Ratio Mathematica Volume 43, 2022

Multi-block two repeated fixed burst error correcting linear codes

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Abstract

During the digital transmission of information, errors are bound to occur. The errors may be random or burst errors. In this paper, we have obtained necessary and sufficient conditions for the existence of linear codes over $GF(q)$ that are capable of correcting 2-repeated burst errors of length b_1(fixed) and 2-repeated burst errors of length b_2(fixed) within sub-blocks of unequal lengths. We have also constructed a $(14, 22)$ parity-check matrix for such a code.
Keywords: Linear codes; parity-check matrix; 2-repeated burst of length b(fixed); blockwise correction; syndromes.
2020 AMS subject classifications: 94B20; 94B65. [1]

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[1] Received on June 03, 2022. Revised on October 04, 2022. Accepted on October 19, 2022. Published on October 21, 2022. DOI: 10.23755/rm.v43i0.792. ISSN: 1592-7415. eISSN: 2282-8214.

1 Introduction

Today, we live in a world flooded with digital information and communication channels. A good communication system is one that conveys information from one point to another point without any distortion. When information is transmitted through digital channels, noise gets added which corrupts the information sent. So, we need to develop codes that are capable of correcting such distorted information so that the information received by the receiver is accurate.

It was seen that when data is transmitted, errors occur in adjacent positions rather than random positions. This led to the development of codes that could correct single errors and double adjacent errors by Abramson [1]. In these types of errors, digits in adjacent or consecutive positions are in error. Further, when the data is transmitted, lots of bits are transmitted in a fraction of seconds while the noise is for a longer period, so the probability of occurrence of errors in the adjacent positions is more than the probability of occurrence of random errors. So it becomes very important to develop the codes that can correct burst errors. A *burst of length* b, also known as Open loop burst error, has been defined by Fire [8] as a vector whose only non-zero components are among some b consecutive components, the first and the last of which are non-zero.

Another category of burst errors was introduced by Chien and Tang [3]. They defined *CT-burst of length* b as a vector whose only non-zero components are confined to some b consecutive positions, the first of which is non-zero. This definition was further modified by Dass [4] and he defined a *burst of length* b*(fixed)* as an n-tuple whose only non-zero components are confined to b consecutive positions, the first of which is non-zero and the number of its starting positions is among the first $n - b + 1$ components. For example, (001011000) is a burst of length 4 but can also be regarded as a burst of length upto 7(fixed) over $GF(2)$.

As more and more information is getting transmitted, the communication channels become extremely busy. This leads to the repetition of errors where errors may be random or burst errors. This led to the definition of repeated bursts of length b or less [2]. A *2-repeated burst of length* b *or less* is a vector whose non-zero components are in the form of two non-overlapping sets of b or less consecutive digits. For example, (001011100) is a 2-repeated burst of length 3 or less over $GF(2)$. In this paper, we consider repeated bursts of length b(fixed) [6].

A 2*-repeated burst of length* b*(fixed)* is defined as an n-tuple whose only non-zero components are confined to 2 distinct sets of b consecutive digits, the first component of each set is non-zero and the number of its starting positions is among the first $n - 2b + 1$ components [6]. We denote a 2-repeated burst of length b(fixed) by $2 - RB_f(b)$. For example, (10010000) is a 2-repeated burst of length atmost 3(fixed) and (001000001000) is a 2-repeated burst of length upto 4(fixed) over $GF(2)$.

The development of codes dealing with repeated bust errors improves the efficiency of the communication channels as the requirement of the parity-check digits is lesser in this case as compared to when we consider such errors as simple burst errors.

In this paper, we are dealing with (n, k) linear codes over $GF(q)$, where q is some power of a prime. The code of length n consists of two subblocks of length n_1 and n_2 such that $n = n_1 + n_2$. Also, the code of length n consists of $k = n - r$ information digits, where r denotes the number of parity check digits. The distance between two codewords is taken to be their Hamming distance [9]. The codes under consideration are able to correct all burst errors of length b_1(fixed) in a block of length n_1 and all burst errors of length b_2(fixed) in a block of length n_2. For an (n, k) linear code that corrects $2 - RB_f(b_1)$ in a sub-block of length n_1 and also corrects $2 - RB_f(b_2)$ in a sub-block of length n_2, following conditions must be satisfied:

1. The syndromes of all $2 - RB_f(b_1)$ occurring in the first sub-block (which is of length n_1) must be non-zero and distinct.
2. The syndromes of all $2-RB_f(b_2)$ occurring in the second sub-block (which is of length n_2) must be non-zero and distinct.
3. The syndromes of all $2 - RB_f(b_1)$ occurring in the first sub-block must be different from the syndromes of all $2 - RB_f(b_2)$ occurring in the second sub-block.

