Science & Philosophy Volume 8(2), 2020 pp. 157-163

Helix hyperoperation in teaching research

Souzana Vougioukli[1]

Abstract

Interaction between sciences has always been an optimum. Within this frame of interdisciplinary approach, an attempt is made to apply a method, a *mathematical model*, used in the *Hyperstructure Theory*, in teaching and research procedure. More specifically, when dealing with a great amount of data arranged in a 'linear' disposal, it is quite difficult to teach. This is the case when the Helix Model is suggested to be used. With the Helix Model, every single piece of data is present and every element maintains its independence.

Keywords: Helix hyperoperation; hyperoperation; teaching.[2]

[1] Aristotle University of Thessaloniki, Greece. elsouvou@gmail.com.

[2] Received on November 1st, 2020. Accepted on December 10th, 2020. Published on December 31st, 2020. doi: 10.23756/sp.v8i2.557. ISSN 2282-7757; eISSN 2282-7765.

1 Introduction on hyperstructures

Interaction between sciences has always been an optimal goal among scientists all over the world. We can see that in every specific science there are methods and applications that can be used both within different branches of the same discipline and in other sciences as well. In this paper we present a method, a Mathematical Model, used in the *hyperstructure theory*, for possible application in teaching and research procedure. Interestingly, the method is likely to be subconsciously used in teaching and research procedure.

The largest class of hyperstructures are called *H_v-structures* and were introduced in 1990 by T. Vougiouklis. They satisfy the *weak axioms* where the non-empty intersection replaces the equality.

Basic definitions, see [1], [3], [8], [9], [10], [11], [14]:

In a set H equipped with a hyperoperation (abbreviation *hyperoperation=hope*)

$$\cdot : H\times H \to P(H)-\{\emptyset\} : (x,y) \to x\cdot y \subset H$$

we call ($\cdot$) *weak associative* if $(xy)z\cap x(yz)\neq\emptyset,\ \forall x,y,z\in H$, and we call it *weak commutative* if $xy\cap yx\neq\emptyset,\ \forall x,y\in H$.

The hyperstructure (H,·) is called *H_v-semigroup* if it is weak associative, it is called *H_v-group* if it is reproductive H_v-semigroup, i.e. $xH=Hx=H,\ \forall x\in H$. The hyperstructure (R,+,·) is called *H_v-ring* if both (+) and (·) are weak associative, the reproduction axiom is valid for (+) and (·) is *weak distributive* with respect to

$$(+):\ x(y+z)\cap(xy+xz)\neq\emptyset,\ (x+y)z\cap(xz+yz)\neq\emptyset,\ \forall x,y,z\in R.$$

Let (R,+,·) be H_v-ring, (M,+) be weak commutative H_v-group and there exists an external hope

$$\cdot :\ R\times M \to P(M): (a,x) \to ax$$

such that, $\forall a,b\in R$ and $\forall x,y\in M$, we have

$$a(x+y)\cap(ax+ay)\neq\emptyset,\ (a+b)x\cap(ax+bx)\neq\emptyset,\ (ab)x\cap a(bx)\neq\emptyset,$$

then M is called an *H_v-module* over F. In the case of an H_v-field F, which is defined later, instead of an H_v-ring R, then the *H_v-vector space* is defined.

Let (H,·), (H,*) be H_v-semigroups. (·) is *smaller* than (*) iff there exists

$$f\in Aut(H,*):\ xy\subset f(x*y),\ \forall x,y\in H.$$

Theorem 1.1 *(The Little Theorem)* [3], [9], [10]. Greater hopes than the ones which are weak associative or weak commutative, are also weak associative or weak commutative, respectively.

The main tool to study hyperstructures is the *fundamental relations* β*, γ* and ε*, which are defined, in H_v-groups, H_v-rings and H_v-vector spaces, respectively, as the smallest equivalences so that the quotient to be group, ring and vector space, respectively.

Theorem 1.2 Let $(H,\cdot)$ be an H_v-group and denote by U the set of all finite products of elements of H. Define β in H by setting $x\beta y$ iff $\{x,y\}\subset u$ where $u\in U$. Then β^* is the transitive closure of β.

An element is called *single* if its fundamental class is singleton.

More general structures can be defined by using the fundamental structures.

Definition 1.3 An H_v-ring $(R,+,\cdot)$ is called *H_v-field* if R/γ^* is a field.

Definition 1.4 The H_v-semigroup $(H,\cdot)$ is called *h/v-group* if the H/β^* is a group.

Similarly, the *h/v-rings*, h/v-fields, *h/v-vector spaces* etc, are defined.

