

Influence of information on behavioral effects in decision processes

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Abstract

Rational models in decision processes are marked out by many anomalies, caused by behavioral issues. We point out the importance of information in causing inconsistent preferences in a decision process. In a single or multi agent decision process each mental model is influenced by the presence, the absence or false information about the problem or about other members of the decision making group. The difficulty in modeling these effects increases because behavioral biases influence also the modeler. Behavioral Operational Research (BOR) studies these influences to create efficient models to define choices in similar decision processes.

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1 Introduction

Some failures occur when dealing with traditional Discounted Utility (DU) model from both normative and descriptive setting. Indeed, some studies, especially in psychology and neuroeconomics (a more specialized field of decision neuroscience), point out anomalies that violate some axioms in the traditional model (Sec. 2).

Bechara et al. [1] show positive effects of anomalies in intertemporal choices and the use of hyperbolic delay discounting (declining as the length of the delay increases) to represent inconsistent preferences. On the other hand, the negative

effects of emotions mainly stem from impulsivity. To properly describe individual differences in intertemporal choices, derived from impulsivity and inconsistency, behavioral economists have proposed the Q -exponential Delay Discount Function and a Multiple Selves Model (quasi-hyperbolic discount model) (Sec. 3).

Mental models of each person, based on different assumptions and preferences, influence the effects of emotions (positives and negatives) and impulsivity. To control impulsivity Strotz [11] proposed two strategies that might be employed by a person who foresees how his preferences will change over time, and Thaler and Shefrin [12] proposed a model in which the individual is treated as if he contained two distinct psyches denoted as *planner and doer* (Sec. 4).

The information held by the agent plays an important role on the delineation of his mental model. In a multi agent decision context all people involved have their mental model and influence other mental models. Hence, in these strategic decisions information about others and about the problem definitely influence the final choice (Sec. 5). If there is no information about other players, as shown in an experiment of Engelmann and Strobel [4], people weight their own decisions more heavily than that of a randomly selected person from the same population (false consensus effect). This happens in non-cooperative decision problems, not properly modeled in OR. Indeed, because false consensus effect and impulsivity not always lead each agent towards the best strategy according to the theory of games, so obtaining a common decision is only a chance (Sec. 6). On the contrary, when all information is explicit people can consider the choices of others as more informative than their own (excess of consensus or overconfidence). An example is a cooperative decision problem, modeled by OR with cooperative games, in which final decision is based on mental models of the participants and their tendency to overconfidence (Sec. 7).

However, the final decision is influenced not only by intrinsic characteristics of every one, but also by the way in which information is passed: misunderstandings and manipulations (above all for self-interest) change people's reactions (Sec. 8).

At last, also the modeler is influenced by his mental models: creating a model to predict a decision making process is itself a decision making process. A new branch of research (BOR, Behavioral Operational Research) studies human impacts of using OR models in decisional processes (Sec. 9).

2 Effects of behavioral aspects: violations of traditional Discount Utility Model

Operational Research (OR) has modeled human behavior in intertemporal choice in terms of DU model, which assumes an exponential temporal discount-

ing function and a constant discount rate: this represents the individual's pure rate of time preference. An important implication of constant discount rate and exponential discounting function is that a person's intertemporal preference is time-consistent.

However, decision neuroscience, whose goal is to integrate research in neuroscience and behavioral decision-making, highlights that there are a number of behavior patterns that violate rational choice theory. Several empirical studies on individual behavior, when discounting real or hypothetical rewards, stress the existence of violations of the traditional discounting model [2].

Theory and algorithms of OR models are free of behavioral effects but as soon as we use them in real life problem solving behavioral effects will be present. Hence behavioral perspective is essential in decision analysis [6]. Research in psychology has reported many types of cognitive and motivational biases as well as heuristics which relate to human behavior and may significantly distort the decision analysis generating inconsistent preferences in intertemporal choices. Delay effect, magnitude effect and sign effect are among the relevant anomalies in intertemporal choice, we will deal with (see also [13]).