2 Necessary Condition

In this section, we obtain a necessary condition for existence of an (n, k) linear code over $GF(q)$ that corrects $2-$repeated bursts of length b_1(fixed) $(2-RB_f(b_1))$ in a block of length n_1 and also corrects $2-$repeated bursts of length b_2(fixed) $(2 - RB_f(b_2))$ in a block of length n_2, where $n = n_1 + n_2$. For a code that corrects any type of error patterns, the error patterns must lie in distinct cosets of the standard array or in other words, must have distinct syndromes. So, we get the following theorem.

Theorem 2.1. *For an (n, k) code over $GF(q)$ that corrects $2 - RB_f(b_1)$ in a block of length n_1 and also corrects $2 - RB_f(b_2)$ in a block of length n_2, where $(n = n_1 + n_2)$*

$$\log_q \left\{ 1 + \sum_{i=1}^{2} (q-1)^i \left\{ \binom{n_1 - ib_1 + i}{i} q^{i(b_1-1)} + \binom{n_2 - ib_2 + i}{i} q^{i(b_2-1)} \right\} \right\} \tag{1}$$

number of parity-check digits are necessary.

Proof. For $n = n_1 + n_2$, consider an (n, k) linear code that corrects $2 - RB_f(b_1)$ in a block of length n_1 and $2 - RB_f(b_2)$ in a block of length n_2. For a code to be able to correct such errors, the syndromes of these errors should be distinct. Also, the total number of syndromes is q^{n-k}. So, q^{n-k} must be greater than the number of syndromes corresponding to such errors.
Now, the number of syndromes corresponding to $2 - RB_f(b_1)$ in the first block of length n_1 is the same as the number of such errors in a vector of length n_1, which is [5]

$$\sum_{i=1}^{2}(q-1)^i\binom{n_1 - ib_1 + i}{i}q^{i(b_1-1)}. \tag{2}$$

Further, the number of syndromes corresponding to the error $2 - RB_f(b_2)$ in the second block of length n_2 is the same as the number of such errors in a vector of length n_2, which is [5]

$$\sum_{i=1}^{2}(q-1)^i\binom{n_2 - ib_2 + i}{i}q^{i(b_2-1)}. \tag{3}$$

Now, the total number of syndromes, including the all zero syndrome is q^{n-k}. So, for the desired code, we must have

$$q^{n-k} \geq 1 + \text{expression } (2) + \text{expression } (3).$$

Taking log, we get the desired result.
Remark: For $n_1 = n_2$ and $b_1 = b_2$ we obtain that

$$1 + 2\left\{\sum_{i=1}^{2}(q-1)^i\binom{n_2 - ib_2 + i}{i}q^{i(b_2-1)}\right\} \tag{4}$$

number of parity-check digits are necessary for existence of a code that corrects $2 - RB_f(b_1)$ in a block of length n_1 and also corrects $2 - RB_f(b_2)$ in a block of length n_2, which coincides with the necessary number of parity-check digits required for the location and correction of such errors in a code of length n with 2 subblocks of length $n_1 (= n_2)$ each [7].

3 Sufficient Condition

We now obtain an upper bound on the number of parity-check digits required for existence of the code considered in Theorem 2.1. The proof uses the technique of

Varshmov-Gilbert-Sacks bound [11, 10, 12]. This technique not only helps us to construct a parity-check matrix for the required code but also ensures the existence of such a code.

