Theta open sets in N-topology

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Abstract

This article elaborates about a novel approach on defining θ-open sets and its properties in N-topological space. We establish various continuous maps and discuss the relationship with θ-continuous maps using N-topology. We procure important results in θ-irresolute maps by θ open sets. Finally, we develop the idea of θ-connectedness properties and some new type of separation axioms in this space.

Keywords: N-topology, $N\tau\theta$-open, $N^{\#}\theta$-irresolute, $N\tau\theta$-connected, $N\tau\theta$-hausdorff, $N\tau\theta$-normal, $N\tau\theta$-regular.

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1. Introduction

J.C. Kelly [1] introduced bitopological space and its essential role to study the shapes of objects. In topological spaces, The notion of α-open sets was initiated by Njastad [2] and θopen sets, θclosed sets and θclosure are established by Velicko [3]. Lellis Thivagar et al. [4] originated the structure of N-topology and its continuous properties, weak forms [5] which are all helps to form more topologies on a non-empty set.

Lellis Thivagar et al. [6] introduced $N\tau\theta$-open sets and $N\tau\theta$-interior concepts in N-topology using $N\tau\theta$-closed sets. In this article, we explored θ-open sets in N-topological space in a different way and introduced some other properties. We defined its continuous maps and investigated the relationship with other continuous maps. Also, we derive important results in θ- irresolute maps by θ open sets. Further, introduced the concept of θ-connectedness and examined their properties. Finally, some sorts of separation axioms in N-topological space were discussed.

2. Preliminaries

In this part, we recollect some definitions and results which are beneficial in the sequent.

Definition 2.1 [1] A bitopological space (Z, P, Q) is a set Z with two topologies, P and Q on it.

Definition 2.2 [2] If Z is a non empty set and $\tau_1, \tau_2, \tau_3, \ldots, \tau_N$ are N arbitrary topologies defined on Z and the collection

$N\tau = \{P \subseteq Z : P = (\bigcup_{i=1}^{n} A_i) \cup (\bigcap_{i=1}^{n} B_i),\ A_i, B_i \in \tau_i\}$ persuades the axioms:

(i) $Z, \emptyset \in N\tau$

(ii) $\bigcup_{i=1}^{\infty} P_i \in N\tau$ for all $\{P_i\}_{i=1}^{\infty} \in N\tau$

(iii) $\bigcap_{i=1}^{n} P_i \in N\tau$ for all $\{P_i\}_{i=1}^{n} \in N\tau$.

Then $(Z, N\tau)$ is claimed as N topological space and $N\tau$ is called N topology.

Definition 2.3 [3] In $(Z, N\tau)$, $U \subseteq Z$ is said to be

(i) $N\tau$semi-open set if $U \subseteq N\tau cl(N\tau int(U))$

(ii) $N\tau$ pre-open set if $U \subseteq N\tau\ int(N\tau\ cl(U))$

(iii) $N\tau\alpha$-open set if $U \subseteq N\tau\ int(N\tau\ cl(N\tau\ int(U)))$

(iv) $N\tau\beta$-open set if $U \subseteq N\tau\ cl(N\tau\ int(N\tau\ cl(U)))$

Proposition 2.4 [2] Let the two N topological spaces are $(Z, N\tau)$ and $(Y, N\sigma)$. A mapping $\eta : (Z, N\tau) \rightarrow (Y, N\sigma)$ is N^*continuous on Z if $\eta^{-1}(V) \in N\tau O(Z)$ for every $V \in N\sigma O(Y)$.

Definition 2.5 [3] Let $(Z, N\tau)$ and $(Y, N\sigma)$ are two N topological spaces. A mapping η : $(Z, N\tau) \rightarrow (Y, N\sigma)$ is $N^*\alpha$ continuous (resp. N^*semi continuous, N^*pre continuous, $N^*\beta$ continuous) on Z if the inverse image of every $N\sigma$ open set in Y is a $N\tau\alpha$ open set (resp. $N\tau$ semi open, $N\tau$ pre open, $N\tau\ \beta$ open) in Z.

Definition 2.6 [6] Suppose $(X, N\tau)$ be a N topological space and $P \subseteq X$. An element $s \in X$ is said to be $N\tau$-θ cluster point of P if $P \cap N\tau\ cl(G) \neq \emptyset$ for every $N\tau$ open set G containing$s$. The set of all $N\tau$- θ cluster points of P is called $N\tau$- θ closure of P and is denoted by $N\tau$- $cl_\theta(P)$. A subset P of X is said to be $N\tau\theta$ closed in X if $N\tau\ cl_\theta(P) = P$. The complement of $N\tau\theta$closed set is $N\tau\theta$ open.

Definition 2.7 [6] Let $P \subseteq (X, N\tau)$ and an element $x \in P$ is $N\tau\theta$-interior point of P if $N\tau\ \theta cl(G) \subseteq P$ for some $N\tau$ open set G containing x. The set of all $N\tau\theta$ interior points of P is called $N\tau\theta$-interior of P and is denoted by $N\tau$- $int_\theta(P)$.

3. $N\tau\theta$-open sets and its properties

In this segment, we strive to define $N\tau\theta$-open sets in a different way and study its common properties which are θ-neighborhood, θ-limit point, θ-derived set, θ-frontier, θ-exterior in N topological space. We introduce $N\tau$-regular open and its connection with $N\tau\theta$-open sets.

Definition 3.1 A set $S \subseteq (P, N\tau)$ is called $N\tau\theta$-open if for every $p \in S$, $\exists$ a $N\tau O(\mathcal{G})$ such that $p \in \mathcal{G} \subseteq N\tau\ cl(\mathcal{G}) \subseteq S$.

Example 3.2 Let $P = \{1,2,3,4\}$, For $N = 3$, $\tau_1 = \{\emptyset, \{1,2\}, \{1,2,3\}, P\}$, $\tau_2 = \{\emptyset, \{3\}, \{3,4\}, P\}$ $\tau_3 = \{\emptyset, \{1,2\}, \{3\}, \{1,2,3\}, P\}$ and 3τ=$\{\emptyset, \{3\}, \{1,2\}, \{3,4\}, \{1,2,3\}, P\}$. Then $3\tau\theta O(P)$ = $\{\emptyset, \{1,2\}, \{3,4\}, P\}$.

