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Attribute Reduction in Students Selection for a Merit cum Means Based Scholarship Scheme using Basis in Nano Topological Spaces

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Abstract

The objective of this paper is to identify the key attributes that have a significant impact on the merit cum means-based scholarship selection process, ensuring that the allocation of resources is fair and efficient. Scholarships aid students with limited resources in their pursuit of a college education. The complexity of selecting suitable candidates with multiple characteristics has made it harder to determine the essential factors for deciding whether or not an applicant is eligible. To reduce attributes, we use the concepts of basis, lower and upper approximations of Nano Topological Spaces to identify the most relevant traits that affect scholarship eligibility. By using a systematic approach, decision-making is minimized and resources are allocated more effectively. Administrators can ensure that scholarships reach deserving students in need by focusing on essential attributes. The research will help to improve scholarship awarding methods and increase inclusion and accessibility in higher education.

Keywords: Upper Approximations, Lower Approximations, Nano Topology, Basis, Core, Attributes.

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1. Introduction

The concept of nano topology was originally introduced by Lellis Thivagar et al. [4] and since then it has proven to be a valuable tool in various applications. Lellis Thivagar used nano topological basis to identify the key factors for adult dietary needs, Jayalakshmi [1] and Aggour [2] used it in a medical diagnosis problem. In this study, we adapted and extended this concept to the area of scholarship selection to help university administrators identify the most deserving students who are truly in need of financial aid. A Nano topological basis provides a sophisticated framework for analyzing subsets of the given universe U. This concept uses lower and upper approximations and boundary regions defined by an equivalent relation to represent and study data. In reducing attributes, as well as identifying the main factors for selecting eligible students on a given attribute basis, the concepts of basis in Nano Topological Spaces are applied.

The merit-based and means-based scholarship program aims to provide financial assistance to deserving students from economically disadvantaged backgrounds. However, selecting the most deserving candidates from a pool of applicants can be difficult due to the multitude of attributes that must be taken into account. By leveraging this fundamental concept in the nano topological space, we aim to streamline and optimize this selection process.

Providing financial support to deserving students pursuing higher education has always been an important aspect of promoting an inclusive and equitable education system. The merit cum means-based scholarship program is one such initiative to support deserving students from economically disadvantaged backgrounds. In recent years, the process of selecting the most suitable students for scholarships has become increasingly complicated. College administrators face the challenge of screening a large pool of applicants, each with their own unique backgrounds and qualifications. The traditional approach of considering a few standard criteria often falls short of accurately assessing the true needs and merit of individual students.

The merit cum means-based scholarship scheme for meritorious students is usually awarded to a needy student with a good educational background. The scheme aims to provide financial assistance to poor, deserving students enrolled in college. To select a suitable student for this purpose, it is necessary to take into account a separate set of parameters that may be applied when granting such scholarships. The parameters are based on

(i) The eligibility criteria for availing of the scholarship is the minimum aggregate marks greater than 50% in the last examination or in the qualifying exam, which is specially conducted for scholarship selection.

(ii) Family background, which includes family annual income, parents' education, occupation of parents (government/private), number of siblings, and whether the family owns any agricultural property.

(iii) Medium of previous study, which can be any regional language or English.

(iv) Residential area, that is, students belonging to rural areas or from the city side, or the student is a hosteller or day scholar.
(v) Involvement in extra-curricular/co-curricular activities and
(vi) Awarded any other scholarship or the student is a first-generation learner.

This paper presents a promising methodology that leverages the concept of Nano Topological Spaces to enhance the scholarship selection process. By considering a broader range of parameters and incorporating individual circumstances, this approach will aid college administrators in identifying the most suitable candidates for the scholarship, thus promoting greater inclusivity and social mobility within the education system.

2. Preliminaries

Definition 2.1[4]**.** Given X is a nonempty finite set of objects and R_X is an equivalence relation on X called the indiscernibility relation. The X is divided into disjoint equivalence classes. The elements of the same equivalence class are said to be indiscernible with each other. The pair (X, R_X) is called an approximation space. Let $A \subseteq X$, then
(i) The lower approximation of A with respect to R_X is $L_{R_X}(A) = \bigcup_{x \in X}\{R_X(x)/ R_X(x) \subseteq A\}$, where $R_X(x)$ is the equivalence class determined by $L_{R_X}(A)$.
(ii) The upper approximation of A with respect to R_X is $U_{R_X}(A) = \bigcup_{x \in X}\{R_X(x)/ R_X(x) \cap A \neq \phi\}$.
(iii) The boundary region of A is $B_{R_X}(A) = U_{R_X}(A) - L_{R_X}(A)$.