Theorem 3.1. *For existence of an (n,k) code over $GF(q)$ that corrects $2-RB_f(b_1)$ in a block of length n_1 and also corrects $2-RB_f(b_2)$ in a block of length n_2, where $(n=n_1+n_2)$,*

$$Max\Bigg[q^{b_2-1}\sum_{i=0}^{3}\binom{n_2-(i+1)b_2+i}{i}(q-1)^i q^{i(b_2-1)},$$
$$q^{b_1-1}\sum_{i=0}^{3}\binom{n_1-(i+1)b_1+i}{i}(q-1)^i q^{i(b_1-1)}$$
$$+\{q^{b_1-1}\{1+(n_1-2b_1+1)(q-1)q^{b_1-1}\}$$
$$\times\sum_{i=1}^{2}\binom{n_2-ib_2+i}{i}(q-1)^i q^{i(b_2-1)}\}\Bigg]$$

number of parity-check digits are sufficient.

Proof. We construct a parity-check matrix of the desired code. In the process of construction, we first add columns of the block of length n_2 (second block) followed by the columns of the block of length n_1 (first block), to obtain a preliminary matrix H'. After completing the process of addition of columns, we reverse the order of the columns of H' to obtain the desired parity-check matrix H.
Suppose first $j-1$, $j\leq n_2$, columns have been appropriately added to the second block. Since, the code is capable of correcting $2-RB_f(b_2)$ in a block of length n_2, so jth column is chosen based on the following condition:
For the correction of $2-RB_f(b_2)$ in second block of length n_2, we should have the syndromes of all $2-RB_f(b_2)$ in this block to be distinct. This ensures that the sum or difference of two bursts is not a code word. So, an $n-k$ tuple can be added as the jth column, h_j, provided that it is not a linear combination of immediately preceding b_2-1 columns together with any three distinct sets of b_2(fixed) consecutive columns out of the first $j-b_2$ columns of the second block i.e.

$$\begin{aligned}h_j\neq{}&(\alpha_1h_{j-b_2+1}+\alpha_2h_{j-b_2+2}+\cdots+\alpha_{b_2-1}h_{j-1})+\\&+(\beta_1h_{r_1+1}+\beta_2h_{r_1+2}+\cdots+\beta_{b_2}h_{r_1+b_2})\\&+(\gamma_1h_{r_2+1}+\gamma_2h_{r_2+2}+\cdots+\gamma_{b_2}h_{r_2+b_2})\\&+(\delta_1h_{r_3+1}+\delta_2h_{r_3+2}+\cdots+\delta_{b_2}h_{r_3+b_2}),\end{aligned}\tag{5}$$

where either all β_r's, γ_r's, δ_r's are zero or if the last non-zero coefficient amongst these is say θ_k, then $b_2\leq k\leq j-b_2$.

Number of ways α's may be chosen is q^{b_2-1}. To count β_r's, γ_r's, δ_r's is equivalent to enumerating the number of $3 - RB_f(b_2)$ in a vector of length $j - b_2$ which, including the zero vector, is [5]

$$\sum_{i=0}^{3} \binom{j-(i+1)b_2+i}{i} (q-1)^i q^{i(b_2-1)}. \tag{6}$$

So, the total number of linear combinations on R.H.S. of (5) is

$$q^{b_2-1} \sum_{i=0}^{3} \binom{j-(i+1)b_2+i}{i} (q-1)^i q^{i(b_2-1)}.$$

Proceeding like this we can add the last column h_{n_2} provided it is not equal to

$$q^{b_2-1} \sum_{i=0}^{3} \binom{n_2-(i+1)b_2+i}{i} (q-1)^i q^{i(b_2-1)} \tag{7}$$

linear combinations.
After adding the last column, n_2 of second block we proceed further to add the (n_2+1)th, (n_2+2)th, $\dots$ columns to H' such that the code is able to correct $2 - RB_f(b_1)$ in a block of length n_1 and also corrects $2 - RB_f(b_2)$ in a block of length n_2.
For that, we should ensure that in the code so constructed:

1. The syndromes of all $2 - RB_f(b_1)$ in a block of length n_1 are not equal.

2. The syndromes of any two 2-repeated bursts(fixed), one of which is a $2 - RB_f(b_1)$ in a block of length n_1 and the other is a $2 - RB_f(b_2)$ in a block of length n_2 (columns corresponding to which have already been chosen), are different.