Remark 3.3 The collection of all $N\tau\theta$-open sets produces a N topology and is signified as $N\tau_\theta$.

Definition 3.4 In $(P, N\tau)$, M is said to be $N\tau$-θ neighborhood of u if $\exists\ \mathcal{L} \in N\tau_\theta$ such that $u \in L \subseteq M$ where $u \in P$ and $M \subseteq P$.

Definition 3.5 Let $S \subseteq (P, N\tau)$ and $p \in P$ is called $N\tau\theta$ limit point of S, if for each $N\tau\theta$ open set $\mathcal{H}$ containing p , $\mathcal{H} \cap \{S - \{p\}\} \neq \emptyset$.

Definition 3.6 The collection of all $N\tau\theta$ limit points of S is called $N\tau\theta$-derived set of S and it is signified as $N\tau\ \theta_D(S)$.

Theorem 3.7 If $P, Q \subseteq (Z, N\tau)$ then

(i) $N\tau_D(P) \subset N\tau\ \theta_D(P)$ where $N\tau_D(P)$ is the $N\tau$ derived set of P

(ii) If $P \subset Q$ then $N\tau\,\theta_D(P) \subset N\tau\,\theta_D(Q)$
(iii) $N\tau\,\theta_D(P) \cup N\tau\,\theta_D(Q) = N\tau\,\theta_D(P \cup Q)$
(iv) $N\tau\,\theta_D(P \cap Q) \subset N\tau\,\theta_D(P) \cap N\tau\,\theta_D(Q)$
(v) $N\tau\,\theta_D(N\tau\,\theta_D(P)) - P \subset N\tau\,\theta_D(P)$
(vi) $N\tau\,\theta_D(P \cup N\tau\,\theta_D(P)) \subset P \cup N\tau\,\theta_D(P)$.

Proof:

(i) It is enough to perceive that every $N\tau\theta$-open set is $N\tau$open.
(ii), (iii), (iv) are obvious.
(v) If $x \in N\tau\,\theta_D(N\tau\,\theta_D(P)) - P$ and $V \in N\tau\theta O(Z)$ and $x \in V$ then $V \cap (N\tau\theta_D(P) - \{x\}) \neq \emptyset$.If $y \in V \cap N\tau\,\theta_D(P) - \{x\}$ then $y \in V$ and $y \in N\tau\,\theta_D(P)$. $V \cap (P - \{y\}) \neq \emptyset$. Now if $z \in V \cap (P - \{y\})$ then $z \neq x$ for $z \in P$ and $x \notin P$. $V \cap (P - \{x\}) \neq \emptyset$. Hence $x \in N\tau\,\theta_D(P)$.
(vi) Let $x \in N\tau\,\theta_D(P \cup N\tau\,\theta_D(P))$. If $x \in P$ then the result is true. Now, If $x \in N\tau\,\theta_D\big(P \cup N\tau\,\theta_D(P)\big) - P$ then for $N\tau$- θ open set V containing x then $V \cap (P \cup N\tau\,\theta_D(P)$-$\{x\}) \neq \emptyset$. So $V \cap (P - \{x\}) \neq \emptyset$ or $V \cap (N\tau\,\theta_D(P) - \{x\}) \neq \emptyset$. Now it arrives by (v) that $V \cap (P - \{x\}) \neq \emptyset$. Hence $x \in N\tau\,\theta_D(P)$ and $N\tau\,\theta_D\big(P \cup N\tau\,\theta_D(P)\big) \subset P \cup N\tau\,\theta_D(P)$.

Theorem 3.8 If $S \subseteq (P, N\tau)$ then $S \cup N\tau\,\theta_D(S) \subset N\tau$- $cl_\theta(S)$.

Proof: Since $N\tau\,\theta_D(S) \subset N\tau$- $cl_\theta(S)$, $S \cup N\tau\,\theta_D(S) \subset N\tau$- $cl_\theta(S)$.

Definition 3.9 Let $(P, N\tau)$ be a N topological space. The θ-border of a set $S \subset P$, denoted by $N\tau\,\theta_b(S)$ is $S - (N\tau$- $int_\theta(S))$

Theorem 3.10 If $S \subseteq (P, N\tau)$ then the following statements hold:
(i) $N\tau_b(S) \subset N\tau\,\theta_b(S)$ where $N\tau_b(S)$ denotes the $N\tau$-border of S
(ii) $S = N\tau$- $int_\theta(S) \cup N\tau\,\theta_b(S)$
(iii) $N\tau$- $int_\theta(S) \cap N\tau\,\theta_b(S) = \emptyset$
(iv) S is a $N\tau\,\theta$-open set if and only if $N\tau\,\theta_b(S) = \emptyset$
(v) $N\tau$- $int_\theta(N\tau\,\theta_b(S)) = \emptyset$
(vi) $N\tau\,\theta_b(N\tau\,\theta_b(S)) = N\tau\,\theta_b(S)$
(vii) $N\tau\,\theta_b(S) = S \cap N\tau$- $cl_\theta(P - S)$.

Proof: Obvious

Definition 3.11 Let $(P, N\tau)$ be a N topological space. The θ-frontier of a set $S \subset P$, signified by $N\tau\,\theta_{Fr}(S)$ is defined as $N\tau\,\theta_{Fr}(S) = N\tau$- $cl_\theta(S) - N\tau$- $int_\theta(S)$.

Example 3.12 Let $P = \{1,2,3,4\}$, For $N = 3$, $\tau_1 = \{\emptyset, \{1,2\}, \{1,2,3\}, P\}$, $\tau_2 = \{\emptyset, \{3\}, \{3,4\}, P\}$, $\tau_3 = \{\emptyset, \{1,2\}, \{3\}, \{1,2,3\}, P\}$ and $3\tau = \{\emptyset, \{3\}, \{1,2\}, \{3,4\}, \{1,2,3\}, P\}$.Let $S = \{1,2,3\}$ and $3\tau\theta$-$cl(S) = P$, 3τ- $int_\theta(S) = \{1,2\}$. Then $3\tau\,\theta_{Fr}(S) = \{3,4\}$.

Proposition 3.13 If $S \subseteq (P, N\tau)$ then the following statements hold:

(i) $N\tau_{Fr}(S) \subset N\tau\ \theta_{Fr}(S)$ where $N\tau_{Fr}(S)$ denotes the $N\tau$-frontier of S.