Definition 2.2[4]**.** Let X be the universe, R_X is an equivalence relation on X and $A \subseteq X$, $\tau_{R_X}(A) = \{\phi, X, L_{R_X}(A), U_{R_X}(A), B_{R_X}(A)\}$. Then $\tau_{R_X}(A)$ satisfies the axioms
(i) X and ϕ are in $\tau_{R_X}(A)$.
(ii) The arbitrary union of the elements of $\tau_{R_X}(A)$ is in $\tau_{R_X}(A)$.
(iii) The finite intersection of any finite sub collection of $\tau_{R_X}(A)$ is in $\tau_{R_X}(A)$.
Here $\tau_{R_X}(A)$is a topology on X called the nano topology on X with respect to A and $(X, \tau_{R_X}(A))$ is called a nano topological space. The elements of $\tau_{R_X}(A)$ are called the nano open sets and the complement of the nano open sets are called nano closed sets.

Definition 2.3[3]**.** Let $\mathcal{M} = \{U, A, V, f\}$ is a set valued information, where U is a non-empty finite set of objects, A is a finite set of attributes, $V = \cup V_a$ is the domain of attribute 'a' and $f: U \times A \to \wp(V)$ is a function such that for every $x \in U$ and $a \in A$, $f(x, a) \subseteq V_a$. The attribute set is divided into two subsets a set C of condition attributes and a decision attribute d, where $C \cap \{d\} = \emptyset$

Definition 2.4[3]**.** The relation $R_A = \{(x, y) \in U \times U: f(x, a) \supseteq f(y, a)$ for every $a \in A\}$ is called the dominance relation on U.

Definition 2.5[3]**.** The dominance class of $x \in U$ is defined as $[x]_A^R = \{y \in U: (y, x) \in R_A\} = \{y \in U: f(y, a) \supseteq f(x, a)$ for every $a \in A$. Here $E_A^R = \{[x]_A^R : x \in U\}$ is the family of dominance classes on U.

Definition 2.6[3]**.** If $\mathcal{M} = \{U, A, V, f\}$ is a set valued information and a subset X of U, the upper approximation of X is defined as $U_{R_X}(X) = \{x \in U: [x]_A^R \cap X \neq \emptyset\}$ and the lower approximation is defined as $L_{R_X}(X) = \{x \in U: [x]_A^R \subseteq X\}$. The boundary region of X is denoted by $B_{R_X}(X) = U_{R_X}(X) - L_{R_X}(X)$.

Definition 2.7[5]**.** Given a set valued ordered information system $\mathcal{M}$, a subset B of A is said to be a criterion reduction of S if $R_A = R_B$ and $R_M \neq R_A$ for any $\mathcal{M} \subset B$. Criterion reduction is a minimal attribute set B such that $R_A = R_B$.

Definition 2.8[3]**.** If $\tau_R(X)$ is a nano topology on U with respect to X, then the set $\beta_R(X) = \{U, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.

Definition 2.9[5]**.** The criterion reduction of a set valued information system is a minimal attribute subset B of the attribute set A such that $\beta_B = \beta_A$.

Definition 2.10[5]**.** The core of the attribute set A is $\mathrm{CORE(A)} = \{\mathrm{a} \in \mathrm{A}: \beta_{\mathrm{A}} \neq \beta_{\mathrm{A}-\{\mathrm{a}\}}\}$. That is CORE(A) is the intersection of the criterion reduction.

3. Attribute Reduction Using Nano Topological Basis

Consider the following information table, which provides information about eight students. The eligibility criterion for availing of the merit-based and means-based scholarship schemes includes the minimum requirement of aggregate marks greater than 50% in the last examination, in the qualifying exam, or in the first semester.

Suppose a batch of students who joined a higher education institution in a particular year are examined for awarding the scholarship. In the selection process, the following attributes are taken into consideration: the students' mark, which is Honor or Medium; the family income, which is less than 3 lakhs or more than 3 lakhs; the medium of previous study of the student, which is either English or any other regional language; the family residence, which is either rural or urban; the student's involvement in extracurricular activities like sports or not involved in sports; and other scholarships availed by the students.