   So as the first requirement, kth $(k > n_2)$ column can be chosen provided that it is not a linear combination of immediately preceding $b_1 - 1$ columns together with any three distinct sets of b_1(fixed) consecutive columns out of the first $k - n_2 - b_1$ columns of the already added second block. i.e.

$$\begin{aligned} h_k \neq\ & (\zeta_1 h_{k-b_1+1} + \zeta_2 h_{k-b_1+2} + \cdots + \zeta_{b_1-1} h_{k-1}) + \\ & + (\lambda_1 h_{l_1+1} + \lambda_2 h_{l_1+2} + \cdots + \lambda_{b_1} h_{l_1+b_1}) \\ & + (\mu_1 h_{l_2+1} + \mu_2 h_{l_2+2} + \cdots + \mu_{b_1} h_{l_2+b_1}) \\ & + (\nu_1 h_{l_3+1} + \nu_2 h_{l_3+2} + \cdots + \nu_{b_1} h_{l_3+b_1}), \end{aligned} \tag{8}$$

where either all λ_l's, μ_l's, ν_l's are zero or if the last non-zero coefficient amongst these is say τ_p, then $b_1 \le p \le k - b_1$, and ζ_l's, λ_l's, μ_l's, ν_l's $\in GF(q)$.

Number of ways ζ's may be chosen is q^{b_1-1}. Choosing λ_l's, μ_l's, ν_l's is equivalent to enumerating the number of $3 - RB_f(b_1)$ in a vector of length $k - n_2 - b_1$. So this number, including the vector of all zeros, is [5]

$$\sum_{i=0}^{3} \binom{k - n_2 - (i+1)b_1 + i}{i} (q-1)^i q^{i(b_1-1)}. \tag{9}$$

Now, according to the second condition, kth column should not be a linear combination of immediately preceding $b_1 - 1$ columns $h_{k-b_1+1}, h_{k-b_1+2}, \ldots, h_{k-1}$, together with b_1 consecutive columns from previous $k - b_1$ columns and two sets of distinct b_2 columns from the second block. So,

$$\begin{aligned} h_k \ne\ & (\phi_1 h_{k-b_1+1} + \phi_2 h_{k-b_1+2} + \cdots + \phi_{b_1-1} h_{k-1}) + \\ & + (\chi_1 h_{l_1+1} + \chi_2 h_{l_1+2} + \cdots + \chi_{b_1} h_{l_1+b_1}) \\ & + (\psi_1 h_{r_1+1} + \psi_2 h_{r_1+2} + \cdots + \psi_{b_2} h_{r_1+b_2}) \\ & + (\omega_1 h_{r_2+1} + \omega_2 h_{r_2+2} + \cdots + \omega_{b_2} h_{r_2+b_2}), \end{aligned} \tag{10}$$

where h_l's are columns corresponding to the first block, h_r's are columns corresponding to the second block. Also, here either all χ_l's are zero or if χ_p is the last non-zero coefficient amongst these, then $b_1 \le p \le k - b_1$. Also not all of ψ_r's and ω_r's are zero and which so ever is the last non-zero coefficient amongst these, say ρ_p then $b_2 \le p \le n_2$.

Number of ways in which ϕ's may be chosen is q^{b_1-1}. Number of ways in which χ_r's, can be chosen is equivalent to the number of bursts of length b_1(fixed) in a vector of length $k - n_2 - b_1$. This number, including the zero vector is [4]

$$1 + (k - n_2 - 2b_1 + 1)(q-1)q^{b_1-1}. \tag{11}$$

So, first two expressions of (10) represent

$$q^{b_1-1}\left\{1 + (k - n_2 - 2b_1 + 1)(q-1)q^{b_1-1}\right\} \tag{12}$$

linear combinations.

Further, number of linear combinations corresponding to ψ_r's and ω_r's is the same as the number of $2 - RB_f(b_2)$ in a vector of length n_2. So, this number is (not including the zero vector) [5]

$$\sum_{i=1}^{2} \binom{n_2 - ib_2 + i}{i} (q-1)^i q^{i(b_2-1)}. \tag{13}$$

So, the total number of linear combinations to which h_k can not be equal is

$$q^{b_1-1}\sum_{i=0}^{3}\binom{k-n_2-(i+1)b_1+i}{i}(q-1)^i q^{i(b_1-1)}$$
$$+\{q^{b_1-1}\{1+(k-n_2-2b_1+1)(q-1)q^{b_1-1}\}$$
$$\times\sum_{i=1}^{2}\binom{n_2-ib_2+i}{i}(q-1)^i q^{i(b_2-1)}\}.$$

Proceeding like this, we can add the nth column provided h_n is not equal to

$$q^{b_1-1}\sum_{i=0}^{3}\binom{n_1-(i+1)b_1+i}{i}(q-1)^i q^{i(b_1-1)}$$
$$+\{q^{b_1-1}\{1+(n_1-2b_1+1)(q-1)q^{b_1-1}\}$$
$$\times\sum_{i=1}^{2}\binom{n_2-ib_2+i}{i}(q-1)^i q^{i(b_2-1)}\} \tag{14}$$

number of linear combinations.

