

Dear Professor Maturo,

I wish to submit a new manuscript entitled “New structure of norms on $\mathbb{R}^n$ and their relations with the curvature of the plane curves” for consideration by Ratio Mathematica. This paper is a combination of mathematical analysis and differential geometry. I confirm that this manuscript has not been published elsewhere or presented elsewhere in part or in entirety, and is not under consideration by another journal. There are no conflicts of interest to declare. My colleague and I believe that the findings of this study are relevant to the scope of your journal and will be of interest to its readership.

In this paper, we report on new constructions of norms on $\mathbb{R}^n$ and their relations with the curvature of the plane curve. First, we consider n fixed nonzero real-valued functions $f_1, f_2, \dots, f_n$ on $\mathbb{R}$ (the real numbers) and put $\varphi_n(X_n) = (x_1^2 f_1^2 + x_2^2 f_2^2 + \dots + x_n^2 f_n^2)^{\frac{1}{2}}$, where $X_n = (x_1, x_2, \dots, x_n) \in \mathbb{R}^n$. We show that φ_n has properties similar to a norm on the normed linear space. Although φ_n is not a norm on $\mathbb{R}^n$ in general, it induces a norm on $\mathbb{R}^n$. For the nonzero function $F : \mathbb{R}^2 \rightarrow \mathbb{R}$, a curvature formula for the implicit curve $G(x, y) = F^2(x, y) = 0$ at any regular point is given. A similar result is presented when F is a nonzero function from $\mathbb{R}^3$ to $\mathbb{R}$. Next, we concentrate on the norm $F(x, y) = \int_a^b \varphi_2(x, y) dt$. It is shown that the curvature of $F(x, y) = c$ when $c > 0$ is a positive multiple of c^2 . Particularly, we observe that $F(x, y) = \int_0^{\frac{\pi}{2}} \sqrt{x^2 \cos^2 t + y^2 \sin^2 t} dt$ is an elliptic integral of the second kind.

It is better to be reminded that in order to comply with the standards of the journal, we have mentioned only the references used in the text of the article.

We also recommend the following people to review the article

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Thank you so much for your consideration of this manuscript.

Yours sincerely,

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