Role of information in decision process

In many decision processes the information held by the agent and the way in which they are represented play an important role, above all in multi agent decision problems, in which all the people involved have their intrinsic mental models, intentions, expectations and cultural habits, and emotions of each agent can be contagious and influence group behavior, modifying their mental models.

In this process the way the interaction and communication is carried out becomes important and has an effect on the dynamics of the problem solving process. An OR process can get opposite results depending on the way the phenomenon is described and how the questions are phrased and graphs used. This can influence the behavior and preferences of the participants. As a result, we need to pay attention to the way we communicate.

In a multi agent decision model the influence of communication depends, first of all, on whether the information is absent or present.

6 False consensus effect for lack of information in a non-cooperative decision problem

As observed in social psychology, people with a certain preference tend to make higher judgments of the popularity of that preference in others, compared to the judgments of those with different preferences. This empirical result has been termed the *false consensus effect*. Consequently, as pointed out in several experiments, in a multi agent decision problem each decision maker overestimates his own opinion.

However, this effect becomes more pervasive when people lack necessary data to base their judgments [10] about the choice of other members of their own group, there are influences in opposite direction to a false consensus effect, while results of experiment are in line with a false consensus effect in all groups in which the information were implicit. This shows that most subjects are unwilling or unable to use information that is not handed to them on a silver platter [4].

As a consequence, in multi agent decision problem without information about others members and about the problem, the *false consensus effect* produces partial objectivity and incomplete impartiality [10].

Mathematical instrument used to describe strategic interactions, as a multi agent decision problem, is the theory of games, and a non-cooperative game can be assimilated to situations in which information about decision of other members of decision group is absent, so implicit. In this kind of interaction it is not possible to implement some precommitment to control the doer's actions (the impulsive part that represents the effects of emotions), as a consequence it is not possible to recognize the best choice on a rational base [7].

If we analyze a non-cooperative multi agent decision problem like the traditional *prisoner's dilemma*, on one temporal interval and with only two alternatives, we note that the agents achieve common decision, and this is the best strategy, because each doer wants to obtain the higher advantage which is the same and, for the false consensus effect, each one thinks that other make the same. The doer of each prisoner will choose the strategy of "do not confess". In the traditional version of the game, the police arrest two suspects (A and B) and interrogate them in separate rooms. Each can either confess, thereby implicating the other, or keep silent. In terms of years in prison the payoff for each strategy are these:

		Agent A	
		Confess (C)	Do not confess (NC)
Agent B	Confess (C)	5,5	10,0
	Do not confess (NC)	0,10	1,1

According to the theory of games, given this set of payoffs, in absence of information there is a strong tendency for each to confess (optimal decision in terms of Pareto), implying two rational players with consistent preferences. This creates the paradoxical situation that rational players lead to a poorer outcome than irrational players. Actually, when each player has to choose the best strategy every doer drives his agent to make decision that leads him a greater advantage, believing that the other will do the same due to the effect of the false consensus. Consequently, the decision made by each leads to optimal decision in terms of Pareto, because both have the same utility function and both doers choose the only action that is the best strategy.

However, it is just a coincidence that the two players have achieved a common strategy. In other types of non-cooperative problems this can not happen, with the result that you will never achieve a joint decision without a prior agreement, if there is no information. Consider, for example, a multi agent decision problem in which the agents set to save money to realize a common purchase. Even agent has a fixed income, Y_A and Y_B and a nonnegative level of saving, S_A and S_B. The planner of each agent choose the best strategy which maximize his utility function of saving (thinking for future), but the present doer of each agent wants to obtain the highest advantage now, so it would consume Y and therefore choose$= 0$, with a degree$= 1$. Indeed, the doers are impulsive, each one assigns weight$= 1$ to one preference and weight$= 0$ to all the others, thinking that everybody will make in the same way for effect of false consensus. In this case it is not possible obtain a common decision.

The plan made in advance by a group of agents (to realize a common purchase) is not feasible if they don't set some rule or some method to alter the incentives for the doers. This type of problem can be represented in the following scheme:

		Agent A	
		Save (S)	Do not save (NS)
Agent B	Save (S)	10,10	5,5
	Do not save (NS)	5,5	-10,-10

where the payoff represent the utility of each agent for each strategy.

According to the rational choice, the Nash equilibrium coincides with the best strategy (S, S). However false consensus effect and impulsivity lead each agent to the worst equilibrium, because utility functions of the agents are different among them (each agent prefers consumptions to savings). This causes the lack of consensus on a common decision.

In conclusion, in a non-cooperative multi agent decision problem, there are two situations:

1) the doers of each agent have the same preference and they will reach a common decision that is given by the unanimous choice (doers don't affect),

2) the doers have different preferences and do not assign any weight to the other preferences, so it is not possible to aggregate the preferences. Hence, we can affirm that in a non-cooperative decision problem it is only a chance obtaining a common decision.

7 Excess of consensus in a cooperative decision problem

According to Engelmann and Strobel's experiment there is no false consensus effect if representative information is highly prominent and retrievable without any effort. On the contrary, there is a significant effect in the opposite direction, indicating that subjects consider others' choices more informative than their own [4]. This is the overconfidence o "groupthink", a psychological phenomenon which can occur in highly trained cohesive groups. Hence, in the extreme case in which all is known in decision making process, the interplay between different subjects involves anyway other behavioral effects, as the excess of consensus, apart from influence of mental models and all behavioral effects of each individual.

In the OR field this kind of decision making process can be modeled with cooperative games where the rationality of the equilibrium choice is saved by the possibility of making an agreement among agents, which represents a pure rule to maintain self-control at later time in Thaler and Shefrin's model. Moreover with an arrangement the agents have explicit information about the choices of other members, so the lack of false consensus effect is in line with the result of Engelmann and Strobel's experiment. However, only the decision of one member will prevail, and this is influenced by the strength of each mental model. An example of cooperative game is a coordination game, when players choose the strategies by a consensus decision making process [7].

Consider the classic example of coordination game: the "battle-of-the sexes". In this game an engaged couple must choose what to do in the evening: the man prefers to attend a baseball game and the woman prefers to attend an opera. In terms of utility the payoff for each strategy is as follows:

		Man	
		Opera (O)	Baseball (B)
Woman	Opera (O)	3,1	0,0
	Baseball (B)	0,0	1,3

In this example there are multiple outcomes that are equilibriums: (B, B) and (O, O). However both players would rather do something together than go to separate events, so no single individual has an incentive to deviate if others are conforming to an outcome. In this context, a consensus decision making process can be considered as an instrument to choose the best strategy in a coordination game. A common final decision is achievable only if the man and the woman have explicit information, then only if there is cooperation [7]. If we follow the Thaler and Shefrin's model, we can analyze choices in a cooperative game in this way: at period-one the planner of each agent states his preference, which is the best strategy because the planner wants to maximize his utility function. Then, the influence of doers, that want to obtain an immediate gratification, can be avoided, because agents can enforce contracts through parties at period-one, what eliminate the problem of loss of self-control, because they eliminate all choices. However, only one will maximize his utility function and this is not known in advance because it depends on the different strength of each mental models.

8 Strategic communication modify behavioral effects

In a multi agent decision problem, information held by the participants can be wrong for two causes, not independent of each other:

1) *misunderstandings* and

2) *manipulations*.

In the first case, the different reaction of people at the same problem with the same information do not reflect people's lack of cognitive abilities but the way the situation is described in the communication. In the second case, there is the will to misrepresent the problem for self-interest, from one member of the group. Human cognitive processes relate strongly to motivational issues which interplay between people in social contexts. Self-interest is the primary cause of biases especially in participatory processes with multiple stakeholders. Self-interest is the driver of strategic behavior, which produce above all priming and framing effects. As a result, some biases can be unintentional consequences of cognitive limitations, others can be motivated by omissions or over or under emphasization of aspects, strategically or not [6].

9 BOR: Behavioral Operational Research

In considering the behavioral effects we should take a humble approach and accept the fact that we are not likely be able to produce a "perfect" model but still could find one that is useful. However, modeling is not about models only, but it matters how we choose the models and how we work with the models [6]. Creating a model to manage or solve problems is a process composed of many phases and human behavior moderates each stage of the process and mediates the progression through stages [5]. Hence the behavioral lens needs to be integrated in the practice of OR as an additional perspective. Behavioral Operational Research (BOR) considers the human impact on the process of using operational research (OR) methods in problem solving and decision support as well as using OR methods to model human behavior [6].

Not only decision makers but also modelers are subject to cognitive and motivational biases and the way the decision problems are framed. Moreover, the cumulative effects of biases in a modeling process can also result in path dependency (a phenomenon where the order in which steps are taken in the modeling process can have an impact on the resulting model). In large models the initial modeling choices can be very hard to change later and these can have a crucial impact on the path the modeling process will proceed. The loss aversion effect in decision making can also have an effect on modeling in general. Theoretically it can be equivalent to use and label variables as losses or gains but in the interpretation of the model results there can be a difference. A somewhat related effect is the so called *action bias* where people choose to foster improvement rather than prevent deterioration [6]. Also communication is an important part of modeling. The modeler should not only be focused only on the perfection of the accuracy of the model, the process and communication counts a lot too. Modelers should recognize the possibility of strategic behavior of the participants. Such behavior can mean, for example, the misrepresentation of preferences or data in an environmental participation process. Finally, also modelers are guided by self-interest. The purpose for which the model is developed is reflected in the parameters and scales as well as in the level of detail used. There is not a single valid model fitting every purpose [6]. Hence biases exist in each aspect of a problem solving process and in each phase of modeling of these processes, however finding ways to avoid them is an open research field.

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The Impact of the Financial Crisis on Earnings Management: Empirical Evidence from the Top 5,000 Non-Listed Stock Italian Companies

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Abstract

Account manipulation has been the subject of accounting discussions not only in the U.S. but across the world, especially during times of financial crises. This paper investigates the impact of the recent financial crisis on account manipulation probability by adopting the Beneish Model (1999, 2013) of eight performance ratios. The analysis has been conducted using the Top 5,000 Non-Listed Stock Italian Companies (Società Per Azioni) ranked by revenues during the time period 2005-2012. We use the AIDA Bureau Van Dijk database. We test the existence of earnings management (EM) within the Top Non-Listed Stock Italian Companies through a comparison between the pre-crisis period (2005-2008) and the crisis period (2009-2012). Our findings show that the number of firms with a higher likelihood of earnings manipulation decreases by 4.53% from the pre-crisis to crisis period. As a consequence, we argue that EM increases when the crisis is weak while EM decreases during the crisis period.

Keywords: Financial Crisis, Earnings Management, Earnings Manipulation, Transparency, Italian stock companies.

2010 AMS subject classification: 97M30.

1 Introduction

Company managers engage in account manipulation, including earnings management, to meet stakeholders' expectations resulting in financial reporting that may not fairly present the firms' operations. Moreover, Stolowy and Breton [28] contend that account manipulation can lead to inefficient capital markets. Extant accounting research [8], [3], and [16] states that executives' acknowledge the importance of meeting earnings to achieve targets (i.e. loss avoidance or analysts' forecasts) as well as recognize that earnings attainment represents a relevant motivation for accounting manipulation [29]. Stolowy and Breton [28] define account manipulation as management's discretionary decision to make accounting choices that may affect the transfer of wealth between companies, the company and capital providers, the company and managers or managers. One form of account manipulation is earnings management (EM).

The objective of this paper is to assess whether managers do manipulate accounts more often during the time of financial crisis than otherwise.

To this end we study the group of about Top 5,000 Non-Listed Stock Italian Companies, and we compute the eight ratios as defined by Beneish [5]. Beneish [4] finds that his eight ratios capture financial statement distortions and provide timely assessments of the likelihood of distortions[1] especially when considered in conjunction with management incentives. So, for each firm-year from 2005 to 2012, we compute the Beneish ratios and consider management's incentive. Then we group these observations as pre-crisis or crisis-period in order to assess whether companies have a high probability of EM or with a low probability for EM. That is, we compare the final scores across two different time periods: pre-crisis (2005-2008) and crisis-period (2009-2012), assuming 2009 as the year of financial crisis in the U.S. and worldwide.