The delay effect. As waiting time increases, the discount rates tend to be higher in short intervals than in longer ones. We can set out this effect as follows:

$$(x, s) \sim (y, s) \text{ but } (x, s + h) < (y, t + h), \text{ for } y > x, s < t \text{ and } h > 0$$

The magnitude effect. Larger outcomes are discounted at a lower rate than smaller outcomes. This effect can be formulated as follows:

$$(x, s) \sim (y, s) \text{ implies } (ax, s) < (ay, t), \text{ for } y > x > 0, s < t$$

and

$$(-x, s) \sim (-y, s) \text{ implies } (-ax, s) > (-ay, t)$$

The sign effect. Gains are discounted at a higher rate than losses of the same magnitude. This anomaly implying that, changing the sign of an amount from gains to losses, the weight of this amount increases:

$$(x, s) \sim (y, s) \text{ implies } (-x, s) > (-y, t) \text{ for } y > x > 0 \text{ and } s < t$$

3 Effects of emotions: violations of traditional Discount Utility model

In a series of studies (see, e. g., ([1], [3], [8])) using a gambling task, it emerges that individuals with emotional dysfunction tend to perform poorly compared with those who are endowed with intact emotional processes. Bechara et

al. [1] demonstrated that normal people possess anticipatory SCRs (*Skin Conductance Response*) – indices of somatic states – which represent unconscious biases that are linked to prior experiences with reward or punishment and produce inconsistent preferences. These biases alarm the normal subject about selecting a disadvantageous course of action, even before the subject becomes aware of the goodness or badness of the choice he is about to make. As a consequence there is considerable agreement among psychologists and economists that the notion of exponential discounting should be replaced by some form of hyperbolic discounting, which can point out the delay effect (or present bias), that is the tendency of the individuals to increasingly choose a smaller-sooner reward over a larger-later reward as the delay occurs sooner in time.

Many authors proposed different hyperbolic discount functions, in which temporal discount function increases with the delay to an outcome. One of these proposed functions has the following form:

$$d(t) = \left(\frac{1}{1 + \alpha t} \right)^{\beta/\alpha}$$

where $\beta > 0$ is the degree of discounting and $\alpha > 0$ is the departure from exponential discounting. Hyperbolic discounting has been applied to a wide range of phenomena, including consumption-saving behavior. Consistent with hyperbolic discounting, people's investment behavior exhibits patience in the long run and impatience in the short run [13].

A second type of empirical support for hyperbolic discounting comes from experiments on dynamic inconsistency. Studies and empirical evidences show that delay effect can derive in preference reversal between two rewards as the time-distance to these rewards diminishes. A hyperbolic discount model can clarify this; in fact, hyperbolic time-preference curves can cross [11] and consequently the preference for one future reward over another may change with time [13].

However, in some contexts individuals deprived of normal emotional reactions might actually make better decisions than normal individuals, because of the loss of self-control, as Damasio found when studying behavior of people with ventromedial prefrontal damage [3]. Temptations are manifestations of loss of self-control and in many cases induce disadvantageous behavior. Indeed, as far as temptation increases the best long run interest of the problem solver conflicts with his short run desires, moreover impulsive behavior may fail to evaluate the consequences of his behavior appropriately [13].

Other evidences suggest that even relatively mild negative emotions that do not result in a loss of self-control can play a counterproductive role among normal individuals in some situations. When gambles that involve some possible loss are presented one at a time, most people display extreme levels of risk aversion toward

the gambles, a condition known as myopic loss aversion. Shiv et al. [9] show that individuals deprived of normal emotional reactions might, in certain situations, make more advantageous decisions than those not deprived of such reactions; so the lack of emotional reactions may lead to more advantageous decisions.

Inconsistent preference is the greatest contradiction of rational theory in intertemporal choice. This behavior can be typically seen in psychiatric disorders (alcoholism, drug abuse), but also in more ordinary phenomena (overeating, credit card debt) [13]. Neuroeconomics has found that addicts are more myopic (have large time-discount rates) in comparison with non-addicted population.

However, the preference for more immediate rewards per se is not always irrational or inconsistent; addicts' behavior is clinically problematic, but economically rational when their choices are time-consistent (if they have large discount rates with an exponential discount function). But addicts also discount delayed outcomes hyperbolically, suggesting the intertemporal choices of addicts are time-inconsistent, resulting in a loss of self-control: they act more impulsively at the moment of the choice, against their own previously intended plan. Moreover if large discount rates are due to habitual drug intake, it is expected that discount rates decreased after long-term abstinence.