(ii) $N\tau\text{-}\ cl_\theta(S) = N\tau\text{-}\ int_\theta(S) \cup N\tau\ \theta_{Fr}(S)$

(iii) $N\tau\text{-}\ int_\theta(S) \cap N\tau\ \theta_{Fr}(S) = \emptyset$

(iv) $N\tau\theta_b(S) \subset N\tau\ \theta_{Fr}(S)$

(v) $N\tau\ \theta_{Fr}(S) = N\tau\text{-}\ cl_\theta(S) \cap N\tau\text{-}\ cl_\theta(P - S)$

(vi) $N\tau\ \theta_{Fr}(S) = N\tau\ \theta_{Fr}(P - S)$

(vii) $N\tau\ int_\theta(S) = S - (N\tau\ \theta_{Fr}(S))$.

(viii) $N\tau\ \theta_{Fr}(S)$ is $N\tau$-closed.

Definition 3.14 The θ-exterior of a set $S \subseteq (P, N\tau)$, signified by $N\tau\ \theta_{Ext}(S)$ is determined as $N\tau\ \theta_{Ext}(S) = N\tau\ int_\theta(P - S)$.

Example 3.15 Let $P = \{1,2,3,4\}$ and $S = \{1,2,3\}$. For $N = 3$, $\tau_1 = \{\emptyset, \{1,2\}, \{1,2,3\}, P\}$, $\tau_2 = \{\emptyset, \{3\}, \{3,4\}, P\}$, $\tau_3 = \{\emptyset, \{1,2\}, \{3\}, \{1,2,3\}, P\}$ and 3τ=$\{\emptyset, \{3\}, \{1,2\}, \{3,4\}, \{1,2,3\}, P\}$. Then $3\tau\ \theta_{Ext}(S) = \emptyset$**.**

Proposition 3.16 If $P, Q \subseteq (Z, N\tau)$ then the following statements hold:

(i) $N\tau\ \theta_{Ext}(P) \subset N\tau_{Ext}(P)$ where $N\tau_{Ext}(P)$ denotes the $N\tau$-exterior of P.

(ii) $N\tau\ \theta_{Ext}(P)$ is $N\tau$-open

(iii) $N\tau\ \theta_{Ext}(P) = N\tau\ int_\theta(Z - P) = Z - (N\tau\text{-}\ cl_\theta(P))$

(iv) $N\tau\ \theta_{Ext}\ (N\tau\ \theta_{Ext}\ (P)) = N\tau\ int_\theta(N\tau\text{-}\ cl_\theta(P))$

(v) If $P \subset Q$, then $N\tau\ \theta_{Ext}\ (P) \supset N\tau\ \theta_{Ext}\ (Q)$

(vi) $N\tau\ \theta_{Ext}\ (P \cup Q) = N\tau\ \theta_{Ext}\ (P) \cup N\tau\ \theta_{Ext}\ (Q)$

(vii) $N\tau\ \theta_{Ext}\ (P \cap Q) = N\tau\ \theta_{Ext}\ (P) \cap N\tau\ \theta_{Ext}\ (Q)$

(viii) $N\tau\ \theta_{Ext}\ (Z) = \emptyset$

(ix) $N\tau\ \theta_{Ext}\ (\emptyset) = Z$

(x) $N\tau\ \theta_{Ext}\ (Z - N\tau\ \theta_{Ext}\ (P)) \subset N\tau\ \theta_{Ext}\ (P)$

(xi) $N\tau\ int_\theta(P) \subset N\tau\ \theta_{Ext}\ (N\tau\ \theta_{Ext}\ (P))$

(xii) $Z = N\tau\ int_\theta(P) \cup N\tau\ \theta_{Ext}\ (P) \cup N\tau\ \theta_{Fr}\ (P)$.

Definition 3.17 A set E is $N\tau$-regular open if E=. $N\tau\text{-}int(N\tau\text{-}cl(E))$.

Theorem 3.18 If $\mathcal{H} \subset (P, N\tau)$ is $N\tau\theta$-open and $x \in \mathcal{H}$ then $\exists$ a $N\tau$-regular open set $\mathcal{G}$ $\ni: x \in \mathcal{G} \subset N\tau\text{-}cl(\mathcal{G}) \subset \mathcal{H}$.

Proof: Since H is $N\tau\text{-}\theta$ open, there exist an $N\tau$open set $W \ni: x \in W \subset N\tau\text{-}cl(W) \subset H$. But $N\tau\text{-}int(N\tau\text{-}cl(W)) = G$ is $N\tau$ -regular open and if follows that $x \in \mathcal{G} \subset N\tau\text{-}cl(\mathcal{G}) \subset \mathcal{H}$ due to the fact that $N\tau\text{-}cl(W) \subset \mathcal{H}$.

Corollary 3.19 The set $\mathcal{H}$ is $N\tau\theta$-open $\Leftrightarrow$ for each $y \in \mathcal{H}$, there exist a $N\tau$-regular open $\mathcal{G}$ such that $y \in \mathcal{G} \subset N\tau\text{-}cl(\mathcal{G}) \subset \mathcal{H}$.

Definition 3.20 A space $(P, N\tau)$ is $N\tau$ almost-regular if $\forall\, p \in P$ and $N\tau$-regular closed set $\mathcal{G}$ not containing p, there exists disjoint $N\tau$ open sets L and $M \ni: x \in L$ and $\mathcal{G} \subset M$.

4. $N^*\theta$-continuous maps

In this segment, we establish various continuous maps and discuss the relationship with θ-continuous maps in N topological space. Moreover, explored Urysohn space, θ–irresolute and its properties using N-topology. Throughout this segment P, Q and R represents the N-topological spaces $(P, N\tau)$, $(Q, N\sigma)$ and $(R, N\rho)$ respectively.

Definition 4.1 A mapping $\eta: P \to Q$ is N^*almost-continuous at $u \in P \Leftrightarrow$ for all $N\sigma$-open set $\mathcal{H} \subset Q$ containing $\eta(u)$, $N\tau\text{-}cl(\eta^{-1}(\mathcal{H}))$ is a $N\tau$-neighborhood of u. If this convinced for every $u \in P$, then η is said to be N^*almost continuous.