Findings show that within the Top 5,000 Stock Italian companies (non-listed on the Italian financial markets), the number of firms with a higher likelihood of earnings manipulation decreased by 4.53% from pre-crisis to crisis periods. This means that financial crisis has had a positive impact by lessening earnings manipulation of the Top Stock Italian Companies. We believe that Italian firms have a greater propensity to manipulate and hide wealth creation during non-crisis periods to obtain tax savings and restrain the distribution of wealth. From the opposite point of view, it does not make sense for firms to

[1] We intend distortions as financial statement distortions which capture unusual accumulations in receivables (DSRI, indicative of revenue inflation), unusual growth of Sales (SGI), unusual growth of Selling, General and Administrative Expenses (SGAI), unusual capitalization and declines in depreciation (AQI and DEPI, both indicative of expense deflation), unusual propensity to borrow money (LVGI), deterioration of Gross Margin (GMI) and the extent to which reported accounting profits are supported by cash profits (TATA).

manipulate earnings in times of financial crisis, because there is less earnings in general.

Our analysis is conducted by adopting a reliable model of the likelihood of manipulation of accounts in order to assess the impact of the financial crisis on non-listed Italian stock companies' accounts. Moreover, this study is useful in assessing the reliability of the financial statements of Italian Stock companies. This analysis could also be helpful to banks and other lending and investing entities as it represents an additional tool useful to detect account manipulation and accounting fraud, and to reduce information asymmetry during the period of financial crisis. Finally, the results have implications for future researchers that study managements' incentives concurrently with security offerings.

We assess the impact of the financial crisis (by assuming year 2009 as the trigger point) on EM for the top non-listed Italian Companies sample ranked by sales revenues. We use the Beneish model [5] of eight performance ratios to predict the probability of fraud cases of these Italian companies. In explaining our analyses, the remainder of this paper proceeds as follow. Next we present a literature review of EM studies during the financial crisis followed by an identification of the performance indicators used to determine EM probability as developed by Beneish. Then we present our empirical analyses results of the Top Stock Italian Companies ranked by sales revenues and tests of these probabilities pre-crisis and during the financial crisis. We conclude with comments on our main findings and provide suggestions for further research.

2 Prior Literature and Hypothesis

2.1 Earnings Manipulation

Many accounting scholars have defined and associated earnings manipulation with accrual accounting. Earnings management (EM) has been defined by Schipper [27] as "a purposeful intervention in the external financial reporting process, with the intent of obtaining some private gain." Many scholars have debated the role of EM as resulting in misleading stakeholders about a firm's performance [16]. In this context, EM is an active manipulation of earnings towards a predetermined target [22]. However, our objective is not to argue the merits of accrual accounting. Rather we study EM as a means of achieving a target during non-financial crisis vs financial crisis times.

According to the prior literature, "accruals" are used as a means for EM adjustments that may result in adverse consequences. Accruals may be explained as the difference between cash flows and operating income and is computed as follows [15], [9]:

Accruals = Reported earnings – Cash flows from operations

Healy [15] and De Angelo [28] have used the above model to find evidence of income manipulation in a different setting, adopting non-discretionary accruals. Many accounting scholars have analyzed the relation between EM and accruals estimates driven by the advent of readily calculable EM metrics [18], [11], and policy concerns raised by influential accounting standard setters [24]. The relevant contribution provided by Jones [18] is based on a linear regression approach that uses non-discretionary accrual variables including sales revenue and property, plant and equipment.

Many studies have improved upon EM measurement models. Dechow et al. [11] updated the Jones model by providing the Modified Jones model which has become one of the most widely used models in earnings management research. The Modified Jones model includes an adjustment to sales based on the change in receivables. Peek et al. [23] have recently contributed by comparing abnormal accruals across different countries. By using the two accruals estimation models, the Modified Jones model and the Dechow and Dichev [10], they found that the accruals models exhibit considerable cross-country variation in predictive accuracy and power to detect earnings management.

Other authors stated that EM can be achieved by using accounting methods and estimates (i.e., an accrual-based manipulation) [1] or by undertaking transactions that make reported income closer to some target numbers, rather than maximizing the firm's discounted expected cash flows [26]. In addition, several studies have explored real earnings manipulation in the context of early debt retirements [14]. Some [25], [12], [30] have contributed to this literature by showing that EM can be undertaken through asset sales. In this context, Beneish [5] provides a contribution by concentrating on eight financial indicators (performance ratios), and demonstrating their ability to categorize companies in two different groups: potential and non-potential earnings manipulators.

2.2 Financial Crisis and Earnings Manipulation

One issue of the financial crises in general is the increase of uncertainty among lenders and investors about fundamental values of assets, which leads to a greater volatility in the market prices of assets [29]. According to Trombetta and Imperatore [29], a financial crisis can be defined as a sudden or gradual interruption in the ongoing functioning of financial markets. This situation of uncertainty increases the asymmetry of information and lenders progressively lose confidence in the accuracy of the information they have about borrowers [21], [13].

Under the conditions of financial crises, financial and capital market participants are more skeptical and the investors are willing to sell off their securities, sending a negative signal to the markets as well as to new potential investors who may be reluctant to invest. These investors could also require a higher return as a consequence of the higher levels of capital market risks. Both investors and creditors might have a less propensity to invest or lend money because of the higher probability of the counterpart's default.

Many scholars have discussed the impact of the financial crises on EM. Kasznik and McNichols [19] and Matsumoto [20] have provided a significant contribution by analyzing how executives carry out earnings manipulation policies in order to attain firms' targets and avoid, at the same time, the communication of bad earnings news to markets.

Bartov et al. [2] described how managers manage earnings in order to alter market's evaluation of firm's likelihood to survive and, hence, reduce the average cost of capital. Willekens and Bauwhede [31] and Huijgen and Lubberink [17] state that managers are less likely to manipulate earnings in a situation of stronger litigation risk in order to reduce the external exposure of the litigation. These results imply that during times of financial crisis, regulatory bodies may be more likely to closely regulate firms than in times of non-financial crisis. Therefore firms may be more likely to not manage earnings in financial crisis periods. In considering extant accounting literature, several possibilities are equally likely and we could expect either more or less EM during a financial crisis. Consequently, we consider it relevant for this debate to conduct an analysis of this relationship specifically within the Italian market. We apply the reclassified Beneish Model, also known as Manipulation Score [4], [5], [6], [7], in order to verify whether the impact of the financial crisis on EM is positive or negative during the time-period from 2005 to 2012. Hypothesis for our empirical analysis is stated as follows:

H1: On the one hand, on average, more firms will have a high probability of EM manipulation before the financial crisis: 2005-2008 than otherwise: 2009-2012. On the other hand, on average, fewer firms will have a high probability of manipulation during and immediately after the financial crisis: 2009-2012 than otherwise: 2005-2008.

3. The Beneish Model

The Manipulation Score [4], [5], [6], [7], is a mathematical model based on eight financial ratios used to identify whether a company has a significant likelihood of managing and manipulating its earnings. The variables are obtained from the firms' financial statements and linked together within a score that describes the rate of earnings manipulation and, consequently, the profile of a company as a "potential earnings manipulator." Beneish suggests using the value of -1.78 as a threshold to distinguish which firms have manipulated their earnings. The variables of the model follow (see the respective extended formulas in Appendix 1):

1. DSRI (Days Sales in Receivables Index). It is the indicator of revenue inflation that measures the days' sales in receivables compared to the prior year. A significant increase in days' sales in receivables means a disproportionate increase in receivables relative to sales that suggest revenue inflation. The higher increase in the DSRI the greater likelihood that revenues and earnings are overstated.
2. GMI (Gross Margin Index). The decrease of Gross Margin value can be a negative signal about a company's health and future incomes. A value higher than 1 suggests a deterioration of gross margin and can force managers to manipulate earnings. To sum up, the Gross Margin is related to the change in inventories and other production that can increase the likelihood of manipulation. Thus, Beneish assumes this variable specifically related to production costs and changes in inventory, which can cause earnings manipulation practices.
3. AQI (Asset Quality Index). The Asset Quality indicator is the ratio of non-current assets other than property, plant, and equipment (PPE) to total asset and measures the proportion of total assets for which future benefits are less certain. Beneish expects a positive relationship between AQI and earnings manipulation practices. The higher value of AQI the greater the propensity in deferring and capitalizing costs in order to increase earnings.
4. SGI (Sales Growth Index). "If growth companies face large stock price losses at the first indication of a slowdown, they may have greater incentives than non-growth companies to manipulate earnings" [5]. There would be a strong positive relationship between the growth of Sales and the likelihood of EM because managers may be more incentivized to manipulate earnings.
5. DEPI (Depreciation Index). The DEPI measures the ratio of the depreciation rate in year t-1 to the corresponding rate in year t. If the index is greater than 1, it indicates that the tangible assets are being depreciated at a slower rate. This suggests that the firm might be revising useful asset life assumptions upwards in a way to increase income. There would be a positive correlation between DEPI and the earnings manipulation.
6. SGAI (Sales, General and Administrative Expenses Index). This ratio shows the SGA Expenses in year t relative to the previous year. If there is a disproportioned increase in Selling, General and Administrative expenses compared to Sales Revenues, there would be a negative signal about a

company´s prospects. Beneish expects a strong positive association between the index and the likelihood of manipulation.

7. LVGI (Leverage Index). This ratio shows the Total Debt (Current and Long-term) in year t relative to the previous year. Beneish stated that LVGI was included to capture incentives in debt covenants for earnings manipulation.
8. TATA (Total Accruals to Total Assets). The value of Total Accruals, normalized by Total Assets, is a proxy used to assess the discretionary accounting choices undertaken by managers in order to practice manipulations. There would be thus a positive correlation between Accruals and the EM.

In summary, these ratios have a predictive function and focus on financial statement distortions which capture unusual accumulations in receivables (DSRI, indicative of revenue inflation), unusual growth of Sales (SGI), unusual growth of Selling, General and Administrative Expenses (SGAI), unusual capitalization and declines in depreciation (AQI and DEPI, both indicative of expense deflation), unusual propensity to borrow money (LVGI), deterioration of Gross Margin (GMI) and the extent to which reported accounting profits are supported by cash profits (TATA).

4. Data Collection and Model Reclassification

The analysis was conducted using the Top 5,000 Non-Listed Stock Italian Companies ranked by Sales Revenues during the time period 2005-2012. These companies have been selected based on meeting a sales revenue threshold of € 50 million and the resulting sample is exactly made of 4,898 companies.

Stock Italian Companies (with Sales Revenues > € 50 mln)	4,898	*100%*
Companies in liquidation sale and Companies with no more than 2 missing values	1,126	*23%*
Observed companies	**3,772**	***77%***

Table 1: Top Stock Italian Companies with Available Data 2005-2012

Table 1 illustrates the sample selection process. We gathered accounting data from the AIDA Bureau Van Dijk database of firm-year observations from 2005 to 2012. Since several financial data variables were not available from this database and some companies were in liquidation sale during the observation period, we eliminated all the firms with more than two years of missing values and those in liquidation sale during the above mentioned period. Then we attained the coverage percentage by dividing the number of companies included in the study (3,772) by the number of the entire sample (4,898) with sales revenue of at least € 50 million. The coverage is shown as shown in Table 1 is about 77%.

Beneish model has been developed within the US environment and given that there are many differences between U.S. GAAP and Italian Accounting standards, we propose a reclassification of the Beneish model by adapting the financial accounting data to the Italian scenario (see Appendix 2 - Indicators legend and Reclassification).

According to the Italian Accounting principles, “Selling, General and Administrative expenses” do not appear separately on financial statements, since their value would result from a classification of expenses by function (as provided for by U.S. GAAP), while Italian financial statements, according to the Civil Code, classify expenses and revenues by nature. For this reason, in this analysis, we use the neutral value equal to 1 for SGAI index since the Income Statement Reclassification, which follows the Italian GAAP, does not show Selling, General and Administrative Expenses .

We use the “full version” of the Reclassified Beneish Model (8M-Score) in order to monitor the impact of the financial crisis on EM before and after the financial crisis periods. Therefore, we expect for Italian firms a negative correlation between the financial crisis and the number of non-listed stock companies with a high probability of being manipulated.

The eight diagnostic tools have been reclassified according Italian GAAP (see Appendix 2) into the M-Score formula in order to achieve the final score that will be later compared to the threshold of -1.78 [7]. By applying the reclassified model, it is possible to categorize companies into two different groups: firms with a low probability of EM, and firms with a high probability of EM.