Behavioral neuroeconomics and econophysical studies have proposed two discount models, in order to better describe the neural and behavioral correlates of impulsivity and inconsistency in intertemporal choice.

Q-exponential discount model. This function has been proposed and examined for subjective value $V(D)$ of delayed reward:

$$V(D) = \frac{A}{\exp_q(k_q D)} = A/[1 + (1 - q)k_q]D^{\frac{1}{1-q}}$$

where D denotes a delay until receipt of a reward, A the value of a reward at $D = 0$, and k_q a parameter of impulsivity at delay $D = 0$ (q -exponential discount rate) and the q -exponential function is defined as:

$$\exp_q(x) = (1 + (1 - q)x)^{\frac{1}{1-q}}$$

The function can distinctly parametrize impulsivity and inconsistency [13].

Quasi-hyperbolic discount model. Behavioral economists have proposed that the inconsistency in intertemporal choice is attributable to an internal conflict between “multiple selves” within a decision maker. As a consequence, there are (at least) two exponential discounting selves (with two exponential discount rates) in a single human individual; and when delayed rewards are at the distant future (> 1 year), the self with a smaller discount rate wins, while delayed rewards approach

to the near future (within a year), the self with a larger discount rate wins, resulting in preference reversal over time. This intertemporal choice behavior can be parametrized in a quasi-hyperbolic discount model (also as a $\beta - \delta$ model). For discrete time τ (the unit assumed is one year) it is defined as:

$$F(\tau) = \beta\delta^\tau \text{ (for } \tau = 1, 2, 3, \dots) \quad \text{and} \quad F(0) = 1 \quad (0 < \beta < \delta < 1)$$

A discount factor between the present and one-time period later (β) is smaller than that between two future time-periods (δ). In the continuous time, the proposed model is equivalent to the linearly-weighted two-exponential functions (generalized quasi-hyperbolic discounting):

$$V(D) = A[w \exp(-k_1 D) + (1 - w) \exp(-k_2 D)]$$

where w , $0 < w < 1$, is a weighting parameter and k_1 and k_2 are two exponential discount rates ($k_1 < k_2$). Note that the larger exponential discount rate of the two k_2 , corresponds to an impulsive self, while the smaller discount rate k_1 corresponds to a patient self [13].

4 Mental models: self-control against impulsivity

Behavioral issues fit in each phase of the problem solving process, both if it is a single agent decision process or a multi agent one. Every individual choice is influenced by impulsivity and by all positive and negative biases derived from it. The impulsive choices derived from mental models, which are informal models, quickly constructed by problem solvers, which go on constantly during problem solving.

Mental models help us to relate cause and effect, but often in a highly simplified and incomplete way. They are always influenced by our preferences and our personal experiences. So they can be extremely limiting. This explains why emotions do not have always positive or negative effects on decision process and why impulsivity generates sometimes positive and sometimes negative effects.

Strotz proposed two strategies that might be employed by a person who foresees how his preferences will change over time [11]:

- 1) The “strategy of precommitment”: a person can commits to some plan of action;
- 2) The “strategy of consistent planning”: an individual take into account future changes in the utility function and reject any plan that he will not follow through. His problem is then to find the best plan among those she will actually follow.

Hyperbolic discounting predicted a number of mechanisms of self-control. However, the hyperbolic model, as well as the exponential one, is only a special case of interpreting reality. Common sense highlights how people, when are in front of identical short term opportunities, perform only sometimes self-control, independently of the use of one's Strotz strategy. In the setting of Multiple Selves Models, to control impulsivity, Thaler and Shefrin proposed a "planner-doer" model which draws upon principal agent theory [12]. They deal with an individual as if he contained two distinct psyches: one planner, which pursue longer-run results; and multiple doers, that are concerned only with short-term satisfactions, so they care only about their own immediate gratification (and have no affinity for future or past doers). For example, consider an individual with a fixed income stream, where which has to be allocated over the finite interval $(0, T)$. The planner would choose a consumption plan to maximize his utility function

$$V(Z_1, Z_2, \dots, Z_T) \quad \text{subject to} \quad \sum_{t=1}^t c_t \leq Y$$

in which is a utility function of consumption level in t .