Definition 4.2 A mapping $\eta: P \to Q$ is $N^*\theta$-continuous if $\eta^{-1}(\mathcal{H}) \in N\tau_\theta\ \forall\, \mathcal{H} \in N\sigma$.

Definition 4.3 A mapping $\eta: P \to Q$ is N^*weakly continuous at $u \in P$ if for every $N\sigma$ open set $\mathcal{H}, \eta(u) \in \mathcal{H} \subset Q\ \exists$ an $N\tau$ open set $\mathcal{G}, u \in \mathcal{G} \subset P \ni: \eta(\mathcal{G}) \subseteq N\sigma\text{-}cl(\mathcal{H})$. If this convinced for every $u \in P$ then η is N^*weakly continuous.

Definition 4.4 A N topological space $(P, N\tau)$ is $N\tau$-urysohn if for every $u \neq v \in P$, $\exists$ an $N\tau$ open sets $\mathcal{G}$ and $\mathcal{H}$, $u \in \mathcal{G}$ and $v \in \mathcal{H} \ni: N\tau\text{-}cl\,(\mathcal{G}) \cap N\tau\text{-}cl\,(\mathcal{H}) = \emptyset$.

Proposition 4.5 If η, γ be N^*weakly-continuous from P to a $N\sigma$ urysohn space Q then $A = \{u \in P: \eta(u) = \gamma(u)\}$ is a $N\tau$ closed set.

Corollary 4.6 Let η, γ be N^*weakly-continuous from P to a $N\sigma$-urysohn space Q and if η, γ coincide on a $N\tau$ dense subset of P then $\eta = \gamma$ everywhere.

Corollary 4.7 Let η, γ be $N^*\theta$-continuous from P to a $N\sigma$-urysohn space Q and if η, γ agree on a $N\tau\theta$ dense subset of P then $\eta = \gamma$ everywhere.

Corollary 4.8 Let η, γ be N^*strongly θ-continuous from P to a $N\sigma$ Hausdorff space Q. If η, γ agree on a $N\tau\theta$ dense subset of P then $\eta = \gamma$ all over.

Definition 4.9 A mapping $\eta: P \to Q$ is N^* faintly continuous if $\eta^{-1}(U)$ is $N\tau$-open for every $N\sigma\text{-}\theta$ open set $U \in Q$

Definition 4.10 A mapping $\eta: P \to Q$ is N^*quasi θ-continuous if $\eta^{-1}(U)$ is $N\tau\theta$-open for every $N\sigma\theta$-open set $U \in Q$.

Proposition 4.11 If a mapping $\eta: P \rightarrow Q$ is N^{*}weakly continuous then η is N^{*}faintly continuous.

Proof: If $u \in P$ and G is a $N\sigma\theta$ open $\ni$: $\eta(u) \in G \subseteq Q$ then $\exists$ an $N\sigma$ open set $M \ni$: $\eta(u) \in M \subset N\sigma\text{-}cl(M) \subset G$. Since, η is N^{*}weakly-continuous then $\exists$ an $N\tau$ open set K containing $u \ni$: $\eta(K) \subset N\sigma\text{-}cl(M) \subset G$. Consequently η is N^{*} faintly-continuous.

Proposition 4.12 Every N^{*}weakly θ-continuous is N^{*}quasi θ-continuous.

Theorem 4.13 Every N^{*}weakly-continuous need not be $N^{*}\theta$-continuous.

Proof: This can be proved by an simple example.

Let $P = \{l, m, n, o\}$. For $N = 3$, $\tau_1 = \{\emptyset, \{l, m, n\}, P\}$, $\tau_2 = \{\emptyset, \{n\}, P\}$ $\tau_3 = \{\emptyset, \{n, o\}, P\}$ and 3τ = $\{\emptyset, \{l, m, n\}, \{n\}, \{n, o\}, P\}$. Let $Q = \{r, s, t, u\}$. For $N = 3$, $\sigma_1 = \{\emptyset, \{r, s\}, \{u\}, \{r, s, u\}, Q\}$, $\sigma_2 = \{\emptyset, \{s, u\}, \{s, t, u\}, Q\}$, $\sigma_3 = \{\emptyset, \{s\}, \{r, s\}, Q\}$ and 3σ = $\{\emptyset, \{r, s\}, \{s\}, \{u\}, \{s, u\}, \{r, s, u\}, \{s, t, u\}, Q\}$. Define $\gamma: P \rightarrow Q$ by $\gamma(l) = r$, $\gamma(m) = s, \gamma(n) = t$, $\gamma(o) = u$. Here γ is 3^{*}weakly continuous but not a $3^{*}\theta$continuous.

Lemma 4.14 If $\eta: P \rightarrow Q$ is N^{*}faintly continuous and $\gamma: P \rightarrow Q$ is N^{*}quasi θ continuous (res.N^{*}strongly θ continuous) then $\gamma \circ \eta: P \rightarrow R$ is N^{*}faintly continuous (res.,N^{*} continuous).

Lemma 4.15 If $\eta: P \rightarrow Q$ is N^{*}continuous and $\gamma: Q \rightarrow R$ N^{*}faintly-continuous then $\gamma \circ \eta: P \rightarrow R$ is N^{*}faintly continuous.

Lemma 4.16 If $\eta: P \rightarrow Q$ is a N^{*}quasi θ-continuous and $\gamma: Q \rightarrow R$ is N^{*} quasi θcontinuous then $\gamma \circ \eta: P \rightarrow R$ is N^{*}quasi θcontinuous. (res. N^{*} strongly θ-continuous).

Proposition 4.17 If $\eta:(P, N\tau) \rightarrow (Q, N\sigma)$ is a mapping then the following are similar:

(1) η is $N^{*}\theta$-continuous

(2) The inverse image of each $N\sigma$ closed set in Q is a $N\tau\text{-}\theta$ closed set in P

(3) $N\tau\ cl_{\theta}[\eta^{-1}(\mathcal{H})] \subseteq \eta^{-1}[N\sigma\text{-}cl(\mathcal{H})]$ for every $\mathcal{H} \subseteq Q$

(4) $\eta[N\tau\text{-}\ cl_{\theta}(\mathcal{G})] \subseteq N\sigma\text{-}cl[\eta(\mathcal{G})]$, for every $\mathcal{G} \subseteq P$

(5) $\eta^{-1}(N\sigma\text{-}int(\mathcal{G})) \subseteq N\tau\ int_{\theta}(\eta^{-1}(\mathcal{G}))$ for every $\mathcal{G} \subseteq Q$.