Students	**Group-I**	**Group-II**	**Group-III**	**Group-IV**	**Group-V**	**Group-VI**	**Decision (D)**
	(MA)	**(FI)**	**(ME)**	**(RE)**	**(EC)**	**(OS)**	
T_1	H	< 3L	E	Ur	NS	Not Availed	Selected
T_2	H	< 3L	R	Ru	S	Availed	Selected
T_3	H	> 3L	R	Ru	S	Availed	Selected
T_4	M	> 3L	R	Ur	NS	Not Availed	Not Selected
T_5	M	< 3L	E	Ru	NS	Not Availed	Selected
T_6	H	> 3L	R	Ru	S	Availed	Not Selected
T_7	H	> 3L	R	Ur	NS	Not Availed	Selected
T_8	M	< 3L	E	Ru	NS	Not Availed	Not Selected

Table.3.1

In table.3.1,

Group-I: Marks (MA) → Honor (H), Medium (M)
Group-II: Family Income (FI) → Income < 3 lakhs(<3L), Income > 3 lakhs(>3L)
Group-III: Medium (ME) → English (E), Regional Language (R)
Group-IV: Residence (RE)→ Rural (Ru), Urban (Ur)
Group-V: Extra-Curricular (EC)→ Sports (S) / Non-Sports (NS)
Group-VI: Other Scholarship (OS)→ Availed / Not Availed

Given $\mathcal{M} = \{U, A, V, f\}$ is the set valued information system, $U = \{T_1, T_2, T_3, T_4, T_5, T_6, T_7, T_8\}$ is the non-empty finite set of objects, and $A = \{MA, FI, ME, RE, EC, OS, D\}$ is the finite set of attributes, $V = \cup V_a$ is the domain of attribute 'a'.
Here, $V = \{H, M, < 3L, > 3L, E, R, Ru, Ur, S, NS, Availed, Not\ Availed\}$. The function f is defined by $f: U \times A \to \wp(V)$ is a function such that for every $x \in U$ and $a \in A$, $f(x, a) \subseteq V_a$, where $\wp(V)$ is the power set of V. Here $f(S_1, MA) = H$, $f(S_3, RE) = Ru$ and so on. The attribute set is divided into two subsets a set C condition attributes and a decision attribute D, where $C \cap \mathrm{D} = \emptyset$ and the decision attribute D describes whether the student is selected for the scholarship or not. The concept of elimination of attributes in Nano Topology is applied to identify the key factors for selecting eligible students to get the scholarship.

The family of dominance classes is given by

$$U_C/R = \{\{T_1\},\{T_2\},\{T_3,T_6\},\{T_4\},\{T_5,T_8\},\{T_7\}\}$$

<u>Case-1</u>: Students who received the Scholarship

Assume $X = \{T_1,T_2,T_3,T_5,T_7\}$, the set of students selected for scholarship. The lower and upper approximations are $L_C(X) = \{T_1,T_2,T_7\}$, $U_C(X) = \{T_1,T_2,T_3,T_5,T_6,T_7,T_8\}$ and the boundary region is $B_C(X) = \{T_3,T_5,T_6,T_8\}$. The nano topology on U is given by $\tau_C(X) = \{\emptyset, U, \{T_1,T_2,T_7\},\{T_3,T_5,T_6,T_8\}, \{T_1,T_2,T_3,T_5,T_6,T_7,T_8\}$ and the basis for $\tau_C(X)$ is $\beta_C(X) = \{U,\{T_1,T_2,T_7\},\{T_3,T_5,T_6,T_8\}\}$.

Step-1:
Let $B_1 = C - \{MA\}$. Then $U_{B1}/R = \{\{T_1\},\{T_2\},\{T_3,T_6\},\{T_4,T_7\},\{T_5,T_8\}\}$ and the lower and upper approximations are $L_{B1}(X) = \{T_1,T_2\}$, $U_{B1}(X) = \{T_1,T_2,T_3,T_4,T_5,T_6,T_7,T_8\}$. The corresponding nano topology and the basis are given by $\tau_{B1}(X) = \{\emptyset, U, \{T_1,T_2\},\{T_3,T_4,T_5,T_6,T_7,T_8\}\}$, $\beta_C(X) = \{U,\{T_1,T_2\},\{T_3,T_4,T_5,T_6,T_7,T_8\}\}$. $\beta_{B1}(X) \neq \beta_C(X)$.