Complete matrix H' can be constructed if both the blocks are added. So, for the existence of the matrix we require $q^{n-k} > Max[(7),(14)]$. Further, the parity-check matrix H is then obtained by reversing the order of the columns of H'.

4 Illustration: Parity-check Matrix

In this section, we have constructed a $(14,22)$ parity-check matrix H, for a $(22,8)$ linear code over $GF(2)$ that can correct $2-RB_f(2)$ in the first sub-block of length 9 and $2-RB_f(3)$ in the second sub-block of length 13. The construction is based on the synthesis process discussed in Theorem 3.1. The null space of this matrix is the required code.

$$H = \begin{bmatrix}
0&0&0&0&0&0&0&0&1&1&0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0\\
1&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&1&1&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&1&0&1&0&1&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0\\
1&1&0&0&0&0&1&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
1&1&0&1&1&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
1&1&0&1&0&1&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
1&0&1&0&0&0&1&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
1&0&0&1&1&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&1&1&0&1&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&1&1&1&0&0&1&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&1&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0
\end{bmatrix}$$

The matrix has been constructed using Python. The following also has been verified using Python.

(a) The syndromes of all $2 - RB_f(2)$ within the first sub-block and $2 - RB_f(3)$ within the second sub-block are all different from zero.

(b) The syndromes of all $2 - RB_f(2)$ occurring in the first sub-block are all different.

(c) The syndromes of all $2 - RB_f(3)$ occurring in the second sub-block are all distinct.

(d) The syndromes of all $2 - RB_f(2)$ occurring in the first sub-block are different from syndromes of all $2 - RB_f(3)$ occurring within the second sub-block.

The error vectors and syndromes are listed in Table 1.

Table 1
Error Patterns - Syndrome vectors

Error Vector	Syndrome	Error vector	Syndrome
1010000000000000000000	00010011101110	0100011000000000000000	00100001010101
1001000000000000000000	00111110010111	0100001100000000000000	00100101110000
1000100000000000000000	00111010110000	0100000110000000000000	10100111100010
1000010000000000000000	00110111011100	0111100000000000000000	00000011010010
1000001000000000000000	00110001101010	0110110000000000000000	00001010011001
1000000100000000000000	00110011111001	0110011000000000000000	00000001000011
1110000000000000000000	00110100001101	0110001100000000000000	00000101100110
1101000000000000000000	00011001110100	0110000110000000000000	10000111110100
1100100000000000000000	00011101010011	0010100000000000000000	00101001011110
1100010000000000000000	00010000111111	0010010000000000000000	00100100110010
1100001000000000000000	00010110001001	0010001000000000000000	00100010000100
1100000100000000000000	00010100011010	0010000100000000000000	00100000010111
1011000000000000000000	00011110000001	0011100000000000000000	00100100110001
1001100000000000000000	00110111011111	0011010000000000000000	00101001011101
1000110000000000000000	00111110010100	0011001000000000000000	00101111101011
1000011000000000000000	00110101001110	0011000100000000000000	00101101111000
1000001100000000000000	00110001101011	0010110000000000000000	00101101111010
1000000110000000000000	10110011111001	0010011000000000000000	00100110100000
1111000000000000000000	00111001100010	0010001100000000000000	00100010000101
1101100000000000000000	00010000111100	0010000110000000000000	10100000010111
1100110000000000000000	00011001110111	0011110000000000000000	00100000010101
1100011000000000000000	00010010101101	0011011000000000000000	00101011001111
1100001100000000000000	00010110001000	0011001100000000000000	00101111101010
1100000110000000000000	10010100011010	0011000110000000000000	10101101111000
0101000000000000000000	00101010001100	0001010000000000000000	00001001001011
0100100000000000000000	00101110101011	0001001000000000000000	00001111111101
0100010000000000000000	00100011000111	0001000100000000000000	00001101101110
0100001000000000000000	00100101110001	0001110000000000000000	00000000000011
0100000100000000000000	00100111100010	0001101000000000000000	00000110110101
0111000000000000000000	00001010011010	0001100100000000000000	00000100100110
0110100000000000000000	00001110111101	0001011000000000000000	00001011011001
0110010000000000000000	00000011010001	0001001100000000000000	00001111111100
0110001000000000000000	00000101100111	0001000110000000000000	10001101101110
0110000100000000000000	00000111110100	0001111000000000000000	00000010010001
0101100000000000000000	00100011000100	0001101100000000000000	00000110110100
0100110000000000000000	00101010001111	0001100110000000000000	10000100100110