Proof: (1) $\Longrightarrow$ (2) Let $\mathcal{H} \subseteq Q$ be an $N\sigma$ closed. Since η is $N^{*}\theta$ continuous $\eta^{-1}(Q - \mathcal{H})$= $P - \eta^{-1}(\mathcal{H})$ is $N\tau\ \theta$ open. Hence $\eta^{-1}(\mathcal{H})$ is $N\tau\text{-}\theta$closed in P.

(2) $\Longrightarrow$ (3) Since $N\sigma\text{-}cl(\mathcal{H})$ is $N\sigma$ closed $\forall \mathcal{H} \subseteq Q$ then $\eta^{-1}(N\sigma\text{-}cl(\mathcal{H}))$ is $N\tau\theta$-closed. Hence $\eta^{-1}(N\sigma\text{-}cl(\mathcal{H})) = N\tau -\ cl_{\theta}\ [\eta^{-1}(N\sigma\text{-}cl(\mathcal{H}))] \supseteq N\tau\ - cl_{\theta}[\eta^{-1}(\mathcal{H})]$.

(3) $\Longrightarrow$ (4) Let $\mathcal{G} \subseteq P$ and $\eta(\mathcal{G})$=V. Then $N\tau -\ cl_{\theta}\big(\eta^{-1}(V)\big) \subseteq \eta^{-1}(N\sigma\text{-}cl\ (V))$. Thus $N\tau\text{-}\ cl_{\theta}(\mathcal{G}) \subseteq N\tau\text{-}\ cl_{\theta}\ [\eta^{-1}(\eta(\mathcal{G}))] \subseteq \eta^{-1}(N\sigma\text{-}cl\ (\eta(\mathcal{G})))$. Hence, it follows that $\eta(N\tau\text{-}cl_{\theta}(\mathcal{G})) \subseteq N\sigma\text{-}cl(\eta(\mathcal{G}))$.

(4) $\Longrightarrow$ (5) Let $\mathcal{G} \subseteq Q$. By (4) $\eta[N\tau\text{-} cl_\theta(P - \eta^{-1}(\mathcal{G})] \subseteq N\sigma\text{-} cl_\theta(\eta(P - \eta^{-1}(\mathcal{G}))$ and $\eta[P - (N\tau\text{-} int_\theta(\eta^{-1}(\mathcal{G})))] \subseteq N\sigma\text{-}cl(Q\text{-} \mathcal{G}) = Q - (N\sigma\, int(\mathcal{G}))$. Therefore $P - [N\tau\text{-} int_\theta(\eta^{-1}(\mathcal{G}))] \subseteq \eta^{-1}(Q - (N\sigma\text{-} int(\mathcal{G})))$ and hence $\eta^{-1}(N\sigma\, int(G)) \subseteq N\tau\, int_\theta\, (\eta^{-1}(\mathcal{G}))$ for every $\mathcal{G} \subseteq Q$.
(5) $\Longrightarrow$ (1) If $\mathcal{G} \subseteq Q$ be an $N\sigma$ open set and $\eta^{-1}(N\sigma\text{ -}int(\mathcal{G}) \subseteq N\tau\, int_\theta(\eta^{-1}(\mathcal{G}))$. Then $\eta^{-1}(\mathcal{G}) \subseteq N\tau\, int_\theta(\eta^{-1}(\mathcal{G}))$. But $N\tau\, int_\theta(\eta^{-1}(\mathcal{G})) \subseteq \eta^{-1}(\mathcal{G})$. Hence $N\tau\, int_\theta(\eta^{-1}\,(\mathcal{G})) = \eta^{-1}(\mathcal{G})$. Therefore $\eta^{-1}(\mathcal{G})$ is $N\tau\theta$-open in P.

Proposition 4.18 Let $S \subseteq Q \subseteq P$, Q is $N\tau\theta$-open in P and S is $N\sigma\theta$-open in Q. Then S is $N\tau\ \theta$- open in P.

Proposition 4.19 If a mapping $\eta: (P, N\tau) \to (Q, N\sigma)$ and $\{U_v: v\ \in I\}$ is a cover of $P \ni : U_v\ \in N\tau_\theta$ for every $v\ \in I$. Suppose $\eta|U_v: U_v \to Q$ is $N^*\theta$continuous for every $v \in I$ and then η is $N^*\theta$continuous.

Proof: Let $\mathcal{H} \subseteq Q$ be $N\sigma$ open set. Now $(\eta|U_v)^{-1}(\mathcal{H})$ is $N\tau\theta$-open in U_v for every $v \in I$. We know that $U_v$is $N\tau\theta$-open in P for each $v \in I$. By proposition 4.18, $(\eta|U_v)^{-1}(\mathcal{H})$ is $N\tau\text{-}\theta$ open in P for every $v \in I$. But $\eta^{-1}(\mathcal{H}) = \cup\,\{(\eta|U_v)^{-1}(H): v \in I\}$ then $\eta^{-1}(\mathcal{H}) \in N\tau_\theta$ since $N\tau_\theta$ is N-topology on P. This implies η is $N^*\theta$-continuous.

Proposition 4.20 If $(P, N\tau)$ and $(Q, N\sigma)$ are N topological spaces and $\eta : P \to Q$ is a mapping then the following are identical.

(a) $\eta: P \to (Q, N\sigma)$ is N^*faintly continuous.
(b) $\eta: P \to (Q, N\sigma_\theta)$ is N^*continuous.
(c) $\eta^{-1}(U)$ is $N\tau$ open for every $N\sigma\theta$-open set $U \in Q$.
(d) $\eta^{-1}(U)$ is $N\tau$ closed for every $N\sigma\theta$-closed set $U \in Q$.

Proof: The implication follows from definitions.

Theorem 4.21 If a mapping $\eta: P \to Q$ is N^*weakly continuous then η is N^*faintly continuous.

Proposition 4.22 A N^*faintly continuous map need not be N^*weakly continuous.