On the other hand, an unrestrained doer 1 would borrow $Y - y_1$ on the capital market and therefore choose $c_1 = Y$; the resulting consequence is naturally $c_2 = c_3 = \dots = c_T = 0$. Such action would suggest a complete absence of psychic integration. The model proposes two instruments that the planner can use to control the behavior of the doers:

- (a) he can impose *rules* on the doers' behavior, which operate by altering the constraints imposed on any given doer;

or

- (b) he can use *discretion* accompanied by some method of altering the incentives or rewards to the doer without any self-imposed constraints [13].

5 Role of information in decision process

In many decision processes the information held by the agent and the way in which they are represented play an important role, above all in multi agent decision problems, in which all the people involved have their intrinsic mental models, intentions, expectations and cultural habits, and emotions of each agent can be contagious and influence group behavior, modifying their mental models.

In this process the way the interaction and communication is carried out becomes important and has an effect on the dynamics of the problem solving process. An OR process can get opposite results depending on the way the phenomenon is described and how the questions are phrased and graphs used. This

can influence the behavior and preferences of the participants. As a result, we need to pay attention to the way we communicate.

In a multi agent decision model the influence of communication depends, first of all, on whether the information is absent or present.

6 False consensus effect for lack of information in a non-cooperative decision problem

As observed in social psychology, people with a certain preference tend to make higher judgments of the popularity of that preference in others, compared to the judgments of those with different preferences. This empirical result has been termed the *false consensus effect*. Consequently, as pointed out in several experiments, in a multi agent decision problem each decision maker overestimates his own opinion.

However, this effect becomes more pervasive when people lack necessary data to base their judgments [10] about the choice of other members of their own group, there are influences in opposite direction to a false consensus effect, while results of experiment are in line with a false consensus effect in all groups in which the information were implicit. This shows that most subjects are unwilling or unable to use information that is not handed to them on a silver platter [4].

As a consequence, in multi agent decision problem without information about others members and about the problem, the *false consensus effect* produces partial objectivity and incomplete impartiality [10].

Mathematical instrument used to describe strategic interactions, as a multi agent decision problem, is the theory of games, and a non-cooperative game can be assimilated to situations in which information about decision of other members of decision group is absent, so implicit. In this kind of interaction it is not possible to implement some precommitment to control the doer's actions (the impulsive part that represents the effects of emotions), as a consequence it is not possible to recognize the best choice on a rational base [7].

If we analyze a non-cooperative multi agent decision problem like the traditional *prisoner's dilemma*, on one temporal interval and with only two alternatives, we note that the agents achieve common decision, and this is the best strategy, because each doer wants to obtain the higher advantage which is the same and, for the false consensus effect, each one thinks that other make the same. The doer of each prisoner will choose the strategy of "do not confess". In the traditional version of the game, the police arrest two suspects (*A* and *B*) and interrogate them in separate rooms. Each can either confess, thereby implicating the other, or keep silent. In terms of years in prison the payoff for each strategy are these:

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		Agent A	
		Confess (C)	Do not confess (NC)
Agent B	Confess (C)	5,5	10,0
	Do not confess (NC)	0,10	1,1

According to the theory of games, given this set of payoffs, in absence of information there is a strong tendency for each to confess (optimal decision in terms of Pareto), implying two rational players with consistent preferences. This creates the paradoxical situation that rational players lead to a poorer outcome than irrational players. Actually, when each player has to choose the best strategy every doer drives his agent to make decision that leads him a greater advantage, believing that the other will do the same due to the effect of the false consensus. Consequently, the decision made by each leads to optimal decision in terms of Pareto, because both have the same utility function and both doers choose the only action that is the best strategy.

However, it is just a coincidence that the two players have achieved a common strategy. In other types of non-cooperative problems this can not happen, with the result that you will never achieve a joint decision without a prior agreement, if there is no information. Consider, for example, a multi agent decision problem in which the agents set to save money to realize a common purchase. Even agent has a fixed income, Y_A and Y_B and a nonnegative level of saving, S_A and S_B . The planner of each agent choose the best strategy which maximize his utility function of saving (thinking for future), but the present doer of each agent wants to obtain the highest advantage now, so it would consume Y and therefore choose= 0, with a degree= 1. Indeed, the doers are impulsive, each one assigns weight= 1 to one preference and weight= 0 to all the others, thinking that everybody will make in the same way for effect of false consensus. In this case it is not possible obtain a common decision.