Manipulation Score = -4.840 + 0.920*DSRI + 0.528*GMI + 0.404*AQI + 0,892*SGI + 0.115*DEPI – 0.172*SGAI – 0.327*LVGI + 4.679*TATA

The final manipulation score for each firm is obtained by computing the average scores separately between the pre-crisis period (2005-2008) and the crisis period (2009-2012).

5 Main findings

Using on the list of available companies from the AIDA Bureau Van Dijk database, the Top 5,000 Italian Stock Companies ranked by Sales Revenues (see

Table 1) are 4,898 and among them 3,772 report the variables needed to develop the manipulation score.

	PRE-CRISIS (2005-2008)	**CRISIS (2009-2012)**
high probability of EM (N° of Companies)	1,929	1,758
low probability of EM (N° of Companies)	1,843	2,014
Total Companies	3,772	3,772
High probability of EM (% of Companies)	***51.14%***	***46.61%***
Low probability of EM (% of Companies)	***48.86%***	***53.39%***

Table 2: Probability of EM Pre-crisis and Crisis Periods

Table 2 illustrates the probability of EM during the pre-crisis and crisis periods. Using a threshold of -1.78 [7], 51.14% of companies have a high probability of manipulating earnings while the 48.86% have a low probability of EM in the pre-crisis period. With the starting of the financial crisis in 2009, there is a decrease in the percentage of companies with a high probability of EM (from 51.14 % to 46.61%) and an increased percentage of companies with low probability of EM (from 48.86% to 53.39%). That is the number of firms with a higher likelihood of earnings manipulation decreased from the pre-crisis to crisis period similar to our overall findings.

	BOTH PRE-CRISIS AND CRISIS PERIODS	**% OF TOTAL COMPANIES**
Companies with LOW probability of EM	1,426	***37.80%***
Companies with HIGH probability of EM	1,341	***35.55%***

Table 3: Number of companies with the same probability (high or low) before and after crisis

Table 3 highlights the number of companies with the same probability of EM consistently (either high or low) throughout the database period. Within the observed sample (3,772 companies) there are 1,426 companies that always have a low probability of EM both in the pre-crises and crisis periods (a percentage of 37.80% of the total companies), and 1,341 companies which have high probability of EM in both periods (a percentage of 37.80% of the total companies). This means that for these companies the financial crisis had no impact on increasing or decreasing their probability of EM.

	Pre-Crisis 2005-2008	**Crisis 2009-2012**	% OF TOTAL COMPANIES
Companies which manipulate only in PRE-CRISIS period	588		**15.59%**
Companies which manipulate only in POST-CRISIS period		417	**11.06%**

Table 4: Number of companies which changes probability (from high to low and vice-versa) from pre-crisis to crisis period

Table 4 illustrates EM results for those other companies of the sample that consistently manipulate accounts in the pre-crisis period different from the crisis period. Table 4 highlights that there are 588 companies which have a high probability of performing manipulated accounting data but only in the pre-crisis period (15.59% of the total companies) while 417 with a high probability of account manipulation only in the crisis period (11.06%). This means that for these 1,005 (588+417) or 26.64% of the companies studied, hypothesis H1 is confirmed.

Appendix 3 and 4 show the range of EM scores which is vast. Therefore we provide additional descriptive statistics both for the set of top 100 firms ranked by sales revenues and for the set of worst 100 firms based on sales revenues.

5.1 Top 100 and Worst 100 Firms ranked by Sales Revenues in the pre-crisis and crisis periods.

	Pre-Crisis 2005-2008	**Crisis 2009-2012**
Number of top 100 firms with a high probability of EM	**42**	**36**
Number of top 100 firms with a low probability of EM	**58**	**64**
Total number of top 100 firms	**100**	**100**

Table 5: Manipulation Scores on Top 100 by Sales

	Pre-Crisis 2005-2008	**Crisis 2009-2012**
Number of worst 100 firms with a high probability of EM	**51**	**52**
Number of worst 100 firms with a low probability of EM	**49**	**48**
Total number of worst firms	**100**	**100**

Table 6: Manipulation Scores on Worst 100 by Sales

Table 5 presents the EM scores of the Top 100 firms ranked by sales revenues and Table 6 presents the manipulation scores of the worst 100 firms ranked by sales revenues (included to the Top 5,000) in the pre-crisis and crisis periods. See Appendix 3 and 4 for more details on Tables 5 and 6.

The Top 100 Firms included in Table 5 show a decrease in the number of potential manipulators by 14.29% (from 42 to 36) but regarding the Worst 100 Firms (Table H) the number of companies with a high probability of being manipulated increases by 1.96% (from 51 to 52). These findings show that the average reducing percentage of potential manipulators between pre-crisis and crisis period has been impacted by financial crisis stronger for companies with higher level of revenues than for companies performing lower revenues.

We believe and discussed previously that during the pre-crisis period (2005-2008), there is a greater propensity for manipulating earnings in the Italian Market which has a tendency to hide the wealth creation through the income boost years to obtain tax savings and to restrain the distribution of wealth. From the opposite point of view, the EM policy has a tendency to decrease because the tax burden tends to decrease based on the natural reduction of earnings as a result of the crisis itself. This is to say that it does not make sense to manipulate earnings in times of financial crisis, because there are less earnings in general.

On the other hand, while the results of the specific analysis on the top 100 companies (by sales revenues) confirms our hypothesis, the results regarding the worst 100 companies showing a slight increase of the likelihood of EM during the crisis period, could be explained as a necessity of those firms to keep constant values of their main performance indicators, compared with those of previous periods, after that the crisis may have impacted too negatively on firm revenues and financial equilibrium.

6 Suggestion for further research

Suggestions for future contributions are based on expanding the data in terms of number of companies. For example, future studies could include analyses of all the Limited Italian Companies (*Società a Responsabilità Limitata*) as well as Partnerships (*Società di Persone*), and assessing the difference in EM between the two types of companies. It would be useful to focus on a multiple country-setting (EU-nations as well as no EU countries) in order to analyze the impact of crisis on EM in different contexts. Furthermore, it would be useful to consider other parameters in addition to sales revenues and ranking firms. For example, the sample could be analyzed base on differences in legal origin, whether IFRS or some other accounting standard is used, culture, market infrastructure or whether tax and financial reporting regulations are similar.

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Appendix 1: The Eight Indicators of Beneish Model

$$DSRI = \frac{(Receivables_t)/(Sales_t)}{(Receivables_{t-1})/(Sales_{t-1})}$$

$$GMI = \frac{(Sales_{t-1} - Cost\ of\ Goods_{t-1})/Sales_{t-1}}{(Sales_t - Cost\ of\ Goods_t)/Sales_t}$$

$$AQI = \frac{1-(Current\ Asset_t + PPE_t)/Total\ Assets_t}{1-(Current\ Asset_{t-1} + PPE_{t-1})/Total\ Assets_{t-1}}$$

$$SGI = \frac{Sales_t}{Sales_{t-1}}$$

$$DEPI = \frac{Depreciation_{t-1}/(Depreciation_{t-1} + PPE_{t-1})}{Depreciation_t/(Depreciation_t + PPE_t)}$$

$$SGAI = \frac{(SGA\ Expenses_t)/(Sales_t)}{(SGA\ Expenses_{t-1})/(Sales_{t-1})}$$

$$LVGI = \frac{(LTD_t + Current\ Liabilities_t)/Total\ Assets_t}{(LTD_{t-1} + Current\ Liabilities_{t-1})/Total\ Assets_{t-1}}$$

$$TATA = \frac{(Cur.\ Ass._t + Cash_t) - (Cur.\ Liab.t - Cur.\ Matur.of\ LTD\ t - Income\ Tax\ t\ ) - Depreciation\&Amortization\ t}{Total\ Assets_t}$$

Appendix 2: Indicators legend and Reclassification

Receivables consist of a series of short and long-term accounting transactions dealing with the billing of a customer for goods and services they have ordered. In AIDA they named as "Crediti vs Clienti entro 12 mesi ed oltre 12 mesi".

Sales are the act of selling a product or service in return for money or other compensation. In AIDA they named as "Ricavi di Vendite e Prestazioni".

Cost of Goods Sold is computed as "cost of beginning inventory + cost of goods purchased (net of any returns or allowances) – cost of ending inventory". In AIDA they named as "Costo del Venduto = Rimanenze Iniziali + Costo delle materie prime – Rimanenze Finali"

Current Assets consists of any asset reasonably expected to be sold, consumed, or exhausted through the normal operations of a business within the current fiscal year or operating cycle. In AIDA, they named as "Attivo Circolante".

PPE (Property, Plant and Equipment) consists of "Tangible Assets" that are included in Fixed Assets. In AIDA they named as "Immobilizzazioni Materiali".

Total Assets is computed as the sum of Current Assets and Fixed Assets. In AIDA, they named as "Totale Attivo".

Depreciation is the decrease in value of Tangible Assets (Property, plant and equipment) while "Amortization" is the decrease of Intangible Assets. In AIDA, they named as "Ammortamento dei beni materiali".

SGA expenses (Selling, General and Administrative expenses) is the sum of all direct and indirect selling expenses and all general and administrative expenses of a company. AIDA doesn't show this cost category. We assume the value of 1.

LTD (Long Term Debts) is the sum of all long term borrowings of a company. AIDA doesn´t show this cost category. In AIDA, the named as "Totale Debiti oltre l'esercizio".

Current Liabilities consists of all debts or obligations that are due within one year. In AIDA, they named as "Passivo Corrente".

Cash consists of Legal tender or coins that can be used in exchange goods, debt, or services. In AIDA, they named as "Totale Disponibilitá Liquide".

Current Maturity of LTD consists of the amount of LTD that expired within one year. This item is included in the general area of "Passivo Corrente". So that, "Passivo Corrente = Current Liabilities + Current Maturity of LTD".

Income Tax Payable comprised of taxes that must be paid to the government within one year. In AIDA, this is computed as "Imposte Correnti + Imposte Differite – Imposte Anticipate".

Depr.&Amort. are decrease in value of both Tangible and Intangible Assets. From AIDA, this is computed as "Ammortamento beni materiali + Ammortamenti beni immateriali".