Table 1 contd. . . .
Error Patterns - Syndrome vectors

Error Vector	Syndrome	Error vector	Syndrome
0000101000000000000000	00001011011010	0000000000000000010000	00001000000000
0000100100000000000000	00001001001001	0000000000000000001000	00010000000000
0000111000000000000000	00001111111110	0000000000000000000100	00100000000000
0000110100000000000000	00001101101101	0000000001010000000000	10000000001010
0000101100000000000000	00001011011011	0000000000101000000000	00000000010100
0000100110000000000000	10001001001001	0000000000010100000000	00000000101000
0000111100000000000000	00001111111111	0000000000001010000000	00000001010000
0000110110000000000000	10001101101101	0000000000000101000000	00000010100000
0000010100000000000000	00000100100101	0000000000000010100000	00000101000000
0000011100000000000000	00000110110111	0000000000000001010000	00001010000000
0000010110000000000000	10000100100101	0000000000000000101000	00010100000000
0000011110000000000000	10000110110111	0000000000000000010100	00101000000000
1000000000000000000000	00110011111000	0000000000000000001010	01010000000000
0100000000000000000000	00100111100011	0000000000000000000101	10100000000000
0010000000000000000000	00100000010110	0000000001100000000000	10000000000110
0001000000000000000000	00001101101111	0000000000110000000000	00000000001100
0000100000000000000000	00001001001000	0000000000011000000000	00000000011000
0000010000000000000000	00000100100100	0000000000001100000000	00000000110000
0000001000000000000000	00000010010010	0000000000000110000000	00000001100000
0000000100000000000000	00000000000001	0000000000000011000000	00000011000000
1100000000000000000000	00010100011011	0000000000000001100000	00000110000000
0110000000000000000000	00000111110101	0000000000000000110000	00001100000000
0011000000000000000000	00101101111001	0000000000000000011000	00011000000000
0001100000000000000000	00000100100111	0000000000000000001100	00110000000000
0000110000000000000000	00001101101100	0000000000000000000110	01100000000000
0000011000000000000000	00000110110110	0000000001110000000000	10000000001110
0000001100000000000000	00000010010011	0000000000111000000000	00000000011100
0000000110000000000000	10000000000001	0000000000011100000000	00000000111000
0000000001000000000000	10000000000010	0000000000001110000000	00000001110000
0000000000100000000000	00000000000100	0000000000000111000000	00000011100000
0000000000010000000000	00000000001000	0000000000000011100000	00000111000000
0000000000001000000000	00000000010000	0000000000000001110000	00001110000000
0000000000000100000000	00000000100000	0000000000000000111000	00011100000000
0000000000000010000000	00000001000000	0000000000000000011100	00111000000000
0000000000000001000000	00000010000000	0000000000000000001110	01110000000000
0000000000000000100000	00000100000000	0000000000000000000111	11100000000000