The plan made in advance by a group of agents (to realize a common purchase) is not feasible if they don't set some rule or some method to alter the incentives for the doers. This type of problem can be represented in the following scheme:

		Agent A	
		Save (S)	Do not save (NS)
Agent B	Save (S)	10,10	5,5
	Do not save (NS)	5,5	-10,-10

where the payoff represent the utility of each agent for each strategy.

According to the rational choice, the Nash equilibrium coincides with the best strategy (S, S) . However false consensus effect and impulsivity lead each agent to the worst equilibrium, because utility functions of the agents are different among them (each agent prefers consumptions to savings). This causes the lack of consensus on a common decision.

In conclusion, in a non-cooperative multi agent decision problem, there are two situations:

- 1) the doers of each agent have the same preference and they will reach a common decision that is given by the unanimous choice (doers don't affect),
- 2) the doers have different preferences and do not assign any weight to the other preferences, so it is not possible to aggregate the preferences. Hence, we can affirm that in a non-cooperative decision problem it is only a chance obtaining a common decision.

7 Excess of consensus in a cooperative decision problem

According to Engelmann and Strobel's experiment there is no false consensus effect if representative information is highly prominent and retrievable without any effort. On the contrary, there is a significant effect in the opposite direction, indicating that subjects consider others' choices more informative than their own [4]. This is the overconfidence or "groupthink", a psychological phenomenon which can occur in highly trained cohesive groups. Hence, in the extreme case in which all is known in decision making process, the interplay between different subjects involves anyway other behavioral effects, as the excess of consensus, apart from influence of mental models and all behavioral effects of each individual.

In the OR field this kind of decision making process can be modeled with cooperative games where the rationality of the equilibrium choice is saved by the possibility of making an agreement among agents, which represents a pure rule to maintain self-control at later time in Thaler and Shefrin's model. Moreover with an arrangement the agents have explicit information about the choices of other members, so the lack of false consensus effect is in line with the result of Engelmann and Strobel's experiment. However, only the decision of one member will prevail, and this is influenced by the strength of each mental model. An example of cooperative game is a coordination game, when players choose the strategies by a consensus decision making process [7].

Consider the classic example of coordination game: the "battle-of-the sexes". In this game an engaged couple must choose what to do in the evening: the man

prefers to attend a baseball game and the woman prefers to attend an opera. In terms of utility the payoff for each strategy is as follows:

		Man	
		Opera (<i>O</i>)	Baseball (<i>B</i>)
Woman	Opera (<i>O</i>)	3,1	0,0
	Baseball (<i>B</i>)	0,0	1,3

In this example there are multiple outcomes that are equilibriums: (*B*, *B*) and (*O*, *O*). However both players would rather do something together than go to separate events, so no single individual has an incentive to deviate if others are conforming to an outcome. In this context, a consensus decision making process can be considered as an instrument to choose the best strategy in a coordination game. A common final decision is achievable only if the man and the woman have explicit information, then only if there is cooperation [7]. If we follow the Thaler and Shefrin's model, we can analyze choices in a cooperative game in this way: at period-one the planner of each agent states his preference, which is the best strategy because the planner wants to maximize his utility function. Then, the influence of doers, that want to obtain an immediate gratification, can be avoided, because agents can enforce contracts through parties at period-one, what eliminate the problem of loss of self-control, because they eliminate all choices. However, only one will maximize his utility function and this is not known in advance because it depends on the different strength of each mental models.

8 Strategic communication modify behavioral effects

In a multi agent decision problem, information held by the participants can be wrong for two causes, not independent of each other:

- 1) *misunderstandings* and
- 2) *manipulations*.

In the first case, the different reaction of people at the same problem with the same information do not reflect people's lack of cognitive abilities but the way the situation is described in the communication. In the second case, there is the will to misrepresent the problem for self-interest, from one member of the group. Human cognitive processes relate strongly to motivational issues which interplay between people in social contexts. Self-interest is the primary cause of biases especially in participatory processes with multiple stakeholders. Self-interest is the driver

of strategic behavior, which produce above all priming and framing effects. As a result, some biases can be unintentional consequences of cognitive limitations, others can be motivated by omissions or over or under emphasis of aspects, strategically or not [6].