2 Helix-hopes

Definition 2.1 [2], [4], [5], [6], [7], [12], [13], [14]. Let $A=(a_{ij})\in M_{m\times n}$ be an $m\times n$ matrix and $s,t\in N$, such that $1\le s\le m$, $1\le t\le n$. Define a mod-like map $\underline{st}$ from $M_{m\times n}$ to $M_{s\times t}$ by corresponding to A the matrix $A\underline{st}=(\underline{a}_{ij})$ which has entries the sets

$$\underline{a}_{ij}=\{a_{i+\kappa s,j+\lambda t}\mid 1\le i\le s,\ 1\le j\le t, \text{ and } \kappa,\lambda\in N,\ i+\kappa s\le m,\ j+\lambda t\le n\}.$$

Thus, we have the map

$$\underline{st}: M_{m\times n}\to M_{s\times t}: A\to A\underline{st}=(\underline{a}_{ij}).$$

We call this multivalued map *helix-projection* of type $\underline{st}$. Thus $A\underline{st}$ is a set of $s\times t$-matrices $X=(x_{ij})$ such that $x_{ij}\in\underline{a}_{ij}$, $\forall i,j$. Obviously $A\underline{mn}=A$.

Let $A=(a_{ij})\in M_{m\times n}$ be matrix and $s,t\in N$ with $1\le s\le m$, $1\le t\le n$. Then we can apply helix-projection on columns and then on rows, the result is the same if we apply the helix-projection on both. Thus,

$$(A\underline{sn})\underline{st}=(A\underline{mt})\underline{st}=A\underline{st}.$$

Definitions 2.2 (a) Let $A=(a_{ij})\in M_{m\times n}$ and $B=(b_{ij})\in M_{u\times v}$ be matrices and $s=\min(m,u)$, $t=\min(n,u)$. We define a hope, called *helix-addition* or *helix-sum*, as follows:

$$\oplus: M_{m\times n}\times M_{u\times v}\to P(M_{s\times t}): (A,B)\to A\oplus B=A\underline{st}+B\underline{st}=(\underline{a}_{ij})+(\underline{b}_{ij})\subset M_{s\times t},$$

where

$$(\underline{a}_{ij})+(\underline{b}_{ij})=\{(c_{ij})=(a_{ij}+b_{ij})\mid a_{ij}\in\underline{a}_{ij} \text{ and } b_{ij}\in\underline{b}_{ij}\}.$$

(b) Let $A=(a_{ij})\in M_{m\times n}$ and $B=(b_{ij})\in M_{u\times v}$ be matrices and $s=\min(n,u)$. We define a hope, called *helix-multiplication* or *helix-product*, as follows:

$$\otimes: M_{m\times n}\times M_{u\times v}\to P(M_{m\times v}): (A,B)\to A\otimes B=A\underline{ms}\cdot B\underline{sv}=(\underline{a}_{ij})\cdot(\underline{b}_{ij})\subset M_{m\times v},$$

where

$$(\underline{a}_{ij})\cdot(\underline{b}_{ij})=\{(c_{ij})=(\textstyle\sum a_{it}b_{tj})\mid a_{ij}\in\underline{a}_{ij} \text{ and } b_{ij}\in\underline{b}_{ij}\}.$$

The helix-addition is an external hope since it is defined on different sets and the result is also in different set. The commutativity is valid in the helix-addition. For the helix-multiplication we remark that we have $A\otimes B=A\underline{ms}\cdot B\underline{sv}$ so we have either $A\underline{ms}=A$ or $B\underline{sv}=B$, that means that the helix-projection was applied only in one matrix and only in the rows or in the columns.

Remark that the helix multiplication is weak associative.

Let us restrict ourselves on the matrices $M_{m\times n}$ where $m<n$. Obviously, we have analogous cases where $m>n$ and for $m=n$ we have the classical theory.

Notation: For given $\kappa\in\mathbb{N}$-{0}, we denote by $\underline{\kappa}$ the remainder resulting from its division by m if the remainder is non zero, and $\underline{\kappa}$=m if the remainder is zero.

A matrix $A=(a_{\kappa\lambda})\in M_{m\times n}$, m<n is called a *cut-helix matrix* if we have $a_{\kappa\lambda}=a_{\kappa\underline{\lambda}}$, $\forall\kappa,\lambda\in\mathbb{N}$-{0}.

Moreover, denote by $I_c=(c_{\kappa\lambda})$ the *cut-helix unit matrix* which the cut matrix is the unit matrix I_m. Therefore, since $I_m=(\delta_{\kappa\lambda})$, where $\delta_{\kappa\lambda}$ is the Kronecker's delta, we obtain that, $\forall\kappa,\lambda$, we have $c_{\kappa\lambda}=\delta_{\kappa\underline{\lambda}}$.