Appendix 3: Ranking of Top 100 Stock Italian Companies based on Sales Revenue

# rank Sales	Companies' List (TOP 100 by SALES REVENUES)	code	MANIPULATION-SCORE per year									
			CRISIS PERIOD				Average	PRE-CRISIS PERIOD				Average
			2012	2011	2010	2009		2008	2007	2006	2005	
1	GESTORE DEI MERCATI ENERGETICI S.P.A.	8299	-2,54	-2,41	-2,74	-2,80	**-2,62**	0,46	-2,36	-2,32	5,26	**0,26**
2	GSE S.P.A.	3510	-2,35	-2,20	-2,79	-2,54	**-2,47**	-3,93	-2,17	5,24	-3,89	**-1,19**
3	KUWAIT PETROLEUM ITALIA S.P.A.	1920	-3,19	-2,10	-3,33	-3,04	**-2,92**	-3,70	-3,45	-3,11	-3,01	**-3,31**
4	ENEL ENERGIA S.P.A.	3511	-1,65	-1,68	-1,87	-2,49	**-1,92**	.	-2,06	-2,09	-4,68	**-2,95**
5	AU S.P.A.	3510	-2,28	-2,49	-2,14	-2,33	**-2,31**	-2,80	-2,75	-2,57	-1,90	**-2,50**
6	ENEL PRODUZIONE S.P.A.	3511	-3,46	-3,27	-3,52	-3,49	**-3,43**	-3,18	-3,36	-2,71	-2,58	**-2,96**
7	ENEL DISTRIBUZIONE S.P.A.	3510	-2,25	-2,96	-2,74	-2,87	**-2,71**	-4,23	-2,94	.	.	**-3,58**
8	ESSELUNGA SPA	4711	-3,16	-3,15	-3,65	-4,17	**-3,53**	-4,18	-3,80	-2,64	-2,09	**-3,18**
9	KRI S.P.A.	1920	-2,32	-3,08	-3,41	-3,60	**-3,10**	-3,22	-3,44	-3,86	-1,53	**-3,01**
10	GDF SUEZ ENERGIA ITALIA S.P.A.	7112	-2,81	3,44	-0,59	227,40	**56,86**	-3,84	-3,48	.	.	**-3,66**
11	TAMOIL ITALIA S.P.A.	1920	-3,37	-3,71	-3,92	-3,62	**-3,66**	-3,42	-3,21	-2,65	-2,18	**-2,87**
12	TRENITALIA S.P.A.	4900	-2,90	-3,37	-2,64	-2,21	**-2,78**	-2,83	-2,98	-1,89	-2,40	**-2,53**
13	WIND TELECOMUNICAZIONI S.P.A.	6100	-3,28	-3,13	-2,98	-3,15	**-3,13**	.	-3,01	-2,90	-2,45	**-2,79**
14	VERSALIS S.P.A.	2010	-2,19	-1,97	-0,87	-1,54	**-1,64**	-2,35	-1,80	-1,44	-1,59	**-1,79**
15	ENOI S.P.A.	3510	.	-1,69	-0,33	-1,75	**-1,26**	0,62	-0,15	-0,52	-0,67	**-0,18**
16	IREN MERCATO S.P.A.	3510	.	.	-2,17	-2,67	**-2,42**	-2,13	-1,26	3,08	-0,57	**-0,22**
17	AUTOSTRADE PER L'ITALIA S.P.A.	5221	-3,09	-2,82	-3,00	-2,43	**-2,84**	-3,11	-2,89	-2,60	-2,73	**-2,83**
18	GS SPA	4711	.	.	-3,51	-4,10	**-3,80**	-3,86	-4,19	-4,19	-3,53	**-3,94**
19	MARCEGAGLIA - S.P.A	2420		.	-1,45	-2,32	**-1,89**	-1,82	-0,99	-0,63	-1,11	**-1,14**
20	NUOVO PIGNONE S.P.A.	2829	-2,24	-2,71	-1,93	-1,80	**-2,17**	-2,93	-3,14	-3,14	-1,66	**-2,72**
21	IES ITALIANA ENERGIA E SERVIZI S.P.A.	1920	.	.	-3,32	-3,58	**-3,45**	-3,04	-2,74	-2,46	-2,33	**-2,64**
22	COSTA CROCIERE S.P.A.	4669	-3,67	-3,88	-3,84	-2,99	**-3,59**	-2,07	-2,19	-2,50	-2,49	**-2,31**
23	LOGISTA ITALIA S.P.A.	4635	-2,63	-2,75	-2,72	-2,55	**-2,66**	-2,63	-2,59	-3,16	-1,02	**-2,35**
24	AUCHAN S.P.A.	4791	.	-4,20	-3,74	-4,45	**-4,13**	-4,34	-3,07	-4,28	-3,70	**-3,85**
25	SEVEL-SPA	2910	.	2,59	-4,08	-2,47	**-1,32**	-4,00	-3,63	-3,75	-3,63	**-3,75**
26	SORGENIA S.P.A.	3510	.	-2,43	-1,73	-2,05	**-2,07**	-1,27	-2,05	-2,14	-1,15	**-1,65**
27	SAMSUNG ELECTRONICS ITALIA S.P.A.	4643	-0,02	-0,80	-1,02	-0,76	**-0,65**	.	.	-0,86	-0,74	**-0,80**
28	FERRERO - SOCIETA' PER AZIONI	1082	-2,89	-2,84	-3,05	-3,15	**-2,98**	-3,39	-3,05	-2,86	-1,88	**-2,80**
29	RAI - RADIOTELEVISIONE ITALIANA SPA	6020	-3,13	-2,58	-3,43	-2,27	**-2,85**	-2,55	-2,75	-2,66	-2,35	**-2,58**
30	CNH INDUSTRIAL ITALIA S.P.A.	2830	.	-2,69	-3,32	-2,58	**-2,86**	-3,19	-2,59	-2,42	-3,23	**-2,86**
31	BARILLA G. E R. FRATELLI - S.P.A.	1073	-2,22	-2,10	-2,86	-2,83	**-2,50**	-3,81	-2,77	-2,50	-2,89	**-2,99**
32	IBM ITALIA S.P.A.	6201	-2,33	-2,35	-2,32	-2,03	**-2,26**	-2,53	-3,36	-3,12	-3,34	**-3,09**
33	RFI S .P.A.	4900	-1,99	-1,98	-2,63	-2,80	**-2,35**	-2,39	-2,80	-2,66	-2,20	**-2,51**
34	AGUSTAWESTLAND S.P.A.	3030	-1,86	-2,08	-1,58	-2,06	**-1,89**	-1,51	.	.	-1,56	**-1,53**
35	MEDIAMARKET SPA	4719	-3,15	-3,39	-3,58	-3,61	**-3,43**	-3,64	-4,49	-4,00	-4,27	**-4,10**
36	SMA S.P.A.	4711	.	-3,62	-4,11	11,63	**1,30**	3,16	-4,71	-4,24	-4,18	**-2,49**
37	ACEA ENERGIA SPA	3510	-2,73	-2,18	-2,03	-2,39	**-2,33**	-1,98	9,18	4,77	-2,03	**2,49**
38	ABB S.P.A.	2790	.	.	-1,71	-1,73	**-1,72**	58,43	2,89	1,05	0,07	**15,61**
39	MERCEDES-BENZ ITALIA S.P.A.	4511	-2,65	-1,97	-2,15	-1,81	**-2,15**	-1,91	-1,72	-1,46	-1,51	**-1,65**
40	FERRARI S.P.A.	2910	-0,83	-1,49	-1,32	-2,76	**-1,60**	-1,82	-2,15	-2,11	-1,85	**-1,98**
41	A2A ENERGIA S.P.A.	3514	.	-0,48	-2,13	-1,32	**-1,31**	-0,84	-0,87	-1,62	-1,16	**-1,12**
42	COMIFAR DISTRIBUZIONE S.P.A.	4646	.	-1,66	-1,94	-1,87	**-1,82**	-1,96	-1,85	-1,90	1,65	**-1,01**
43	PUBLITALIA 80 S.P.A.	7312	-2,07	-1,58	-1,65	-2,21	**-1,88**	.	.	-1,23	-1,20	**-1,22**
44	PAM PANORAMA S.P.A.	4711	-3,16	-3,68	-3,78	-2,82	**-3,36**	-3,14	-2,80	-3,26	-3,01	**-3,05**
45	FIAT POWERTRAIN S.P.A.	2932	.	-4,36	-3,88	-3,68	**-3,97**	-5,23	-3,72	-3,16	-3,14	**-3,81**
46	BURGO GROUP S.P.A.	1712	-2,69	-2,52	-2,35	-2,51	**-2,52**	-2,40	-2,33	-2,98	-2,88	**-2,65**
47	BMW ITALIA SPA	4500	-1,78	-0,73	-1,45	-1,75	**-1,43**	-0,72	1,10	-0,21	0,01	**0,04**
48	IPLOM S.P.A.	1920	-2,63	-3,23	-2,04	-1,72	**-2,41**	-1,24	-2,62	-2,21	-2,30	**-2,09**
49	MICHELIN ITALIANA S.A.M.I.	2211	-3,41	-3,31	-2,14	-2,56	**-2,86**	-2,56	-2,70	-2,40	-2,13	**-2,45**
50	ITALPREZIOSI S.P.A.	4672	-3,12	-3,23	-3,01	-1,59	**-2,74**	-1,60	-5,08	-0,78	-3,17	**-2,66**

51	IPER MONTEBELLO S.P.A.	4711	-2,81	-2,94	-2,93	-3,08	-2,94	-4,35	-4,79	-4,49	-4,22	-4,46
52	METRO ITALIA CASH AND CARRY S.P.A.	4690	-2,43	-2,13	-3,02	-3,54	-2,78	-4,35	7,68	-2,56	-2,26	-0,37
53	GREEN NETWORK S.P.A.	3513	-2,71	-2,95	-1,04	-4,41	-2,78	-2,15	-3,42	-3,99	1,17	-2,10
54	CHIMET - S.P.A.-	2440	0,53	0,51	1,08	0,44	0,64	0,18	-0,81	-0,63	-1,06	-0,58
55	ACCIAIERIA ARVEDI S.P.A.	2430	-4,01	-3,99	-3,35	-3,39	-3,69	-3,56	-2,91	-2,97	-3,26	-3,17
56	BENNET S.P.A.	6810	-3,03	-3,02	-4,27	-3,89	-3,55	-3,08	-3,03	-3,39	-3,06	-3,14
57	FASTWEB SPA	6100	-2,70	-2,67	-3,18	-2,78	-2,83	-2,94	-2,79	-3,58	-2,26	-2,89
58	GIORGIO ARMANI S.P.A.	7410	-0,76	-1,24	-0,61	1,72	-0,22	-2,73	-2,55	-2,15	-2,30	-2,43
59	SIEMENS S.P.A.	2562	0,94	-0,58	-0,96	-1,24	-0,46	-1,37	-2,01	47,06	28,70	18,09
60	MERCK SERONO S.P.A.	2120	.	.	-1,46	-1,94	-1,70	0,53	0,13	0,92	-0,30	0,32
61	GDF SUEZ ENERGIE S.P.A.	3510	-2,70	-2,51	-2,90	-2,55	-2,67	-1,46	1,42	-0,94	-0,39	-0,34
62	REPSOL ITALIA S.P.A.	4730	-1,81	-1,53	-1,26	0,84	-0,94	-2,25	-0,15	-1,36	.	-1,25
63	ALPHA TRADING S.P.A.	4321	-2,84	-2,13	-2,76	-2,62	-2,59	-2,09	-2,54	-2,76	-2,23	-2,41
64	SHELL ITALIA E&P S.P.A	0620	-0,10	1,35	-2,27	-1,68	-0,67	-1,05	-0,87	-0,98	-1,32	-1,05
65	RENAULT ITALIA S.P.A.	4511	-2,19	-2,05	3,44	-3,03	-0,96	-2,08	-0,96	-0,47	-1,94	-1,36
66	SASOL ITALY S.P.A.	1920	-1,40	-0,72	0,02	-1,54	-0,91	5,62	-3,17	-2,61	-1,47	-0,41
67	CARLO COLOMBO S.P.A.	2400	-3,29	-3,28	-3,63	-0,90	-2,78	-3,19	-3,22	-1,93	-2,18	-2,63
68	SANOFI-AVENTIS S.P.A.	2120	-0,19	-0,12	-1,71	-1,17	-0,80	-2,37	0,49	0,90	1,06	0,02
69	ALSTOM FERROVIARIA S.P.A.	3020	.	-1,96	-1,83	-1,85	-1,88	-1,53	-2,09	-1,82	-0,75	-1,55
70	ERICSSON TELECOMUNICAZIONI - S.P.A.	2630	-1,78	-1,52	-1,74	-1,45	-1,62	-1,32	0,44	-0,10	0,21	-0,19
71	CALZEDONIA S.P.A.	4642	-1,83	-1,66	-1,10	-1,83	-1,61	-1,31	-1,70	-1,53	-1,18	-1,43
72	SPESA INTELLIGENTE S.P.A.	4711	-4,59	-3,49	-5,30	-4,70	-4,52	-4,32	-4,50	-3,92	-4,47	-4,30
73	SATA S.P.A.	2910	-1,61	-2,13	.	.	-1,87	-2,06	-2,23	-1,87	-1,15	-1,83
74	ALPIQ ENERGIA ITALIA S.P.A.	3510	-0,82	-1,45	-1,36	-1,85	-1,37	-1,01	-1,18	-1,77	-0,90	-1,22
75	SOCIETA' ITALIANA PER IL GAS PER AZIONI	3521	-2,56	-3,06	-0,26	-3,18	-2,27	.	-3,82	-1,83	-2,18	-2,61
76	E.ON ENERGIA S.P.A.	3523	-1,20	-2,41	-2,73	-0,72	-1,77	-2,25	-2,67	-1,94	.	-2,29
77	ITALIA MARITTIMA S.P.A.	5000	-3,11	-3,46	-3,83	-3,85	-3,56	-2,95	-3,84	-3,96	-3,53	-3,57
78	REPOWER ITALIA S.P.A.	3511	-1,10	-1,28	-1,77	-1,45	-1,40	-2,44	-0,69	.	.	-1,56
79	DALMINE SPA	2420	-1,14	-1,23	-1,29	-1,87	-1,38	-1,69	-1,48	-0,35	-1,10	-1,15
80	FORD ITALIA S.P.A.	4511	-0,55	0,18	-2,50	.	-0,96	.	-1,64	-0,23	-0,81	-0,89
81	NE.IT. S.P.A.	1000	-2,23	.	.	-1,62	-1,93	-1,87	-1,95	-1,77	-2,45	-2,01
82	TI SPARKLE S.P.A.	6100	-2,30	-2,62	.	.	-2,46	-0,76	-0,64	-1,00	-1,36	-0,94
83	TECNIMONT S.P.A.	7110	.	1,84	-1,50	-2,30	-0,66	4,90	-0,61	65,43	6,12	18,96
84	ACCENTURE S.P.A.	6201	-1,18	0,16	-0,27	-0,52	-0,46	0,68	0,07	-0,02	0,09	0,21
85	E.ON PRODUZIONE S.P.A.	7010	-0,47	-2,03	-1,49	-3,03	-1,76	-2,25	-2,89	-2,76	-1,86	-2,44
86	ARVAL SERVICE LEASE ITALIA S.P.A.	7711	-4,16	-4,31	-4,35	-4,11	-4,23	-4,37	-4,36	-3,84	-4,07	-4,16
87	UNICO LA FARMACIA DEI FARMACISTI S.P.A.	4646	.	-2,14	-2,01	-2,12	-2,09	-0,18	-1,68	-1,97	-1,86	-1,42
88	LAVAZZA S.P.A.	1083	-1,79	-1,51	-1,65	-1,76	-1,68	-1,36	-1,50	-1,25	-0,61	-1,18
89	BENIND S.P.A.	5229	-2,01	-1,86	4,19	-2,48	-0,54	-2,27	-1,87	0,07	-2,17	-1,56
90	HENKEL ITALIA S.P.A.	2010	-2,59	-2,11	-1,44	-2,00	-2,04	-1,01	-1,09	-1,39	-0,91	-1,10
91	BRT S.P.A.	4941	-2,69	-2,62	-3,11	-3,06	-2,87	-3,02	-2,98	.	.	-3,00
92	P.A.I. S.P.A.	4511	.	-3,56	1,49	-1,96	-1,34	-3,51	-1,21	-3,12	-2,59	-2,61
93	INDESIT COMPANY S.P.A.	2751	-2,87	-2,48	-2,41	-2,31	-2,52	-2,37	-2,92	-0,72	-3,05	-2,27
94	CITROEN ITALIA S.P.A.	2910	-3,39	-2,12	-2,51	-2,21	-2,56	-2,89	-2,29	-2,59	-3,00	-2,69
95	CONFIRMEC S.P.A.	4614	-2,06	-2,37	-2,36	-2,50	-2,32	-1,69	-1,71	-2,02	-1,92	-1,83
96	SOCIETA' AGRICOLA LA PELLEGRINA S.P.A.	0147	0,04	-1,46	-1,69	-2,10	-1,30	8,52	-0,29	12,38	3,56	6,04
97	ACEA ENERGIA HOLDING S.P.A.	3510	5,76	17,54	-2,26	-2,74	4,57	-3,60	-0,96	.	.	-2,28
98	NOVARTIS FARMA SPA	2120	-1,74	-1,26	-0,83	-0,59	-1,10	-0,71	-1,11	-1,07	-1,17	-1,02
99	TECHINT S.P.A.	7490	2,14	2,79	14,81	57,60	19,33	-4,41	-1,79	-1,95	-2,14	-2,57
100	MAGNETI MARELLI S.P.A.	2931	-2,97	-2,63	-2,49	-2,51	-2,65	22,47	-2,54	-1,71	-2,43	3,95