Let $B_2 = C - \{FI\}$. Then $U_{B2}/R = \{\{T_1\},\{T_2,T_3,T_6\},\{T_4\},\{T_5,T_8\},\{T_7\}\}$ and the corresponding nano topology and basis are $\tau_{B2}(X) = \{\emptyset, U, \{T_1,T_7\},\{T_2,T_3,T_5,T_6,T_8\}, \{T_1,T_2,T_3,T_5,T_6,T_7,T_8\}\}$ and $\beta_{B2}(X) = \{U,\{T_1,T_7\},\{T_2,T_3,T_5,T_6,T_8\}$.
$\beta_{B2}(X) \neq \beta_C(X)$.

If $B_3 = C - \{ME\}$. Then $U_{B3}/R = \{\{T_1\},\{T_2\},\{T_3,T_6\},\{T_4\},\{T_5,T_8\},\{T_7\}\}$ and $\tau_{B3}(X) = \{\emptyset, U, \{T_1,T_2,T_7\}, \{T_3,T_5,T_6,T_8\}, \{T_1,T_2,T_3,T_5,T_6,T_7,T_8\}\} = \tau_C(X)$.
$\beta_{B3}(X) = \{U,\{T_1,T_2,T_7\},\{T_3,T_5,T_6,T_8\}\} = \beta_C(X)$.

If $B_4 = C - \{RE\}$. Then $U_{B4}/R = \{\{T_1\},\{T_2\},\{T_3,T_6\},\{T_4\},\{T_5,T_8\},\{T_7\}\}$ and $\tau_{B4}(X) = \{\emptyset, U, \{T_1,T_2,T_7\}, \{T_3,T_5,T_6,T_8\}, \{T_1,T_2,T_3,T_5,T_6,T_7,T_8\}\} = \tau_C(X)$. Thus, $\beta_{B4}(X) = \beta_C(X)$.

If $B_5 = C - \{EC\}$. Then $U_{B5}/R = \{\{T_1\},\{T_2\},\{T_3,T_6\},\{T_4\},\{T_5,T_8\},\{T_7\}\}$ and $\tau_{B5}(X) = \{\emptyset, U, \{T_1,T_2,T_7\}, \{T_3,T_5,T_6,T_8\}, \{T_1,T_2,T_3,T_5,T_6,T_7,T_8\}\} = \tau_C(X)$.
$\beta_{B5}(X) = \beta_C(X)$.

If $B_6 = C - \{OS\}$. Then $U_{B6}/R = \{\{T_1\},\{T_2\},\{T_3,T_6\},\{T_4\},\{T_5,T_8\},\{T_7\}\}$ and $\tau_{B6}(X) = \{\emptyset, U, \{T_1,T_2,T_7\}, \{T_3,T_5,T_6,T_8\}, \{T_1,T_2,T_3,T_5,T_6,T_7,T_8\}\} = \tau_C(X)$. Then, $\beta_{B6}(X) = \beta_C(X)$.
Hence B_3, B_4, B_5, B_6 are the criterion reducts for the student's selection.

Step-2: Consider $B_3 = C - \{ME\} = \{MA, FI, RE, EC, OS\}$.
If $S_1 = B_3 - \{MA\} = \{FI, RE, EC, OS\}$, then $U_{S1}/R = \{\{T_1\},\{T_2\},\{T_3,T_6\},\{T_4,T_7\},\{T_5,T_8\}\}$

and the lower and upper approximations are $L_{S1}(X) = \{T_1, T_2\}$, $U_{S1}(X) = \{T_1, T_2, T_3, T_4, T_5, T_6, T_7, T_8\}$. The corresponding nano topology and the basis are $\tau_{S1}(X) = \{\emptyset, U, \{T_1, T_2\}, \{T_3, T_4, T_5, T_6, T_7, T_8\}\}$, $\beta_{S1}(X) = \{U, \{T_1, T_2\}, \{T_3, T_4, T_5, T_6, T_7, T_8\}\}$. So, $\beta_{S1}(X) \neq \beta_C(X)$.