Proof: The following example proves this proposition. Let $P = \{0,1\}$, For $N = 2$, $\tau_1 = \{\emptyset, P\}$, $\tau_2 = \{\emptyset, P, \{1\}\}$ and 2τ=$\{\{\emptyset, P, \{1\}\}$ and let $Q = \{l.m.n\}$ with $\sigma_1 = \{\emptyset, Q, \{l\}\}$, $\sigma_2 = \{\emptyset, Q, \{m\}, \{l, m\}\}$ and 2σ = $\{\emptyset, Q, \{l\}, \{m\}, \{l, m\}\}$. Now if $\eta: P \to Q$ is defined as $\eta(0) = l$ and $\eta(1) = m$. Then η is not 2^*weakly continuous at $l = 0$, but η is 2^*-faintly continuous since the only $2\sigma\theta$-open set in Q is Q itself.

Corollary 4.23 If $(Q, N\sigma)$ is $N\sigma$ almost-regular and $\eta: Q \to (Q, N\sigma)$ then the following are equivalent:

(a) η is N^*faintly continuous

(b) η is N^*weakly continuous

(c) η is $N^*\theta$-continuous

(d) η is N^*almost-continuous.

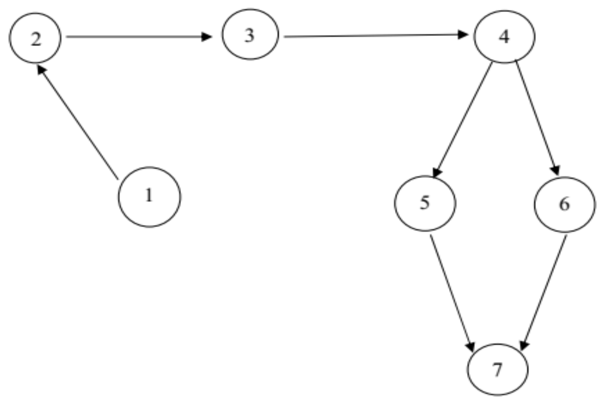
Proposition 4.24 If $\eta: P \to Q$ is N^*faintly continuous and $K \subset P$, then $\eta|K: K \to Q$ is N^* faintly -continuous.

Proposition 4.25 If $\eta: P \to Q$ is $N^*\theta$ continuous and $\gamma: Q \to R$ is $N^*\theta$ continuous then $\gamma o \eta$ is $N^*\theta$-continuous.

Definition 4.26 A mapping $\eta: (P, N\tau) \to (Q, N\sigma)$ is N^*strongly continuous (N^*strongly θ-continuous) at $u \in P$ if for every $N\sigma$ open set $\mathcal{H}, \eta(u) \in H \subset Q$, there exist an $N\tau$-open set $\mathcal{G}, u \in \mathcal{G} \subset P$ such that $\eta(N\tau\text{-}cl\ (\mathcal{G})) \subseteq \mathcal{H}$. If this convinced at every $u \in P$, then η is N^*strongly-continuous (N^*strongly θ-continuous)

Proposition 4.27 Let $\eta: P \to Q$ be N^*strongly θcontinuous and if $\gamma: Q \to R$ is N^*-continuous then $\gamma o \eta$ is N^*strongly θ-continuous.

Proposition 4.28 Let $\eta: P \to Q$ be $N^*\theta$-continuous and if $\gamma: Q \to R$ be N^*strongly θ-continuous then $\gamma o \eta$ is $N^*\theta$-continuous.



1. N^*strongly θ-continuous 2. N^*continuous 3. N^*almost-continuous
4. $N^*\theta$-continuous 5. N^* Quasi θ-continuous
6. N^*weakly continuous 7. N^*faintlycontinuous

Figure 1. The relationship between $N^*\theta$-continuous maps and other continuous maps.

Definition 4.29 A mapping $\eta: (P, N\tau) \to (Q, N\sigma)$ is $N^\#$-θirresolute if $\eta^{-1}(\mathcal{G}) \in N\tau_\theta$ for every $\mathcal{G} \in N\tau_\theta$.

Proposition 4.30 If $\eta: P \to Q$ is a mapping then the following are identical:

(i) η is $N^\#$-θ irresolute

(ii) $\eta^{-1}(U)$ is $N\tau$-θ closed for every $N\sigma$-θ closed set $U \in Q$

(iii) $N\tau$- $cl_\theta[\eta^{-1}(\mathcal{H})] \subseteq \eta^{-1}[\ N\sigma$- $cl_\theta(\mathcal{H})]$ for every $\mathcal{H} \subseteq Q$

(iv) $\eta[N\tau$- $cl_\theta(\mathcal{G})] \subseteq N\sigma$- $cl_\theta[\eta(\mathcal{G})]$ for every $\mathcal{G} \subseteq P$.

(v) $\eta^{-1}[N\sigma\ int_\theta(M)] \subseteq\ N\tau\ int_\theta[\eta^{-1}(Q)]$ for every $M \subseteq Q$.

Proposition 4.31 A mapping $\eta: P \to Q$ is $N^{\#}\theta$-irresolute $\Leftrightarrow$ for every point u in P and every $N\sigma$ -θ open M in Q with $\eta(u) \in M$, there is a $N\tau\ \theta$ open set L in $P \ni: u \in L$, $\eta(L) \subseteq M$.

Proof: Necessity: Let $u \in P$ and $M \in N\sigma_\theta \ni: f(u) \in M$. Let $L = \eta^{-1}(M)$. Since η is $N^{\#}$-θirresolute, L is $N\tau$- θopen in P. Also $u \in \eta^{-1}(M) = L$ as $\eta(u) \in M$. Hence $\eta(L) = \eta(\eta^{-1}(M)) \subseteq M$. S**ufficiency:** Let $M \in N\sigma_\theta$ and let $L = \eta^{-1}(M)$. we prove that L is $N\tau\theta$- open in M. Now, let $x \in L$. It implicit $\eta(x) \in M$. By conjecture, $\exists L_x \in N\tau_\theta \ni: x \in L_x$ and $\eta(L_x) \subseteq M$. Then $L_x \subseteq \eta^{-1}[\eta(L_x)] \subseteq \eta^{-1}(M) = L$. Thus $L = \cup \{L_x : x \in L\}$. These results L is $N\tau\theta$ open in P and η is $N^{\#}\theta$-irresolute.

Proposition 4.32 A mapping $\eta: (P, N\tau) \to (Q, N\sigma)$ is $N^{\#}\theta$-irresolute $\Leftrightarrow$ for every $u \in P$, the inverse image of every $N\sigma\theta$-neighborhood of $\eta(u)$, is a $N\tau\theta$-neighborhood of u.