Specifications:

- Code values represent the industry in which each company operates according to the UK Standard industrial classification of economic activities (SIC) as updated in 2007;
- Values represents the Beneish score for each year while the average value is introduced separately for the pre-crises period (2005-2008) and for the crises period (2009-2012);
- Score values expressed in red font represent those higher than the Beneish threshold for high probability of EM (-1.78)

Appendix 4: Ranking of Worst 100 Stock Italian Companies based on Sales Revenue

# rank Sales	Companies' List (TOP 100 by SALES REVENUES)	code	MANIPULATION-SCORE per year									
			CRISIS PERIOD				Average	PRE-CRISIS PERIOD				Average
			2012	2011	2010	2009		2008	2007	2006	2005	
1	HONDA AUTOMOBILI ITALIA S.P.A.	4511	.	-2,81	48,07	-1,96	**14,43**	-2,10	-1,16	-2,33	-0,68	**-1,57**
2	ABC - ACQUA BENE COMUNE NAPOLI	3600	.	-1,81	-1,96	-2,07	**-1,95**	-2,03	-1,93	-2,29	-2,62	**-2,22**
3	SACER PETROLI S.P.A.	4671	.	.	-2,71	-2,62	**-2,67**	-2,19	-2,75	-1,74	-2,07	**-2,19**
4	V.AR.VIT. - VESCOVINI ARISTIDE VITERIE - BULLONERIE S.P.A.	4674	.	.	0,50	-1,53	**-0,51**	0,06	-0,02	-1,56	-0,62	**-0,54**
5	DURST PHOTOTECHNIK SPA % DURST PHOTOTECHNIK AG	2670	.	-0,77	-0,11	-1,38	**-0,75**	-0,93	-0,39	-0,39	-0,16	**-0,47**
6	HOSPAL S.P.A.	2660	.	.	-0,45	-1,09	**-0,77**	-1,13	-1,67	-1,70	-0,57	**-1,27**
7	ALESSI S.P.A.	2599	.	-0,95	-1,51	-1,90	**-1,45**	-1,57	-0,75	-0,55	-1,46	**-1,08**
8	KERSELF S.P.A.	4674	.	7,01	-5,53	-3,71	**-0,75**	-2,36	-0,52	0,00	-1,71	**-1,14**
9	RIVA FIRE S.P.A.	7010	.	-3,21	-3,37	-4,04	**-3,54**	-3,00	-4,02	-5,29	-2,46	**-3,69**
10	PROGETTO S.P.A.	4520	.	.	-2,95	-3,23	**-3,09**	-2,52	-2,48	-2,91	-2,57	**-2,62**
11	SIFER SPA	4672	.	-1,43	-0,78	-1,18	**-1,13**	-2,14	-2,40	-0,57	9,37	**1,07**
12	DOTT. FORMENTI - S.P.A.	2120	.	.	-1,84	-1,66	**-1,75**	-2,13	-1,77	-2,25	-2,22	**-2,10**
13	PROMATECH S.P.A.	2894	.	.	-3,77	-3,98	**-3,87**	-3,55	-3,48	-3,06	-2,35	**-3,11**
14	D.G.S. S.P.A.	4771	.	.	-3,23	-3,26	**-3,24**	-4,79	-4,53	-2,38	-3,92	**-3,90**
15	ABBOTT PRODUCTS SPA	2120	.	-3,08	-1,97	0,21	**-1,61**	-0,30	-0,67	5,19	-0,24	**0,99**
16	MONDADORI FRANCHISING S.P.A.	4649	.	.	-1,12	-0,63	**-0,87**	-0,64	-0,94	-0,05	-1,58	**-0,80**
17	OMVP S.P.A	2815	.	.	-2,46	-2,10	**-2,28**	-2,58	-1,85	-3,06	-2,88	**-2,59**
18	FITT S.P.A. SOCIETA' UNIPERSONALE	2016	.	-1,34	-1,31	-1,38	**-1,34**	-0,84	-0,36	-1,48	-1,90	**-1,14**
19	SONEPAR ITALIA SUD S.P.A.	4647	.	.	-2,40	-1,71	**-2,05**	-2,10	-1,72	-1,51	-1,50	**-1,71**
20	YKK ITALIA S.P.A.	3299	.	-0,56	1,20	2,02	**0,89**	-0,78	-0,03	-0,69	-0,50	**-0,50**
21	IGAP S.P.A.	3291	.	-1,91	-1,77	-1,88	**-1,85**	-1,23	-1,84	-1,57	0,58	**-1,01**
22	CENTOSTAZIONI S.P.A.	5221	.	-1,56	2,36	-2,05	**-0,42**	-2,83	-1,87	-1,69	-0,27	**-1,66**
23	OTIS SPA	6420	.	.	-3,92	-2,48	**-3,20**	-2,48	-2,97	-2,94	-2,53	**-2,73**
24	RAGALL S.P.A.	2453	.	.	-1,25	-1,11	**-1,18**	-1,71	-1,58	-0,13	-0,20	**-0,91**
25	CANESSA SPA	2562	.	-4,16	-4,82	-0,24	**-3,07**	-3,77	-1,67	-1,79	-1,83	**-2,26**
26	SALUMIFICIO FRATELLI RIVA S.P.A.	1013	.	.	-1,46	-1,62	**-1,54**	-1,72	-1,77	-1,84	-2,15	**-1,87**
27	MONDOLIBRI S.P.A.	4791	.	.	-1,17	-2,49	**-1,83**	-2,06	-1,73	-1,99	-1,25	**-1,76**
28	CELLULAR ITALIA S.P.A.	4652	.	.	-0,68	-1,17	**-0,93**	-1,53	-1,12	-0,71	-2,08	**-1,36**
29	OVIESSE FRANCHISING SOCIETA' PER AZIONI	7740	.	-1,53	-2,11	-1,58	**-1,74**	-2,41	-1,30	-1,24	-1,58	**-1,63**
30	CALCE S PELLEGRINO SPA	2352	.	-2,41	-2,03	-2,19	**-2,21**	-2,18	-1,89	-1,80	-2,09	**-1,99**
31	PASTIFICIO GUIDO FERRARA SPA	1073	.	-2,16	-2,66	-2,49	**-2,44**	-2,49	-2,21	-2,86	-3,53	**-2,77**
32	FONDERIE E OFFICINE MECCANICHE TACCONI S.P.A.	2450	.	-1,63	-1,82	-1,28	**-1,58**	-1,67	0,23	-0,58	-1,60	**-0,90**
33	SUMITOMO CORPORATION ITALIA S.P.A.	4619	.	.	-0,25	-1,02	**-0,64**	-2,16	-1,85	-1,39	0,98	**-1,11**
34	ESTEL OFFICE SPA	3101	.	-2,26	-1,85	-3,20	**-2,44**	-0,74	-1,24	-3,01	.	**-1,66**
35	INGEGNERIA BIOMEDICA SANTA LUCIA S.P.A.	2660	.	-2,08	-2,43	-2,49	**-2,33**	-2,14	-0,45	-2,21	-1,13	**-1,48**
36	JAGUAR ITALIA S.P.A	4511	.	.	-0,59	-1,65	**-1,12**	8,59	-0,49	-3,23	-3,03	**0,46**
37	BSL SPA	5229	.	-1,51	-1,87	-2,25	**-1,88**	-2,15	-2,76	-2,84	-1,58	**-2,33**
38	IN.CAM. S.P.A.	2592	.	.	-0,36	-0,79	**-0,57**	-0,35	-0,90	-1,37	-0,53	**-0,79**
39	FTM S.P.A.	4321	.	-2,50	-3,16	-2,93	**-2,86**	-2,21	-0,36	-2,19	-2,83	**-1,90**
40	FARMACEUTICI RINALDI SPA	4646	.	-0,03	-0,91	-0,79	**-0,58**	-0,22	6,28	-0,57	-0,26	**1,31**
41	SAINT - GOBAIN ISOVER ITALIA S.P.A.	2311	.	.	-1,83	-2,84	**-2,33**	-2,51	-1,48	-2,42	-1,98	**-2,10**
42	GENERALI REAL ESTATE S.P.A.	6832	.	-2,90	-1,12	-2,91	**-2,31**	-1,04	-0,64	1,48	10,20	**2,50**
43	MONTEBOVI SOCIETA PER AZIONI	4636	.	-3,34	-2,72	-3,00	**-3,02**	-2,26	-2,43	-1,86	-1,88	**-2,11**
44	ALUBERG S.P.A.	2511	.	-1,76	-1,63	-1,54	**-1,64**	-1,05	-0,83	-0,72	-0,62	**-0,80**
45	MAQUET ITALIA SOCIETA PER AZIONI	4646	.	-1,60	-1,47	-0,88	**-1,31**	-1,30	-1,02	-0,98	1,19	**-0,53**
46	TREVISANALAT SPA	1051	.	-2,89	-1,87	-1,29	**-2,01**	-2,21	-2,91	-2,86	-2,52	**-2,62**
47	DASTY ITALIA S.P.A.	2041	.	.	-0,99	-1,46	**-1,23**	-1,59	-2,18	-2,64	-1,90	**-2,08**
48	PAPERNET S.P.A.	1720	.	.	-4,60	-3,82	**-4,21**	-3,90	-3,58	-3,26	-4,13	**-3,72**
49	NATIONAL CAN ITALIANA (N.C.I.) - S.P.A.	2592	.	.	-1,70	-1,62	**-1,66**	-1,72	-2,13	-2,03	-1,84	**-1,93**
50	VG HOLDING S.P.A.	4778	.	-2,12	-2,20	-2,99	**-2,44**	-2,49	-2,64	-1,77	-2,25	**-2,29**