If $S_2 = B_3 - \{FI\} = \{MA, RE, EC, OS\}$, then $U_{S2}/R = \{\{T_1, T_7\}, \{T_2, T_3, T_6\}, \{T_4\}, \{T_5, T_8\}\}$ and the corresponding nano topology is $\tau_{S2}(X) = \{\emptyset, U, \{T_1, T_7\}, \{T_2, T_3, T_5, T_6, T_8\}, \{T_1, T_2, T_3, T_5, T_6, T_7, T_8\}\} \neq \tau_C(X)$. Therefore, $\beta_{S2}(X) \neq \beta_C(X)$.

If $S_3 = B_3 - \{RE\} = \{MA, FI, EC, OS\}$, then $U_{S3}/R = \{ \{T_1\}, \{T_2\}, \{T_3, T_6\}, \{T_4\}, \{T_5, T_8\}, \{T_7\}\}$ and the corresponding nano topology is $\tau_{S3}(X) = \{\emptyset, U, \{T_1, T_2, T_7\}, \{T_3, T_5, T_6, T_8\}, \{T_1, T_2, T_3, T_5, T_6, T_7, T_8\}\} = \tau_C(X)$. Thus, $\beta_{S3}(X) = \beta_C(X)$.

If $S_4 = B_3 - \{EC\} = \{MA, FI, RE, OS\}$, then $U_{S4}/R = \{\{T_1\}, \{T_2\}, \{T_3, T_6\}, \{T_4\}, \{T_5, T_8\}, \{T_7\}\}$ and $\tau_{S4}(X) = \{\emptyset, U, \{T_1, T_2, T_7\}, \{T_3, T_5, T_6, T_8\}, \{T_1, T_2, T_3, T_5, T_6, T_7, T_8\}\} = \tau_C(X)$. $\beta_{S4}(X) = \beta_C(X)$.

Similarly, if $S_5 = B_3 - \{OS\} = \{MA, FI, RE, EC\}$. Then $U_{S5}/R = \{\{T_1\}, \{T_2\}, \{T_3, T_6\}, \{T_4\}, \{T_5, T_8\}, \{T_7\}\}$ and the corresponding nano topology is $\tau_{S5}(X) = \{\emptyset, U, \{T_1, T_2, T_7\}, \{T_3, T_5, T_6, T_8\}, \{T_1, T_2, T_3, T_5, T_6, T_7, T_8\}\} = \tau_C(X)$. So, $\beta_{S5}(X) = \beta_C(X)$.

Hence S_3, S_4, S_5 are the criterion reducts for the information system.
CORE(A)=$S_3 \cap S_4 \cap S_5$
=$\{MA, FI, EC, OS\} \cap \{MA, FI, RE, OS\} \cap \{MA, FI, RE, EC\} = \{MA, FI\}$.
CORE={Mark, Family Income}

Step-3: $B_4 = C - \{RE\} = \{MA, FI, ME, EC, OS\}$.

If $V_1 = B_4 - \{MA\} = \{FI, ME, EC, OS\}$. Then $U_{V1}/R = \big\{\{T_1, T_5, T_8\}\big\}, \{T_2\}, \{T_3, T_6\}, \{T_4, T_7\}\}$. The nano topology and the basis are $\tau_{V1}(X) = \{\emptyset, U, \{T_2\}, \{T_1, T_3, T_4, T_5, T_6, T_7, T_8\}\}$, $\beta_{V1}(X) = \{U, \{T_2\}, \{T_1, T_3, T_4, T_5, T_6, T_7, T_8\}\}$. This implies that $\beta_{V1}(X) \neq \beta_C(X)$.

If $V_2 = B_4 - \{FI\} = \{MA, ME, EC, OS\}$, then $U_{V2}/R = \{\{T_1\}, \{T_2, T_3, T_6\}, \{T_4\}, \{T_5, T_8\}, \{T_7\}\}$ and the nano topology is $\tau_{V2}(X) = \{\emptyset, U, \{T_1, T_7\}, \{T_2, T_3, T_5, T_6, T_8\}, \{T_1, T_2, T_3, T_5, T_6, T_7, T_8\}\} \neq \tau_C(X)$.
Thus, $\beta_{V2}(X) = \{U, \{T_1, T_7\}, \{T_2, T_3, T_5, T_6, T_8\}\} \neq \beta_C(X)$.