9 BOR: Behavioral Operational Research

In considering the behavioral effects we should take a humble approach and accept the fact that we are not likely be able to produce a “perfect” model but still could find one that is useful. However, modeling is not about models only, but it matters how we choose the models and how we work with the models [6]. Creating a model to manage or solve problems is a process composed of many phases and human behavior moderates each stage of the process and mediates the progression through stages [5]. Hence the behavioral lens needs to be integrated in the practice of OR as an additional perspective. Behavioral Operational Research (BOR) considers the human impact on the process of using operational research (OR) methods in problem solving and decision support as well as using OR methods to model human behavior [6].

Not only decision makers but also modelers are subject to cognitive and motivational biases and the way the decision problems are framed. Moreover, the cumulative effects of biases in a modeling process can also result in path dependency (a phenomenon where the order in which steps are taken in the modeling process can have an impact on the resulting model). In large models the initial modeling choices can be very hard to change later and these can have a crucial impact on the path the modeling process will proceed. The loss aversion effect in decision making can also have an effect on modeling in general. Theoretically it can be equivalent to use and label variables as losses or gains but in the interpretation of the model results there can be a difference. A somewhat related effect is the so called *action bias* where people choose to foster improvement rather than prevent deterioration [6]. Also communication is an important part of modeling. The modeler should not only be focused only on the perfection of the accuracy of the model, the process and communication counts a lot too. Modelers should recognize the possibility of strategic behavior of the participants. Such behavior can mean, for example, the misrepresentation of preferences or data in an environmental participation process. Finally, also modelers are guided by self-interest. The purpose for which the model is developed is reflected in the parameters and scales as well as in the level of detail used. There is not a single valid model fitting every purpose [6]. Hence biases exist in each aspect of a problem solving process and in each phase of modeling of these processes, however finding ways to avoid them is an open research field.

References

- [1] A. Bechara, H. Damasio, D. Tranel and A. R. Damasio, *Deciding advantageously before knowing the advantageous strategy*, Science 275, (1997), 1293-1295.
- [2] A. Bechara, *The role of emotion in decision-making: evidence from neurological patients with orbitofrontal damage*, Brain and Cognition 55, (2004), 30-40.
- [3] A. R. Damasio, *Descartes' error: emotion, reason, and the human brain*, Grosset/Putnam, New York, 1994.
- [4] D. Engelmann and M. Strobel, *The False consensus effect: deconstruction and reconstruction of an anomaly*, CERGE-EI Working Paper 233, (2004).
- [5] R. P. Hamalainen, J. Luoma and E. Saarinen, *On the importance of behavioral operational research: The case of understanding and communicating about dynamic systems*, European Journal of Operational Research 228, (2013), 623-634.
- [6] R. P. Hamalainen, *Behavioral issues in environmental modeling - The missing perspective*, Environmental Modelling and Software 73, (2015), 244-253.
- [7] A. Longo, M. Squillante, A. G. S. Ventre and V. Ventre, *The intertemporal choice behaviour - The role of emotions in a multi-agent decision problem*, Atti Accad. Pelorit. Pericol. Cl. Sci. Fis. Mat. Nat., 93 (2), C* (2015) [14 pages].
- [8] M. C. Nussbaum, *Upheavals of thought. The intelligence of emotions*, Cambridge University Press, Cambridge, 2001.
- [9] B. Shiv, G. Loewenstein, A. Bechara, H. Damasio and A. R. Damasio, *Investment behavior and the negative side of emotion*, Psychological Science 16, (2005), 435-439.
- [10] M. Squillante and V. Ventre, *Assessing false consensus effect in a consensus enhancing procedure*, International Journal of Intelligent System 25, (2010), 274-285.
- [11] R. H. Strotz, *Myopia and Inconsistency in Dynamic Utility Maximization*, Review of Economic Studies 23(3), (1955-1956), 165-80.
- [12] R. Thaler and H. M. Shefrin, *An economic theory of self-control*, Journal of Political Economy 89(2), (1981), 392-406.
- [13] A. G. S. Ventre and V. Ventre, *The intertemporal choice behavior: classical and alternative delay discounting models and control techniques*, Atti Accad. Pelorit. Pericol. Cl. Sci. Fis. Mat. Nat. 90 (1), C3, (2012) [21 pages].