51	YKK MEDITERRANEO - S.P.A.	3299	.	-1,41	-0,14	3,98	**0,81**	-3,87	-3,57	-2,98	-3,58	**-3,50**
52	CONCERTA S.P.A.	5629	.	-1,81	-1,26	-0,79	**-1,29**	-0,94	-1,66	-2,55	-2,23	**-1,84**
53	OPERA21 S.P.A.	6202	.	-2,73	-2,52	0,03	**-1,74**	-2,05	-1,57	-2,31	-2,06	**-2,00**
54	ALDINET SPA	4643	.	-0,32	-1,80	-2,31	**-1,48**	-2,27	-3,55	-1,51	-2,02	**-2,34**
55	INNSE-BERARDI S.P.A.	2840	.	.	-2,63	-0,55	**-1,59**	-2,95	-2,07	-2,18	-3,10	**-2,57**
56	ISOGAS SPA	3523	.	-0,19	-0,93	-1,61	**-0,91**	-1,69	-2,02	-1,69	1,23	**-1,04**
57	CARL ZEISS SPA	4643	.	-1,38	-1,16	-1,39	**-1,31**	-1,22	-0,98	-1,33	-1,05	**-1,15**
58	BEAUTY POINT S.P.A.	4775	.	.	-2,96	-2,85	**-2,91**	-2,97	-2,72	-2,63	-2,88	**-2,80**
59	SAIP&SCHYLLER SPA	2220	.	-3,09	-3,16	-4,51	**-3,59**	-1,12	-2,52	-3,52	-4,12	**-2,82**
60	AUTOITALIA S.P.A.	4511	.	-2,75	-16,64	-1,37	**-6,92**	-1,41	-1,69	-3,59	0,88	**-1,45**
61	ALUPRESS SPA	2453	.	-1,08	-0,66	-1,47	**-1,07**	-0,48	-0,88	-1,52	-1,63	**-1,13**
62	INVENSYS SYSTEMS ITALIA S.P.A.	3320	.	-2,87	-3,22	-1,57	**-2,56**	-2,69	-3,38	-3,39	-2,63	**-3,02**
63	GOZZO IMPIANTI SOCIETA' PER AZIONI	2790	.	-2,29	-1,64	-2,66	**-2,20**	-1,97	-1,51	-1,71	-2,22	**-1,85**
64	SICES SPA	2820	.	-2,77	-2,60	-2,88	**-2,75**	-2,91	-3,52	-2,40	.	**-2,95**
65	CASA DOLCE CASA S.P.A	4673	.	.	-1,07	-1,35	**-1,21**	-1,82	-1,79	-1,35	-2,22	**-1,80**
66	OFFICINE FERROVIARIE VERONESI S.P.A.	3020	.	-1,78	-1,64	-3,00	**-2,14**	-1,63	-0,82	-1,54	1,75	**-0,56**
67	CAMAR S.P.A.	4511	.	.	-0,18	-1,85	**-1,02**	-2,76	-2,14	-2,26	-1,58	**-2,19**
68	SEPSA SPA SOCPER L ESERCIZIO DI PUBBLICI SERVIZI	4910	.	.	-2,68	-3,02	**-2,85**	-2,16	-2,19	-2,09	-1,89	**-2,08**
69	BIOMASSE ITALIA S.P.A.	3511	.	-4,04	-1,11	-6,03	**-3,73**	-4,29	-4,43	-2,89	-2,34	**-3,49**
70	MARZOLI S.P.A.	2894	.	.	-3,13	-3,48	**-3,30**	-4,03	-2,05	-2,48	-3,57	**-3,03**
71	REGGIANA ALIMENTARI SPA ABBREVIABILE IN REAL SPA	4711	.	-4,64	-4,02	-4,05	**-4,23**	-4,06	-4,93	-4,50	-4,04	**-4,38**
72	MALAVOLTA SPA	2511	.	-1,85	-0,83	-2,13	**-1,61**	-1,80	-1,35	-1,42	-1,83	**-1,60**
73	MIGLIORE SONEPAR S.P.A. - UNIPERSONALE	4669	.	.	-2,25	-2,63	**-2,44**	-2,66	-2,32	-2,42	-2,26	**-2,41**
74	FINI S.P.A.	2813	.	-2,43	-2,50	-2,18	**-2,37**	-2,80	-2,52	-2,46	-2,25	**-2,51**
75	FIN. AL - S.P.A.	2442	.	.	-1,40	-2,29	**-1,84**	-1,46	-1,79	-1,42	-1,83	**-1,62**
76	ANTICA FARMACEUTICA MODENESE S.P.A.	6420	.	35,33	0,47	-0,31	**11,83**	-0,32	-0,09	-0,30	-0,44	**-0,29**
77	COSTRUZIONI DONDI SPA	4311	.	-0,21	-2,05	-1,74	**-1,33**	-2,12	-0,60	-1,16	-1,28	**-1,29**
78	SADA SPA	4941	.	-3,27	-2,99	-3,35	**-3,20**	-2,86	-2,37	-2,38	-2,63	**-2,56**
79	EMERSON INDUSTRIAL AUTOMATION ITALY S.P.A.	2790	.	1,27	1,22	0,43	**0,98**	1,00	1,99	1,54	1,44	**1,49**
80	ISTITUTO GENTILI S.P.A.	2120	.	.	4,41	-0,57	**1,92**	-2,70	-0,56	1,27	-2,12	**-1,03**
81	FEP RIMONDI S.P.A.	4647	.	.	-1,01	-1,63	**-1,32**	-1,19	-1,53	-1,17	-1,31	**-1,30**
82	MOTIA COMPAGNIA DI NAVIGAZIONE S.P.A.	5000	.	-2,98	-2,71	-0,55	**-2,08**	-2,94	-3,17	-2,46	-1,40	**-2,49**
83	PIRCHER OBERLAND S.P.A.	1610	.	-1,25	0,02	0,54	**-0,23**	0,66	-0,07	0,41	0,08	**0,27**
84	DESMET BALLESTRA OLEO S.P.A.	2562	.	.	-2,77	-3,18	**-2,98**	-0,73	2,78	-0,13	-0,05	**0,47**
85	ITALEASE GESTIONE BENI S.P.A.	6810	.	-4,69	-1,77	-5,14	**-3,87**	-4,08	-5,87	-3,53	-1,45	**-3,73**
86	MAIR RESEARCH S.P.A.	3320	.	.	-1,61	-1,83	**-1,72**	-3,15	-2,61	-1,64	-1,08	**-2,12**
87	PARKER HIROSS S.P.A.	2829	.	.	-0,38	-1,08	**-0,73**	-0,58	0,10	0,24	-1,24	**-0,37**
88	TECHINT CIMIMONTUBI SPA	2562	.	.	-0,06	-1,16	**-0,61**	-1,10	-1,66	-3,78	-1,46	**-2,00**
89	M & Z RUBINETTERIE S.P.A. ABBREVIABILE IN	2814	.	-1,81	-1,93	-1,89	**-1,88**	-2,48	-2,29	-1,82	-1,98	**-2,14**
90	LI.SIT. S.P.A.	6201	.	.	-4,39	-4,36	**-4,37**	-5,33	-4,07	-2,83	0,39	**-2,96**
91	GRUPPO BONIFACI SPA	6810	.	-4,62	-2,81	3,38	**-1,35**	-3,83	43,82	9,91	.	**16,63**
92	ZIMMERHOFER SPA % ZIMMERHOFER AG	4100	.	.	-2,93	-2,71	**-2,82**	0,37	-2,17	-2,94	-0,94	**-1,42**
93	RODRIQUEZ CANTIERI NAVALI SPA	3011	.	-1,20	-2,13	-2,92	**-2,08**	-2,78	-3,48	-3,54	-0,63	**-2,61**
94	SIEMENS HOLDING S.P.A.	7010	.	.	-4,47	-4,02	**-4,25**	-4,52	2,24	-4,01	-1,86	**-2,04**
95	SEMPLICE SPA	7740	.	-2,85	4,66	-1,47	**0,11**	-2,19	0,84	-0,86	-0,69	**-0,72**
96	I CASTELLANI S.P.A.	2331	.	.	1,52	-3,13	**-0,81**	-1,33	-1,72	-1,84	-1,92	**-1,70**
97	ORECCHIA S.P.A.	6499	.	.	-2,82	19,01	**8,09**	-0,39	-1,98	-2,14	-2,08	**-1,65**
98	CALA CONTAINER SHIPPING S.P.A.	6820	.	.	-0,48	-1,23	**-0,86**	13,93	-1,39	-3,43	-4,90	**1,05**
99	ORSI MACCHINE TESSILI - S.P.A.	4660	.	-2,44	-3,90	4,30	**-0,68**	4,98	-3,32	-4,23	-0,35	**-0,73**
100	BINDA SPA	7010	.	.	-3,31	-2,14	**-2,73**	-3,82	8,93	-1,33	-1,54	**0,56**

Specifications:

- Code values represent the industry in which each company operates according to the UK Standard industrial classification of economic activities (SIC) as updated in 2007;
- Values represents the Beneish score for each year while the average value is introduced separately for the pre-crises period (2005-2008) and for the crises period (2009-2012);
- Score values expressed in red font represent those higher than the Beneish threshold for high probability of EM (-1.78)

Beneish M-score and detection of earnings management in Italian SMEs

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Abstract
Accounting literature on the reliability of financial information presents several mathematical models whose purpose is to identify the existence of values manipulations. The phenomenon is described as earnings management and presents a broad discussion concerning the search for suitable models to measure the distortions in values. In this respect, the present paper aims to compare the ability of two versions of the same mathematical model of classify the risk of earnings manipulation in a discriminant way.

Keywords: Beneish M-score; accounting ratios; regression analysis; earnings management

2010 AMS subject classification: 03H10; 62P20; 91B02; 91G70.

1 Introduction

The paper aims to compare the ability of two Beneish models, the M-score5 and the modified M-scoreIt, to detect earnings management. These methods have not been evaluated by prior research, and it is unclear which type of model dominates, as each models relies on the same assumptions and only empirically we can verify which of them is more descriptively valid. Davidson, Stickney and Weil (1987) [1] define the earnings management as the process by which managers, staying within accepted accounting principles, try to get at a certain desired level of profit to be marked on the outside. Healy and Wahlen (1999) [2] state that earnings management occurs when "managers use their own judgment in reporting the financial data and in structuring transactions in order to alter financial reports to deceive stakeholders on the fundamental economic performance the company or to influence the consequences of contracts that depend on accounting data reported". This perspective focuses on the matter to the judgment of the managers in the definition of financial data. Technically, earnings management activities include a spectrum of activities ranging from conservative accounting fraud through aggressive accounting and the neutral, through a wide range of accounting choices [3]. There are several ways in which managers can apply judgment to influence the financial reports. For example by means of the estimates that relate to the final value and duration of a certain good, or about possible future expenses are not yet done. For this reason this phenomenon is linked to the discretionary-accruals components present in the financial statement. Literature on earnings management has extensively examined a set of models to estimate discretionary accruals. These models range from the simple mathematical equation, in which total accruals are used as a measure of discretionary accruals, to sophisticated regression models, which decompose accruals into discretionary and nondiscretionary components and aim to forecast the presence of fraud and financial distress. Conversely, other models consider only a set of interrelated accounting ratios, comparing the values among several years in order to find some abnormalities. Attention to earnings management policies comes from the social and financial consequences which produces the distortion of information on the financial results of the company. Famous scandals of major companies are proof, for that reason more than thirty years research on mathematical methods able to adequately identify the phenomenon showed continued growth. Prior studies concluded that managers use discretionary accruals to convey their private information to investors, examining the time-series of discretionary-accruals (Hansen, 1996) [4] or the association between stock returns, discretionary accruals and nondiscretionary earnings [5]. Several studies are focused on listed firms or on financial statements based on US GAAP. In this study we observe a sample of 99 Italian academic spin offs, with homogeneous activity and omogeneous accounting rule system. These firms are mainly small and medium and not listed, for these reasons all the statistic models linked to market price of equity, stock volatility and US GAAP principles may be inadequate in detecting earnings management practices. In the next section we describe the main attribute of academic spin offs, followed by the concept and the consequences of earnings management and from the properties of Beneish model. Descriptive statistics, the comparative calculation and the regression analysis will be presented in section 5.