If $V_3 = B_4 - \{ME\} = \{MA, FI, EC, OS\}$. Then $U_{V3}/R = \{ \{T_1\}, \{T_2\}, \{T_3, T_6\}, \{T_4\}, \{T_5, T_8\}, \{T_7\}\}$ and $\tau_{V3}(X) = \{\emptyset, U, \{T_1, T_2, T_7\}, \{T_3, T_5, T_6, T_8\}, \{T_1, T_2, T_3, T_5, T_6, T_7, T_8\}\} = \tau_C(X)$. Therefore, $\beta_{V3}(X) = \beta_C(X)$.

If $V_4 = B_4 - \{EC\} = \{MA, FI, ME, OS\}$, then $U_{V4}/R = \{\{T_1\}, \{T_2\}, \{T_3, T_6\}, \{T_4\}, \{T_5, T_8\}, \{T_7\}\}$ and $\tau_{V4}(X) = \{\emptyset, U, \{T_1, T_2, T_7\}, \{T_3, T_5, T_6, T_8\}, \{T_1, T_2, T_3, T_5, T_6, T_7, T_8\}\}$ $= \tau_C(X)$. $\beta_{V4}(X) = \beta_C(X)$.

If $V_5 = B_4 - \{OS\} = \{MA, FI, ME, EC\}$, then $U_{V5}/R = \{\{T_1\}, \{T_2\}, \{T_3, T_6\}, \{T_4\}, \{T_5, T_8\}, \{T_7\}\}$ and $\tau_{V5}(X) = \{\emptyset, U, \{T_1, T_2, T_7\}, \{T_3, T_5, T_6, T_8\}, \{T_1, T_2, T_3, T_5, T_6, T_7, T_8\}\}$ $= \tau_C(X)$. $\beta_{V5}(X) = \beta_C(X)$.

Therefore V_3, V_4, V_5 are the criterion reducts.

CORE(A) = $V_3 \cap V_4 \cap V_5 = \{MA, FI, EC, OS\} \cap \{MA, FI, ME, OS\} \cap \{MA, FI, ME, EC\}$
$= \{MA, FI\}$.

So, CORE={Mark, Family Income}

Step-4: $B_5 = C - \{EC\} = \{MA, FI, ME, RE, OS\}$.

If $W_1 = B_5 - \{MA\} = \{FI, ME, RE, OS\}$. Then $U_{W1}/R = \{\{T_1\}, \{T_2\}, \{T_3, T_6\}, \{T_4, T_7\}, \{T_5, T_8\}\}$ and $\tau_{W1}(X) = \{\emptyset, U, \{T_1, T_2\}, \{T_3, T_4, T_5, T_6, T_7, T_8\}\}$, $\beta_{W1}(X) = \{U, \{T_1, T_2\}, \{T_3, T_4, T_5, T_6, T_7, T_8\}\} \neq \beta_C(X)$.

If $W_2 = B_5 - \{FI\} = \{MA, ME, RE, OS\}$. Then $U_{W2}/R = \{\{T_1, T_7\}, \{T_2, T_3, T_6\}, \{T_4\}, \{T_5, T_8\}\}$ and the corresponding nano topology is given by $\tau_{W2}(X) = \{\emptyset, U, \{T_1, T_7\}, \{T_2, T_3, T_5, T_6, T_8\}, \{T_1, T_2, T_3, T_5, T_6, T_7, T_8\}\} \neq \tau_C(X)$. Therefore, $\beta_{W2}(X) \neq \beta_C(X)$.

If $W_3 = B_5 - \{ME\} = \{MA, FI, RE, OS\}$. Then $U_{W3}/R = \{\{T_1\}, \{T_2\}, \{T_3, T_6\}, \{T_4\}, \{T_5, T_8\}, \{T_7\}\}$ and $\tau_{W3}(X) = \{\emptyset, U, \{T_1, T_2, T_7\}, \{T_3, T_5, T_6, T_8\}, \{T_1, T_2, T_3, T_5, T_6, T_7, T_8\}\}$ $= \tau_C(X)$. So, $\beta_{W3}(X) = \beta_C(X)$.

If $W_4 = B_5 - \{RE\} = \{MA, FI, ME, OS\}$. Then $U_{W4}/R = \{\{T_1\}, \{T_2\}, \{T_3, T_6\}, \{T_4\}, \{T_5, T_8\}, \{T_7\}\}$ and $\tau_{W4}(X) = \{\emptyset, U, \{T_1, T_2, T_7\}, \{T_3, T_5, T_6, T_8\}, \{T_1, T_2, T_3, T_5, T_6, T_7, T_8\}\}$ $= \tau_C(X)$. $\beta_{W4}(X) = \beta_C(X)$.