3 Examples on helix-hopes

Example 3.1 Consider the matrices

$$\boldsymbol{A} = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix} \quad \text{and} \quad \boldsymbol{B} = \begin{pmatrix} 0 & 1 & 2 & 1 \\ 3 & 2 & 1 & 3 \\ 1 & 1 & 2 & 1 \end{pmatrix}$$

then:

$$\boldsymbol{A}\otimes\boldsymbol{B} = \begin{pmatrix} \{0,1\} & 2 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 2 & 1 \\ 3 & 2 & 1 & 2 \\ 1 & 1 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 6 & \{4,5\} & \{2,4\} & \{6,7\} \\ 4 & 3 & 3 & 4 \end{pmatrix} =$$

$$= \left\{ \begin{pmatrix} 6 & 4 & 2 & 6 \\ 4 & 3 & 3 & 4 \end{pmatrix}, \begin{pmatrix} 6 & 4 & 2 & 7 \\ 4 & 3 & 3 & 4 \end{pmatrix}, \begin{pmatrix} 6 & 4 & 4 & 6 \\ 4 & 3 & 3 & 4 \end{pmatrix}, \begin{pmatrix} 6 & 4 & 4 & 7 \\ 4 & 3 & 3 & 4 \end{pmatrix}, \begin{pmatrix} 6 & 5 & 2 & 6 \\ 4 & 3 & 3 & 4 \end{pmatrix}, \begin{pmatrix} 6 & 5 & 2 & 7 \\ 4 & 3 & 3 & 4 \end{pmatrix}, \begin{pmatrix} 6 & 5 & 4 & 6 \\ 4 & 3 & 3 & 4 \end{pmatrix}, \begin{pmatrix} 6 & 5 & 4 & 7 \\ 4 & 3 & 3 & 4 \end{pmatrix} \right\}$$

Example 3.2 Consider the matrices

$$\boldsymbol{A} = \begin{pmatrix} -1 & 2 \\ 2 & 0 \\ 3 & 2 \end{pmatrix} \quad \text{and} \quad \boldsymbol{B} = \begin{pmatrix} 1 & 1 \\ -1 & 2 \\ -2 & 1 \\ 0 & 3 \end{pmatrix}$$

then:

$$\boldsymbol{A}\otimes\boldsymbol{B} = \begin{pmatrix} -1 & 2 \\ 2 & 0 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} \{1,-2\} & 1 \\ \{-1,0\} & \{2,3\} \end{pmatrix} = \begin{pmatrix} \{-3,-1,0,2\} & \{3,5\} \\ \{2,-4\} & 2 \\ \{1,3,-8,-6\} & \{7,9\} \end{pmatrix}.$$

There are 128 matrices of type 3×2 in the set!

Example 3.3 A hyper-matrix representation of 4-dimensional non-degenerate case with helix-hope: On the field of real or complex numbers we consider all 3×5 matrices of the type

$$\boldsymbol{A} = \begin{pmatrix} 1 & a & b & 1 & d \\ 0 & 1 & c & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

This set is closed under the helix hope. That means that the helix-hope, (hyper-product) of two such matrices is a 3×5 matrix, of the same type. In fact, we have

$$\boldsymbol{A \otimes A'} = \begin{pmatrix} 1 & a & b & 1 & d \\ 0 & 1 & c & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & a' & b' & 1 & d' \\ 0 & 1 & c' & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & \{a,d\} & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & a' & b' & 1 & d' \\ 0 & 1 & c' & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & \{a+a', d+a'\} & \{b+b'+ac', b+b'+dc'\} & 1 & \{a+d', d+d'\} \\ 0 & 1 & c+c' & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

Therefore, the result is asset with 8 matrices.

Example 3.4 The 3×7 matrix

$$\boldsymbol{A} = \begin{pmatrix} -1 & 2 & -3 & -1 & 2 & -3 & -1 \\ 5 & -2 & 3 & 5 & -2 & 3 & 5 \\ -6 & 0 & -1 & -6 & 0 & -1 & -6 \end{pmatrix}$$

is a cut-helix matrix because the 4^{th} and 7^{th} columns are equal to the 1^{st}, the 5^{th} column is equal to the 2^{nd}, and the 6^{th} column is equal to the 3^{rd}. Notice that if we use cut-helix matrices in helix-hopes then the results are simplified because they are singletons.

4 The meaning of the helix-hope

Now let us try to explain the way that helix-hope acts in order to find out whether we could use it in other sciences in similar circumstances.