Finally we drawn some conclusion about the ability of two versions of Beneish model in detecting earnings management within Italian SME.

2 Academic spin-offs: an overview

The current improvement of the spin-off phenomenon in Europe has provided a treasured approach to spread new technologies and knowledge [6], driving up the business prospects for the academics and other players involved in projects directed to increase the outcome of the university scientific research [7;8]. Simultaneously, the spin-off process from a parent organization, especially from universities, has recently received growing attention both from the academic literature [9;10;11;12] and in the practice [13]. Furthermore, thanks to their capability in generating wealth and inspiring the development of scientific knowledge, policy-makers have showed an emergent interest in the academic spin-offs, considering them an active tool to encourage the development of knowledge-based economies in different institutional settings [14;15] so that their creation has become a crucial matter for policy-makers all around the world [16]. This is also due to the fact that either academicians, policy-makers either practitioners agree about the role played by universities as one of the main sources of innovations and their successful diffusion in the society [17;18]. Indeed, several scholars [19;20;21; 22] underline that the formation of a firm by a research institution is an outstanding method to commercialize the outcomes of the public research, as well as in contributing to the economic and social welfare and to the regional development. Scholars usually highlight the eminence of the foundation and diffusion of knowledge by universities as a noteworthy driving force for technological innovation in an economy, both at local and at national level [23]. The existing literature remarks that the new model of "open innovation", embraced by numerous organizations with the aim to contribute to the dissemination of knowledge, [24;25] has become a critically method in cooperating either with New Technology-Based Firms (NTBFs) either with scientific foundations, such as academics spin-offs, which provide new research settings and a multidisciplinary approach for the development of innovation processes [26;27]. Academic spin-offs (ASOs) are firms generated in order to exploit knowledge originated within universities. More specifically, the current literature defines academic spin-offs as “those companies that germinate from a University, where a group of researchers composes the entrepreneurial unit aiming at the exploitation of skills and results from the research developed within the University” [28] or “company composed by individuals who were former employees of the parent organization, and where the technology and the academic inventors may spin-off both from the institution, or where the technology spins out from the institution but the academic inventor is employed in the University, or, lastly, where only the technology spins out, while the

academic inventor does not maintain relationships with the new firm but may have equity" [29].
The establishment of the knowledge/technology employed by an academic spin off is a multi-stage process. Generally, literature identified three main models of academic spin-off creation and development. In the first model, Ndonzuau, Pirnay, and Surlemont [30] recognized four central stages in the growth of academic spin-offs: i) creating a sustainable business idea, ii) converting the idea into a business process, iii) building a firm and iv) contributing value to customers, employees, investors and all other stakeholders. These four stages are reciprocally dependent, since choices made in the earlier stages may effective influence the later stages. The second model, developed by Shane [7], embraces five stages in typifying a distinctive process to build an academic spin off. The first state is merely academic but the model also allows for tangential technologies that have the prospective to easily enable the development of new products. In cases where the researcher considers that their new technology is an invention which can be commercialized, then, they reveal it to the Technology Transfer Offices (TTO). Next, in the third stage, the prospective for intellectual property protection is estimated and a patent application may be made. Based on the limited monopoly via the patent, the TTO can either license the technology to a foundation firm or the researcher may start an academic start-up. Moving from the models by Ndonzuau et al. and Shane, Vohora, Wright and Lockett [31] provided a new perspective on the expansion of academic spin offs. Their model also has five stages, but it stresses four pivotal junctures that must be overlapped before transitioning to the next stage: i) Research (Opportunity recognition), ii) Opportunity framing (Entrepreneurial commitment), iii) Pre-organization (Threshold of credibility), iiii) Re-orientation, iiiii) Threshold of sustainability (Sustainable returns). Considering the above arguments, it is worthwhile to observe that the awareness demonstrated by literature in the success factors and supporting mechanisms of university entrepreneurship, through ASOs, has increased in the last years [15]. Indeed, several scholars [32;33] deal with the elements fostering their creation and growth, which are classified into different categories. A first category refers to the institutional supporting measures [26] such as government laws, financial and non-financial incentives. A second type is associated to university policies [34] such as business plan competitions, spin-off regulations, university business incubators. The third, instead, refers to the external critical factor of the spin off activity [35] such as, for example, entrepreneurial support mechanisms, venture capital, science parks, proximity to parent organization and prospects available from industry. Finally, a fourth type is related to the technology features [36], e.g. the prospective of commercialization, the appropriability and the value to customers. Often scholars [37] associate features affecting the growth dynamics of academic spin-offs with three different levels of analysis, employed to

investigate the phenomenon with a more comprehensive approach: micro, meso and macro levels. Regarding the first level of analysis, the macro one, the focus is on the national systems of innovation and, above all, on the role that policy-makers may have in the foundation of academic spin-offs [26;32]. Hence, the studies on the creation of academic spin-offs focus on the occurrence of venture capitalists, legal protection of innovations, regional infrastructures and on the business environment in which the universities are regulated but, at the same time, they are less interested in what is happening within the parent organization, which is the university. That's the reason why the theoretical framework that helps to explain the effects generated at the macro level of analysis – particularly as regards the NTBFs, of which the ASOs are a specific typology – is the *Knowledge Spillover Theory of Entrepreneurship* applied to the regional context [38]. As for the meso level of analysis, this is focused on the study of university and the TTO and tries to identify the fostering mechanisms or factors by which universities promote the effective creation/development of academic start-ups [39;40], as well as, it tries to explore the success of spinning out processes such as a university technology transfer mechanism. Frequently, the theoretical framework used to carry out this type of analysis is the *Resource-Based View*, according to which internal factors define or influence the formation of academic spin-offs. Lastly, the last level of analysis, the micro one, concerns the role played by the individual characteristics of the entrepreneurs or the managerial team, jointly with their social ties, in encouraging the spinout foundation process. In this case, the theoretical framework of reference involves the field of *Entrepreneurial Theories* [41;42], which studies the individual characteristics, in conjunctions with the *Resource-Based View* which explores the personal assets influencing and affecting the foundation of the academic spin-offs [43]. Following a theoretical approximation, the first two levels of analysis can be attributed to those ones which the literature defines university fostering mechanisms of academic entrepreneurship [26;32;35], while the second may be included in those contextual elements which form the local context factors in the development of the academic spin-offs [15].

3 Earnings management in SMEs

Earnings management can be loosely defined as a strategy of generating accounting earnings, which "is accomplished through managerial discretion over accounting choices and operating cash flows" [44]. It occurs when managers use judgment in financial reporting and in structuring transactions to alter financial reports to either mislead some stakeholders about the underlying economic performance of the company or to influence contractual outcomes that depend on reported accounting numbers [45]. Earnings management is an umbrella for acts that affect the reported accounting earnings or their interpretation, starting from production and investment decisions that partly determine the underlying economic earnings, going through the choice of accounting treatment and the size of accruals when preparing the periodic reports, and ending in actions that affect the interpretation of the reported earnings. Not all earnings management is misleading. Investors, for example, prefer to separate persistent earnings from one-time shocks. Firms that manage earnings in order to allow investors to better distinguish between the two components do not distort earnings. On the contrary, they enhance the informational value of their reported earnings. Thus, depending on the will to signal of hide the short or long term performance, it can be beneficial, pernicious or neutral [46]. The studies usually relate the level and type of the earning management adopted by firms with the interests of the key players on the financial accounting scene, which can be grouped into three main categories: management, users and gatekeepers or monitors [46]. Management reports earnings, users use earnings as an input to their decision making, and gatekeepers provide valuable signals to other users regarding the credibility and the informational value of the reported earnings [47]. The literature about earnings management has mainly explored the effect of these key players in large firms because financial information published by these firms is easily accessible. Large companies are generally listed companies with publicly available financial information while SMEs are subject to less demand for financial information. In the last years. the literature is focusing on the level and type of earnings management in SMEs as a result of intuition that firm size affects the incentives to this practice. There should be a little interest of management in managing earnings in SMEs for its own advantages because small companies are less subject to agency problems, especially when shareholders and managers are the same people, like in family firms. However, incentives to manage earnings also exist in SMEs when the company needs external financing, for example from banks. Also tax purposes are often advanced to explain accounting choices in small firms, especially when alignment between financial and tax reporting is high [48]. On one hand, several studies tried to explain the objectives of financial reporting in SMEs. Lavigne [49] shows that, according to the managers of Canadian SMEs, financial reporting respond to both internal management and tax purposes. He shows that structural factors, such as firm size, ownership structure and debt also influence accounting policies. In the same context of Canadian firms, Maingot and Zeghal [50] find that the objectives of financial reporting are linked to taxes and debt. The performance of the firm can also influence financial reporting. Saboly [51] shows that managers of small distressed firms can manage earnings to influence stakeholders. In Australia, McMahon [52] finds that financial reporting quality in SMEs is associated with firm size, but not with performance and growth. On the other hand, literature has also focused on the issue of earnings management's intensity and typology in comparative terms between SME's and large firms. Moses [53] finds evidence that large firms have a bigger incentive to smooth earnings than small firms and Michaelson, James, and Charles [54] also find consistent evidence. Differently, Albrecht and Richardson [55] find evidence that large firms have less incentive to smooth earnings than small firms. Burgstahler and Dichev [56] analyze the impact of earnings management on the company's losses, in a sample of 300 companies and the results show that large firms and small ones manage their earnings in order to avoid small losses or small profits decline. Rangan [57] finds a significant relationship between earnings management and performance of experienced equity offerings. He suggests that older and largest firms were maneuvering the current accruals to exaggerate the earnings of the experienced equity offerings. Degeorge, Patel, and Zeckhauser [58] indicate that large companies manipulate the earnings of the company to avoid the negative earnings. Lee and Choi [59] also find that firm size is a variable that could influence a firm's tendency to manage earnings: smaller firms are more likely to manage earnings to avoid reporting losses than larger firms. Barton and Simko [60] show that big companies face more influence to get the analysts' demands to manage earnings more effectively. Nelson, Elliott, and Tarpley [61] showed that sometimes auditors might ignore the earnings management of large sized firms. He argues that, since audit fees increase with client size, the probability of adjustments in the financial statements by the auditor becomes lower when increasing the client size. Ching, Firth, and Rui, [62] examine that whether unrestricted current accruals forecasted the returns and earnings performance and resulted that larger firms manipulate current accruals to overstate earnings than the small sized firms. Siregar and Utam [63] find inconsistent evidence with regard to the impact of firm size on type of earnings management while Persons [64] analysis of frauds reveals evidence of more fraudulent activity in smaller firms. The contributions above outlined testify that literature do not converge towards a homogeneous scenario and demonstrate that there is still much to say about SME's propensity to earnings management.