Proof: Necessity Let $u \in P$ and L be a $N\sigma\theta$-neighborhood of $\eta(u)$. If there exist $M \in N\sigma_\theta$ such that $\eta(u) \in M \subseteq L$. This implies that $u \in \eta^{-1}(M) \subseteq \eta^{-1}(L)$. Since η is $N^{\#}\theta$ -irresolute, so $\eta^{-1}(M) \in N\tau_\theta$. Hence $\eta^{-1}(L)$ is $N\tau\theta$-neighborhood of u. **Sufficiency:** let $L \in N\sigma_\theta$ and $M = \eta^{-1}(L)$. Let $x \in M$ then $\eta(x) \in L$. But then L being $N\sigma\theta$-open set is a $N\sigma\theta$-neighborhood of $\eta(x)$. So by hypothesis $M = \eta^{-1}(L)$ is a $N\tau\theta$-neighborhood of x. By definition, $\exists M_x \in N\tau_\theta \ni: x \in M_x \subseteq M$. Hence $M = \cup \{M_x : x \in M\}$ and M is a $N\tau\ \theta$-open in P. So η is $N^{\#}\theta$ -irresolute.

5. $N\tau\theta$-connected and separation axioms

In this chapter, we establish the ideas of $N\tau\theta$-connected, $N\tau$-θ hausdorff, $N\tau - \theta$ regular, $N\tau$-θ normal spaces. Throughout this section $(P, N\tau)$ represents a N topological space on P.

Definition 5.1 $(P, N\tau)$ is $N\tau\theta$-disconnected if $P = L \cup M$ where $L \cap M = \emptyset$ and L$\neq \emptyset, M \neq \emptyset$. Or else, P is $N\tau\theta$-connected.

Proposition 5.2 In $(P, N\tau)$, the subsequent are identical:

(i) P is $N\tau\theta$-connected.

(ii) $\emptyset, P \subseteq P$ are the only $N\tau\theta$-open and $N\tau\theta$-closed sets.

(iii) $P \neq L \cup M, L \neq \emptyset,\ M \neq \emptyset$; $L \cap M = \emptyset$ where L & M are $N\tau\theta$-open sets.

(iv) There is no $N^*\theta$-continuous function $\eta: P \to E$ is surjective.

Proposition 5.3 $(P, N\tau)$ is $N\tau$ connected $\Leftrightarrow$ it is $N\tau\theta$-connected.

Proposition 5.4 $(P, N\tau)$ is $N\tau\theta$-connected $\Leftrightarrow$ for every $S \subseteq P$, $S \neq \emptyset$ such that $N\tau\ \theta_{Fr}(S) \neq \emptyset$.

Proof: **Necessity:** assuming P is $N\tau\ \theta$-disconnected. Then there exist$A \neq \emptyset$, $B \neq \emptyset$ and $A \cap B = \emptyset$ such that $P = A \cup B$. Here both A and B are $N\tau\theta$-open and $N\tau\theta$-closed in P. Therefore $A' = A^0 = \bar{A}$ but $N\tau\ \theta_{Fr}(A)$=$N\tau$- $cl_\theta(A) - N\tau\ int_\theta(A)$. Hence $N\tau\theta_{Fr}(A) = \emptyset$ and this is contrary to the conjecture. Consequently P must be $N\tau\theta$-connected. **Sufficiency:** let P be $N\tau\theta$-connected and suppose $\exists\, S \subset P, S \neq \emptyset \ni: N\tau\ \theta_{Fr}(S) = \emptyset$. Now $N\tau$- $cl_\theta(S) = N\tau\ int_\theta(S) \cup N\tau\ \theta_{Fr}(S) = S \cup N\tau\ \theta_{Fr}\ (N\tau\text{- } cl_\theta(S))$. Hence $S = N\tau\ int_\theta(S)$= $N\tau$-$cl_\theta(S)$ showing that S is both $N\tau\theta$-open and $N\tau\theta$-closed in P and therefore P is a $N\tau\theta$-disconnected which is a denial. Hence every proper subset of P must have a non empty $N\tau\theta$-frontier.

Definition 5.5 A *N* topological space *P* is

(i) $N\tau$ -θ hausdorff if every two disjoint elements p, q in P then $\exists$ distinct $N\tau$ -θopen sets G, $H \ni: p \in G$ and $q \in H$

(ii) $N\tau$- θ regular if for every $N\tau$ closed set C and every point $x \notin C$, $\exists$ disjoint $N\tau$ -θopen sets $G, H \ni: x \in G$ and $C \subseteq H$

(iii) $N\tau$ –θ normal if for any two distinct $N\tau$closed sets $C, D \subseteq P$, $\exists$ distinct $N\tau$ -θopen sets G, $H \ni: C \subseteq G$ and $D \subseteq H$.

Remark 5.6 Every $N\tau$-θ hausdorff (resp. $N\tau$-θ regular, $N\tau$-θ normal) space is $N\tau$ -hausdorff (res. $N\tau$-regular, $N\tau$-normal).

Proposition 5.7 In $(P, N\tau)$ the subsequent are identical:

(i) P is $N\tau$-θ hausdorff

(ii) let $a \in P$ for $b \neq a$, $\exists$ a $N\tau\theta$-open set G, $a \in G \ni: b \notin N\tau$- $cl_\theta(G)$

(iii) for every $a \in P$, $D = \bigcap\{N\tau\ \theta\text{-}cl(G): G$ is $N\tau\theta$-open and $x \in G\} = \{x\}$.

Proof:

$(i) \Longrightarrow (ii)$ For any different elements $a, b \in P$, $\exists$ disjoint $N\tau\theta$-open sets $G, H \ni: x \in G$ and $y \in H$. Hence $b \notin N\tau$- $cl_\theta(G)$

$(ii) \Longrightarrow (iii)$ Suppose $x \in D$, for each $b \neq a$ $\exists$ a $N\tau\theta$-open set $G, x \in G$ $\ni$: $b \notin N\tau$-$cl_\theta(G)$. Thus $b \notin D$ since b is arbitrary $D = \{a\}$.