Let us take a matrix, with entries positive integers, of type 3×5, as the following

$$\boldsymbol{A} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 1 & 6 & 7 & 8 \\ 1 & 0 & 9 & 1 & 0 \end{pmatrix}$$

The usual product of this matrix from the left, with a matrix of the same type, is not defined. The reason is that our matrix must have 3 columns. If we delete the 4^{th} and 5^{th} columns, then we will lose the information contained in the entries of those two columns. Therefore, we can use the helix-hope in order to define the product using all the entries and at the same time we move from operations to hyperoperations, from single valued to multivalued cases. In order to apply the helix-hope we take the entries from the 4^{th} column and shift them to the row corresponding elements of the 1^{st} column.

We do not add those two elements, but we consider them as elements of the same set. Then we take the next column 5 and shift its entries to the corresponding entries of the 2nd column. In the case that we have more columns, we follow the same procedure. Thus, in our example, we take the following matrix where instead of numbers as entries, we have the elements:

$$\boldsymbol{A}' = \begin{pmatrix} \{1,4\} & \{2,5\} & 3 \\ \{0,7\} & \{1,8\} & 6 \\ 1 & 0 & 9 \end{pmatrix}$$

With this form of the matrix the product is defined. The only change is that now we have sets as entries. In this way we have a hyperproduct.

Here we should remark that if the transferred element is the same with the corresponding entry, we do not have a set, but we have an element. In our example, this can be noticed in positions (3,1) and (3,2).

To recap, we claim that the helix-hope replaces some elements and shift them together along with the corresponding elements, treating them in the same way, as elements of a set. This replacement in mathematics is called modulo-like procedure. One can say that this modulo-like procedure reminds us of the ‘repetition’ in teaching or the ‘motivos’ in music compositions.

5 Conclusion

When dealing with a great amount of data, any type of information, arranged either in a ‘linear’ disposal or without any arrangement, grouping or systematization of any kind, we encounter the serious problem of managing all this data so as that once embedded in our minds, then to be accessible, easy to teach or transfer to another person-receiver. The Mathematical Model Helix might give an easily applicable method- solution in cases like that. This difficult to manage amount of data, information or elements to be taught can be placed in a ‘helix’ way instead of being placed linearly. In this way, every element is present and, most important, every element maintains its independence, appears self-contained and is always easily available. Moreover, especially in the teaching process, the overlapping nature of the Helix-hyperoperation, describes and promotes one of the most important principles of teaching, that of repetition as repeating the encounter fuses it into one’s awareness.

References

[1] Corsini P., Leoreanu V. (2003). Application of Hyperstructure Theory, Klower Academic Publishers.

[2] Davvaz B., Vougioukli S., Vougiouklis T. (2011). On the multiplicative H_v-rings derived from helix hyperoperations, Util. Math., 84, 53-63.

[3] Davvaz D., Vougiouklis T. (2018). A Walk Through Weak Hyperstructures, H_v-Structures, World Scientific.

[4] Vougiouklis S. (2009). H_v-vector spaces from helix hyperoperations, Int. J. Math. Anal. (New Series), 1(2), 109-120.

[5] Vougiouklis S. (2020). Hyperoperations defined on sets of S-helix matrices, J. Algebraic Hyperstructures and Logical Algebras, V.1, N.3, 81-90.

[6] Vougiouklis S., Vougiouklis T. (2008). Helix-Hopes on S-Helix Matrices, Ratio Mathematica, V. 33, 67-179.

[7] Vougiouklis S., Vougiouklis T. (2016). Helix-hopes on Finite H_v-fields, Algebras Groups and Geometries (AGG), V.33, N.4, 2016, 491-506.

[8] Vougiouklis T. (1991). The fundamental relation in hyperrings. The general hyperfield, 4th AHA, Xanthi, Greece (1990), World Scientific, 209-217.

[9] Vougiouklis T. (1994). Hyperstructures and their Representations, Monographs in Mathematics, Hadronic.

[10] Vougiouklis T. (1995). Some remarks on hyperstructures, Contemporary Mathematics, Amer. Math. Society, 184, 427-431.

[11] Vougiouklis T. (1999). On H_v-rings and H_v-representations, Discrete Mathematics, Elsevier, 208/209, 615-620.

[12] Vougiouklis T., Vougiouklis S. (2005). The helix hyperoperations, Italian J. Pure Appl. Math., 18, 197-206.

[13] Vougiouklis T., Vougiouklis S. (2015). Hyper-representations by non square matrices. Helix-hopes, American J. Modern Physics, 4(5), 52-58.

[14] Vougiouklis T., Vougiouklis S. (2016). Helix-Hopes on Finite Hyperfields, Ratio Mathematica, V.31, 2016, 65-78.