Similarly, if $W_5 = B_5 - \{OS\} = \{MA, FI, ME, RE\}$. Then $U_{W5}/R = \{\{T_1\}, \{T_2\}, \{T_3, T_6\}, \{T_4\}, \{T_5, T_8\}, \{T_7\}\}$ and the corresponding nano topology is given by $\tau_{W5}(X) = \{\emptyset, U, \{T_1, T_2, T_7\}, \{T_3, T_5, T_6, T_8\}, \{T_1, T_2, T_3, T_5, T_6, T_7, T_8\}\} = \tau_C(X)$. Thus, $\beta_{W5}(X) = \beta_C(X)$.

Hence W_3, W_4, W_5 are the criterion reducts and CORE(A)=$W_3 \cap W_4 \cap W_5$
$=\{MA, FI, RE, OS\} \cap \{MA, FI, ME, OS\} \cap \{MA, FI, ME, RE\} = \{MA, FI\}$

That is CORE={Mark, Family Income}

Step-5: $B_6 = C - \{OS\} = \{MA, FI, ME, RE, EC\}$.

If $Y_1 = B_6 - \{MA\} = \{FI, ME, RE, EC\}$. Then $U_{Y1}/R = \{\{T_1\}, \{T_2\}, \{T_3, T_6\}, \{T_4, T_7\}, \{T_5, T_8\}\}$ and $\tau_{Y1}(X) = \{\emptyset, U, \{T_1, T_2\}, \{T_3, T_4, T_5, T_6, T_7, T_8\}\}$. The basis is $\beta_{Y1}(X) = \{U, \{T_1, T_2\}, \{T_3, T_4, T_5, T_6, T_7, T_8\}\} \neq \beta_C(X)$

If $Y_2 = B_6 - \{FI\} = \{MA, ME, RE, EC\}$. Then $U_{Y2}/R = \{\{T_1\}, \{T_2, T_3, T_6\}, \{T_4\}, \{T_5, T_8\}, \{T_7\}\}$ and the nano topology is $\tau_{Y2}(X) = \{\emptyset, U, \{T_1, T_7\}, \{T_2, T_3, T_5, T_6, T_8\}, \{T_1, T_2, T_3, T_5, T_6, T_7, T_8\}\} \neq \tau_C(X)$. Therefore, $\beta_{Y2}(X) \neq \beta_C(X)$.

Similarly, for $Y_3 = B_6 - \{ME\} = \{MA, FI, RE, EC\}$, $Y_4 = B_6 - \{RE\} = \{MA, FI, ME, EC\}$ and $Y_5 = B_6 - \{EC\} = \{MA, FI, ME, RE\}$,
$U_{Y3}/R = U_{Y4}/R = U_{Y5}/R = \{\emptyset, \{T_1\}, \{T_2\}, \{T_3, T_6\}, \{T_4\}, \{T_5, T_8\}, \{T_7\}\}$ and
$\tau_{Y3}(X) = \{\emptyset, U, \{T_1, T_2, T_7\}, \{T_3, T_5, T_6, T_8\}, \{T_1, T_2, T_3, T_5, T_6, T_7, T_8\}\} = \tau_{Y4}(X) = \tau_{Y5}(X) = \tau_C(X)$. Thus, $\beta_{Y3}(X) = \beta_{Y4}(X) = \beta_{Y5}(X) = \beta_C(X)$.

Hence Y_3, Y_4, Y_5 are the criterion reducts and CORE(A)=$Y_3 \cap Y_4 \cap Y_5$
=$\{MA, FI, RE, EC\} \cap \{MA, FI, ME, EC\} \cap \{MA, FI, ME, RE\} = \{MA, FI\}$
Therefore, CORE={Mark, Family Income}

<u>Case-2</u>: Students who didn't received the Scholarship

Assume $X = \{T_4, T_6, T_8\}$, the set of students not selected for the scholarship. Then, $U_C/R = \{\{T_1\}, \{T_2\}, \{T_3, T_6\}, \{T_4, T_7\}, \{T_5, T_8\}\}$ and the lower and upper approximations are $L_C(X) = \{T_4\}$,$U_C(X) = \{T_3, T_4, T_5, T_6, T_8\}$. The boundary region is $B_C(X) = \{T_3, T_5, T_6, T_8\}$. The nano topology on U is $\tau_C(X) = \{\emptyset, U, \{T_4\}, \{T_3, T_5, T_6, T_8\}, \{T_3, T_4, T_5, T_6, T_8\}\}$ and the basis for $\tau_C(X)$ is $\beta_C(X) = \{U, \{T_4\}, \{T_3, T_4, T_5, T_6, T_8\}\}$.