$(iii) \Longrightarrow (i)$ If $a, b \in X$ then $\ni$: $a \neq b$. suppose $\exists$ a $N\tau\theta$-open set $G, x \in G$ $\ni$: $b \notin N\tau$-$cl_\theta(G)$. By remark 5.6, $\exists$ a $N\tau\ \theta$-open set $H, b \in H \ni : G \cap H = \emptyset$. Therefore P is $N\tau$ -θ hausdorff.

Proposition 5.8 In $(P, N\tau)$, the subsequent are identical:

(i) P is $N\tau\theta$- regular

(ii) for every $z \in P$ and an $N\tau$ open set G, $z \in G$ then $\exists$ a $N\tau\theta$-open set $H \ni$: $z \in H \subseteq N\tau\text{-}\, cl_\theta(H) \subseteq G$

(iii) for every $z \in P$ and $N\tau$ closed set C with $z \notin C$ then $\exists$ a $N\tau\theta$-open set V, $z \in V \ni$: $D \cap N\tau\text{-}\, cl_\theta\,(H) = \emptyset$.

Proof:

$(i) \Longrightarrow (ii)$ Suppose $z \in P$ and an $N\tau$ open set G, $z \in G$ then $\exists$ disjoint $N\tau\theta$-open sets H,W $\ni: z \in H$ and $P - G \subseteq W$. Since $N\tau\text{-}\, cl_\theta(H) \cap (P - G) \subseteq N\tau\text{-}\, cl_\theta(H) \cap N = \emptyset$ then $N\tau\text{-}\, cl_\theta(H) \subseteq G$. Therefore $z \in H \subseteq N\tau\text{-}\, cl_\theta(H) \subseteq G$.

$(ii) \Longrightarrow (iii)$ Suppose $z \in P$, and C be an $N\tau$ closed set with $z \notin C$ then $\exists$ $N\tau\theta$-open set V, $z \in V \ni: V \subseteq N\tau\text{-}cl_\theta(V) \subseteq P - C$.Hence $N\tau\text{-}cl_\theta(V) \cap C = \emptyset$.

$(iii) \Longrightarrow (i)$ Suppose $z \in P$, and C be an $N\tau$ closed set with $z \notin C$, then $\exists$ $N\tau\theta$-open set V, $z \in V \ni$: $N\tau\text{-}\, cl_\theta(V) \cap C = \emptyset$. Since $P - N\tau\text{-}\, cl_\theta(V)$ is a $N\tau\theta$-open set and $C \subseteq P - N\tau\text{-}\, cl_\theta(V)$. Further, $V \cap P - N\tau\text{-}cl_\theta(V) = \emptyset$. Hence P is $N\tau\theta$-regular.

Proposition 5.9 In $(P, N\tau)$, the subsequent are identical:

(i) P is $N\tau - \theta$ normal

(ii) if for every $N\tau$ closed set G and an $N\tau$ open set $H \supset G$, $\exists$ a $N\tau\theta$-open set K, $G \subseteq K \ni: N\tau\text{-}\, cl_\theta(K) \subseteq H$

(iii) for every two disjoint $N\tau$ closed sets G and L then $\exists$ a $N\tau\theta$-open set K, $G \subseteq K$ $\ni$: $N\tau\text{-}\, cl_\theta(K) \cap L = \emptyset$.

Proof:

$(i) \Longrightarrow (ii)$ Suppose G is $N\tau$ closed set and H is a $N\tau$ open set , $H \supset G$. Now G and $P - H$ are disjoint $N\tau$ closed sets in P. S Then there exist a disjoint $N\tau\theta$-open sets K, $W \ni$ $: G \subseteq K$ and $P - H \subseteq W$. We know that $N\tau\text{-}\, cl_\theta(K) \subseteq N\tau\text{-}\, cl_\theta(P - W) = P - W$. Consequently, $N\tau\text{-}\, cl_\theta(H) \subseteq P - W \subseteq H$.

$(ii) \Longrightarrow (iii)$ Suppose $G, L \subseteq P$ are two distinct $N\tau$ closed sets. Now $G \subseteq P - L$ and $P - L$ is $N\tau$open. As presumption, there exist $N\tau\ \theta$-open sets K, $K \supseteq G \ni$: $N\tau\text{-}\, cl_\theta(K) \subseteq P - L$ and hence $N\tau\text{-}\, cl_\theta(K) \cap L = \emptyset$.

$(iii) \Longrightarrow (i)$ Suppose $G, L \subseteq P$ are two distinct $N\tau$closed sets then there exist a $N\tau\theta$-open sets K containing $G \ni$: $N\tau\text{-}\, cl_\theta(K) \cap L = \emptyset$ and $L \subseteq P - (N\tau\text{-}\, cl_\theta(K))$. Since K and $P - (N\tau\text{-}\, cl_\theta(K))$ are distinct $N\tau\theta$-open sets then P is $N\tau$ normal.

Definition 5.10 A *N* topological space P is $N\tau\ T_1$-space ($res., N\tau T_1^\theta$space) if $\forall\, p, q \in P$ with $p \neq q$, $\exists$ an $N\tau$ open set G, H (res., $N\tau$ -θopen) $\ni$: $p \in G$, $q \notin G$ and $q \in H$, $p \notin V$.

Proposition 5.11 Let $(P, N\tau)$ be a $N\tau\ T_1^\theta$space. The following holds:

(i) if P is $N\tau$-θ regular then P is $N\tau$ -θ hausdorff

(ii) if P is $N\tau$ -θ normal then P is $N\tau$ -θ regular.

Proof:

(i) Let P is $N\tau$-θregular. When P is a $N\tau T_1$-space, $\forall\ a, b\ \in P$ with $a \neq b$ then there exist a distinct $N\tau$ open sets G and $H\ \ni : a \in G,\ b \notin G$ and $b \in H\ ,\ a \notin H$ which implies $a \notin P - G$and $b \notin P - H$. Since P is $N\tau$ –θregular, $\exists$ distinct $N\tau\theta$-open sets A, $B\ \ni$: $a \in A$ and $P - G \subseteq B$. Since $b \in P - G$ then $b \in B$. Thus P is $N\tau$–θ hausdorff.

(ii) Proof is similar.

6. Conclusion

In this article, we explored θ-open sets in N-topological space through a different approach. Moreover, we investigated the relationship of θ-continuous maps, θ-connectedness properties and some new type of separation axioms in this space. Thus, this concept can be used in defining several new topological spaces in addition to several types of mappings.

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