4 Beneish Manipulation-Score for Italian ASOs

Literature on earnings management examines the amount of discretionary and non discretionary accruals within the financial statement, considering these values the main sources of manipulation. The pioneering Healy [65] contribution assumes that profits derive from a cash part and accruals, the increase of which denotes the presence of a not really cashed income and hence more maneuverable. Accruals include revenue and expenditure that have taken place in a certain period, but that did not generate a cash flow during the same period. Discretionary accruals are measured as the accruals that cannot be explained by a change in sales and the level of fixed assets, thus, their measure will capture changes in any number of expenses, some revenues, and changes in various working capital accounts. Marquardt and Wiedman [66] demonstrate that firms issuing equity manage accruals by increasing revenue and decreasing depreciation expense. In other researches [67] emerge that changes to pension assumptions, inventory method, depreciation method and estimates, as well as LIFO liquidations are used to manage earnings. Other researches associate manipulation of results sudden adoption of more favourable credit terms, the increase in product inventories, the increase in discretionary spending such as research and development, advertising and maintenance [68]. As a result of the earnings management research the analyst will understand that some firms manipulate accounting numbers to manage earnings and that the vehicles chosen for manipulation vary in predictable ways. Other than the earnings number, however, it is not known in any given context which numbers are likely to have been manipulated. DeAngelo *et al*. [69] state that abnormal changes in accruals between one year and the other are associated with intentional distortion of income, related to the managers' desire to increase their profit margins in order to achieve their goals. There are different models that estimate accruals, based on statistic index or accounting ratios. The most popular models are the DeAngelo Model (1986), Healy Model (1985), Jones Model (1991) and the Modified Jones Model (Dechow, Sloan, and Sweeney 1995), the Industry Model [45], the Cross-Sectional Jones Model [70] and the Beneish M-score [71]. The first seven models attempt to measure the earning manipulation through the ratio between the discretionary and non discretionary accruals and three of them, the Industry Model, the Healy Model and the Jones Model, are estimated over an eight-year period ending just prior to the event year. In this analysis we use the Beneish model adapted to Italian SMEs by Giunta, Bini and Dainelli [72], which consider the disparate effects on accruals played by the Italian accounting principles. Beneish M-Score is a mathematical model that adopts some financial metrics to identify the extent of a company's earnings. This model observes the value alteration phenomena in non-listed companies, where value emerges mainly from the financial statements. The original *Manipulation score* (M-score) includes an intercept and eight variables that capture the financial statement distortions that can result from earnings manipulation or that indicate a predisposition to engage in earnings manipulation [73]. One advantage of the M-score is that the treatment sample consists of firm that have indeed managed earnings and that determination is independent of abnormal accrual models [71]. The formula is as follows:

1) M-score8= -4,840 + 0.920DSRI + 0.528GMI + 0.0404AQI + 0.892SGI + 0.115DEPI – 0.172SGAI - 0.327LVGI + 4.679TATA

Days Sales in Receivables Index (DSRI) measures the ratio of days that sales are in accounts receivable in a year compared to that of a prior year and an index higher than 1 describes the increased percentage of non cash sales compared to the prior year. A disproportionate increase in accounts receivable may be indicative of inflated revenues. Gross Margin Index (GMI) measures the variation of gross operating margin and when it's greater than 1 shows that the profit has worsened in the period under review with the consequence that the firm is likely to manipulate its revenues. Asset Quality Index (AQI) is the ratio of current (CA) and non current asset (property, plants and equipments-PPE) to total assets in one year to a prior year. An increase in AQI index may represent additional expenses that are being capitalized to preserve profitability [71]. Indeed, an index greater than 1indicates that the firm has potentially increased its cost deferral or increased its intangible assets, implementing a potential earnings manipulation. Sales Growth Index (SGI) is a measure of growth in revenue and if it's greater than 1 there is a positive growth in the year under review. Callen et al. [74] show that the likelihood of revenue manipulation is increasing with the credit loss ratio, leverage and with the volatility of equity returns and with the ratio of accounts receivable to sales. Depreciation Index (DEPI) is the ratio of depreciation expense and gross value of PPE in one year over a prior year. An index above 1 could be a reflection of an upward adjustment of the useful life of PPE. Leverage Index (LVGI) measures the ratio of total debt to total assets, describing the long-term risks of a company. An index of greater than 1 is interpreted as an increase in the gearing of the company and for that matter exposed to manipulation. Total Accruals to Total Assets Index (TATA) measures the quality of cash flows of the firms. The total accruals metric is computed as change in current assets (except cash and equivalent) less depreciation and the current portion of debts. An increasing degree of accruals as part of total assets would indicate a higher chance of manipulation. Another version of the index was empirically derived from the University of Lille with another European companies samples [72]. In this case only 5 variables were significant for the purpose of earning manipulation. The formula assumes the following definition:

2) M-score5 = -6,065 + 0.823DSRI + 0.906GMI + 0.593AQI + 0.717SGI + 0.107DEPI

Empirically, when the M-score5 is greater than -2.22 is high the probability of earning manipulation. Some of these variables (DSRI, GMI and TATA) describe the firms' ability to generate cash and profits from their business operations. Two of them (SGI and LVGI) try to capture the company's skills and motivations that could lead to the manipulation of accounting rules. Finally, the others (AQI, DEPI and SGAI) evaluate investments in assets of the firm and the ability to control costs. The application of models based on the estimation of accounting parameters affected by accounting principles applicable in Italy requires a revision in the calculation of the indicators and in their selection. Applying the initial formula to a sample of Italian listed companies, Giunta et al. [73] found a large number of false positives and a predictive power of less than 47%. For this reason the model has been adapted to the Italian system, dominated by SMEs who base their financial statements on the Civil Code rules, on national accounting principles (OIC) based on the principle of prudent estimates of costs and provisions. Readjustment affected the structure and the number of variables and related weights. SGI and TATA indicators were removed considering their low significance in the sample for the earning manipulation event. Therefore, the formula that we could consider for Italian SMEs is the following [72]:

3) M-scoreIt = - 6,2273 + 0.448DSRI + 0.1871GMI + 0.2001AQI + 0.2819DEPI + 0.6288LVGI

The variable weights were estimated using the maximum likelihood analysis, starting from a sample of manipulative society compared with a control group of non-manipulative firms. In this case, the cut-off value for M-score is -4.14. Giunta et al. (2014) shows that with this value the model reduces the errors for false positive at level 7.14% and correctly identifies the 92% of manipulations. Table 1 describes the formula for each variable considered for M-score; in this analysis we compare M-score5 and M-scoreIt.

Table 1 – Variables description

Code	Name	Formula
DSRI	Days Sales in Receivables Index	(Accounts receivables$_t$/Sales$_t$) / (Accounts receivables$_{t-1}$/Sales$_{t-1}$)
GMI	Gross Margin Index	[(Revenues $_{t-1}$ – Costs of Goods sold $_{t-1}$)/ Revenues $_{t-1}$] / [(Revenues $_t$ – Costs of Goods sold $_t$)/ Revenues $_t$]
AQI	Asset Quality Index	{1 – [(CA $_t$ +PPE $_t$)/Total assets $_t$] } / {1 – [(CA $_{t-1}$ +PPE $_{t-1}$)/Total assets $_{t-1}$]} *where* CA = Current assets PPE = property, plant and equipment
DEPI	Depreciation Index	[Depreciation and amortization$_{t-1}$ / (Depreciation and amortization$_{t-1}$ + PPE $_{t-1}$)] / [Depreciation and amortization$_t$ / (Depreciation and amortization$_t$ + PPE $_t$)]
LVGI	Leverage Index	(Total Debts t / Total Assets t) / (Total Debts t-1 / Total Assets t-1)
SGI	Sales growth Index	Revenues$_t$ / Revenues$_{t-1}$

5 Research model and results

In order to analyse the effects of Beneish model in signalling the manipulative firms, the research observes a sample of Italian academic spin offs born in 2004 and 2005 and existing until 2015, taken from the database of national network of Italian academic spin offs and patents (Netval). This analysis considers the performances during the period 2009-2010, just after the beginning of the financial crisis, that is considered a pivotal event for earning managements. Data were collected through Infocamere database (the national register of Italian companies), AidaBvdep system and from company websites. We excluded the inactive firms, those with no financial statements after the 2010, distressed firms and others in liquidations. The final sample includes 99 firms, around the 12% of those academic spin offs existing on Netval database in 2010 and 66% of those born in 2004-2005. The variation index of the net income in the period $t\text{-}t_{-1}$ is the proxy used to estimate the manipulation risk. Descriptive statistics in Table 2 show the higher volatility of GMI and SGI indicators that affect the value of M-score5. The mean value of In table 2 we compare the M-score5 model with M-scoreIt. Always for SGI index, the median value exceeds the unit, showing for it a high associated risk of earnings manipulation related to the revenues management.

Table 2 – Descriptive statistics

	No.	MIN	MAX	MEAN	STDDEV	MEDIAN
DSRI	99	0	16.61	0.744572	1.957968	0
GMI	99	-83.2	4268.56	42.71517	429.198	0.743818
AQI	99	0	18.45	1.643234	2.580883	0.991328
SGI	99	0	4913.6	50.7929	493.7183	1.079366
DEPI	99	0	3.11	1.043452	0.53463	0.927252
LVGI	99	0	5.9	1.086904	0.711245	0.984005
M-score5	99	-76.7	7385.16	70.75233	742.7017	-3.27986
M-scoreIt	99	-18.41	793.47	3.405278	80.24861	-4.73452
VarProfit t/t-1	99	-8979.53	26.95	-0.03542	0.396172	0.450448

Table 3 – Comparative analysis for M-score

	High Risk	Low risk
M-score5	33%	66%
M-scoreIt	31%	69%

According to model based on 5 variables, the 33% of the sample presents a high risk of earnings manipulation, while the M-scoreIt identifies a lower number of potentially manipulative firms, despite it assumes a lower threshold value. Thinking about possible sources of bias, we may assume that the variable with the greatest impact on the difference of the two scores is associated with fluctuations in sales revenue (SGI), considering the high standard deviation that takes in the sample. The reasons can be adduced both to the fact that SGI is not scaled by total assets, as happens for the other and also for the nature of academic spin offs. In fact, the instability in sales is quite common and frequent in these firms, whereas many of them have to wait long periods before concluding the development of research and bring to market the goods obtained. However, the gap between the two indices is rather small, is to be concluded that the classification to which they lead is quite similar, therefore emerges not a significant contribution from the M-scoreIt model in discriminating manipulative companies compared to the M-score5 based on accruals. Considering that the Beneish M-score is a probabilistic model, its limit is that the ability to detect potential fraud is not with 100% accurancy. For this reasons in this analysis we consider only the risk of profit manipulation, linking the variation of net income to the M-score variables, examining the linear regression as follows:

4) $VarProfit = \beta_0 + \beta_1 DSRI + \beta_2 GMI + \beta_3 AQI + \beta_4 DEPI + \beta_5 LVGI + \beta_6 SGI + \varepsilon_i$

The stepwise procedure (Table 4) shows that only the AQI is significant to explain the variation of net income in the period observed. AQI in the sample assumes a mean value greater than 1 and a median value close to 1 that could indicate that the academic spin offs have potentially increased the deferred cost. The negative coefficient in the regression analysis shows that when the firms increase the capitalization of cost related to intangible assets, such as R&D costs, the variability of profit decreases between one period and another, leaving to hypothesize that the budgeting of costs related to R&D could ensure a certain stability in the level of profit. Therefore, the systematic capitalization of these deferred costs would allow to homogenize the income levels over time, leaving to assume the existence of an earnings management policy.

Table 4 – Regression analysis

Model		non-standardized coefficients		standardized coefficients		
		B	std.error	Beta	t	Sig.
1	Costant	64,384	102,790		,626	,533
	AQI	-92,196	33,884	-,264	-2,721	,008

Stepwise selection: prob F in <=0,050; prob F out>=0,100

Model	**R**	**R-Square**	**R-Square Adj**	**Std. error**
1	,264(a)	,070	,060	866,29650

a. (Constant), AQI

6 Conclusion

Literature on earning management has largely focused on methods able to detect manipulative companies, minimizing classification errors, considering that the inadequacy of the calculation method can lead to important social and economic consequences. If on one hand the statistic accrual prediction models neglect some operational dynamics of the company and don't describe in a significant way the phenomena when the samples are small, on the other hand, the accounting models are less stringent, and built on the basis of accounting standards adopted in selected countries. This paper assumes that in earnings management analysis is important consider the contingent features of the business, of corporate governance, the economic situation and the specific accounting rules of each country. These items affect the business trend of the firms, influencing the accounting policies and favouring opportunistic behaviour. Applying the Beneish M-score model to a sample of Italian SME, in order to detect earnings management, rather than forecasting fraud and financial distress, we didn't found deep differences between the adjusted version of M-scoreIt and the simplified model for the European firms. Regression analysis also confirmed that the typicality of economic activity, from which descend the investment decisions, is the most effective on the variability of profit margins so for the purpose of detection of earnings management should be considered also expressive variables of this situation. An appropriate weighting system could adequately quantify the impact of sectoral differences, as well as the company size, then the complexity of corporate governance.

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