Step-1:

Let $B_1 = C - \{MA\}$. Then $U_{B1}/R = \{\{T_1\}, \{T_2\}, \{T_3, T_6\}, \{T_4, T_7\}, \{T_5, T_8\}\}$ and the lower and upper approximations are $L_{B1}(X) = \emptyset$,$U_{B1}(X) = \{T_3, T_4, T_5, T_6, T_7, T_8\}$ corresponding nano topology and the basis are $\tau_{B1}(X) = \{\emptyset, U, \{T_3, T_4, T_5, T_6, T_7, T_8\}\}$, $\beta_{B1}(X) = \{U, \emptyset, \{T_3, T_4, T_5, T_6, T_7, T_8\}\}$. This implies that $\beta_{B1}(X) \neq \beta_C(X)$.

If $B_2 = C - \{FI\}$. Then $\tau_{B2}(X) = \{\emptyset, U, \{T_4\}, \{T_2, T_3, T_5, T_6, T_8\}, \{T_2, T_3, T_4, T_5, T_6, T_8\}\}$, $\beta_{B2}(X) = \{U, \{T_4\}, \{T_2, T_3, T_5, T_6, T_8\}\}$. Thus $\beta_{B2}(X) \neq \beta_C(X)$.

If $B_3 = C - \{ME\}$. Then $\tau_{B3}(X) = \{\emptyset, U, \{T_4\}, \{T_3, T_5, T_6, T_8\}, \{T_3, T_4, T_5, T_6, T_8\}\}$, $\beta_{B3}(X) = \{U, \emptyset, \{T_3, T_5, T_6, T_8\}\}$=$\beta_C(X)$.

Similarly, if $B_4 = C - \{RE\}$, $B_5 = C - \{EC\}$, $B_6 = C - \{OS\}$, then $\tau_{B4}(X) = \{\emptyset, U, \{T_4\}, \{T_3, T_5, T_6, T_8\}, \{T_3, T_4, T_5, T_6, T_8\}\} = \tau_{B5}(X) = \tau_{B6}(X)$,
Thus $\beta_{B4}(X) = \beta_{B5}(X) = \beta_{B6}(X) = \beta_C(X)$.

Hence B_3, B_4, B_5, B_6 are the criterion reducts for student's selection and CORE=$B_3 \cap B_4 \cap B_5 \cap B_6$= {Marks, Family income}

4. Discussion

In this article, attribute reduction using the basis of nanotopological spaces is introduced and explored to improve the selection process of students eligible for the merit-based scholarship program. Applying this new approach, we sought to identify the key factors that play a central role in determining which deserving students will receive a scholarship. Our analysis revealed that the main attributes that influence the selection process are the students' marks and family income.

The academic performance of the students, as represented by their marks, serves as a critical indicator of their potential and dedication to their studies. On the other hand, family income is a key determinant of the financial needs of students. By considering both of these attributes, administrators can ensure that the scholarship reaches those students who retain the necessary academic merit and financial constraints to truly benefit from the financial assistance.

Applying the nano topological basis concepts in attribute reduction provides an efficient and systematic way to identify the most relevant attributes for stock market selection, identifying suitable crops for a particular season, etc. By focusing on key attributes, administrators can avoid unnecessary complexity and redundancy in the decision-making process, which facilitates efficient and fair resource allocation. Our findings contribute to the field of scholarship awarding and education policy as they provide an objective, data-driven method for identifying the most deserving students. This approach ensures that scholarships are awarded to those who need them and who have demonstrated their academic potential.

In conclusion, the concept of attribute reduction using the basis in Nano Topological Spaces proves to be a valuable tool in the scholarship selection process. By identifying key attributes of student grades and family income, we can ensure a more efficient and effective allocation of the scholarship program based on merit and means. This research serves as a foundation for further exploration and optimization of scholarship awarding methods, ultimately leading to a more inclusive and equitable education system.

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