Table 1 contd. . . .
Error Patterns - Syndrome vectors

Error Vector	Syndrome	Error vector	Syndrome
0000000001001000000000	10000000010010	0000000001010000100000	10000100001010
0000000001000100000000	10000000100010	0000000001010000010000	10001000001010
0000000001000010000000	10000001000010	0000000001010000001000	10010000001010
0000000001000001000000	10000010000010	0000000001010000000100	10100000001010
0000000001000000100000	10000100000010	0000000001011010000000	10000001011010
0000000001000000010000	10001000000010	0000000001010101000000	10000010101010
0000000001000000001000	10010000000010	0000000001010010100000	10000101001010
0000000001000000000100	10100000000010	0000000001010001010000	10001010001010
0000000001001010000000	10000001010010	0000000001010000101000	10010100001010
0000000001000101000000	10000010100010	0000000001010000010100	10101000001010
0000000001000010100000	10000101000010	0000000001010000001010	11010000001010
0000000001000001010000	10001010000010	0000000001010000000101	00100000001010
0000000001000000101000	10010100000010	0000000001011100000000	10000000111010
0000000001000000010100	10101000000010	0000000001010110000000	10000001101010
0000000001000000001010	11010000000010	0000000001010011000000	10000011001010
0000000001000000000101	00100000000010	0000000001010001100000	10000110001010
0000000001001100000000	10000000110010	0000000001010000110000	10001100001010
0000000001000110000000	10000001100010	0000000001010000011000	10011000001010
0000000001000011000000	10000011000010	0000000001010000001100	10110000001010
0000000001000001100000	10000110000010	0000000001010000000110	11100000001010
0000000001000000110000	10001100000010	0000000001011110000000	10000001111010
0000000001000000011000	10011000000010	0000000001010111000000	10000011101010
0000000001000000001100	10110000000010	0000000001010011100000	10000111001010
0000000001000000000110	11100000000010	0000000001010001110000	10001110001010
0000000001001110000000	10000001110010	0000000001010000111000	10011100001010
0000000001000111000000	10000011100010	0000000001010000011100	10111000001010
0000000001000011100000	10000111000010	0000000001010000001110	11110000001010
0000000001000001110000	10001110000010	0000000001010000000111	01100000001010
0000000001000000111000	10011100000010	0000000001101000000000	10000000010110
0000000001000000011100	10111000000010	0000000001100100000000	10000000100110
0000000001000000001110	11110000000010	0000000001100010000000	10000001000110
0000000001000000000111	01100000000010	0000000001100001000000	10000010000110
0000000001011000000000	10000000011010	0000000001100000100000	10000100000110
0000000001010100000000	10000000101010	0000000001100000010000	10001000000110
0000000001010010000000	10000001001010	0000000001100000001000	10010000000110
0000000001010001000000	10000010001010	0000000001100000000100	10100000000110

Table 1 contd. . . .
Error Patterns - Syndrome vectors

Error Vector	Syndrome	Error vector	Syndrome
000000000110101000000	10000001010110	000000000111000010100 0	10010100001110
000000000110010100000 0	10000010100110	000000000111000001010 0	10101000001110
000000000110001010000 0	10000101000110	000000000111000000101 0	11010000001110
000000000110000101000 0	10001010000110	000000000111000000010 1	00100000001110
000000000110000010100 0	10010100000110	000000000111110000000 0	10000000111110
000000000110000001010 0	10101000000110	000000000111011000000 0	10000001101110
000000000110000000101 0	11010000000110	000000000111001100000 0	10000011001110
000000000110000000010 1	00100000000110	000000000111000110000 0	10000110001110
000000000110110000000 0	10000000110110	000000000111000011000 0	10001100001110
000000000110011000000 0	10000001100110	000000000111000001100 0	10011000001110
000000000110001100000 0	10000011000110	000000000111000000110 0	10110000001110
000000000110000110000 0	10000110000110	000000000111000000011 0	11100000001110
000000000110000011000 0	10001100000110	000000000111111000000 0	10000001111110
000000000110000001100 0	10011000000110	000000000111011100000 0	10000011101110
000000000110000000110 0	10110000000110	000000000111001110000 0	10000111001110
000000000110000000011 0	11100000000110	000000000111000111000 0	10001110001110
000000000110111000000 0	10000001110110	000000000111000011100 0	10011100001110
000000000110011100000 0	10000011100110	000000000111000001110 0	10111000001110
000000000110001110000 0	10000111000110	000000000111000000111 0	11110000001110
000000000110000111000 0	10001110000110	000000000111000000011 1	01100000001110
000000000110000011100 0	10011100000110	000000000010010000000 0	00000000100100
000000000110000001110 0	10111000000110	000000000010001000000 0	00000001000100
000000000110000000111 0	11110000000110	000000000010000100000 0	00000010000100
000000000110000000011 1	01100000000110	000000000010000010000 0	00000100000100
000000000111100000000 0	10000000011110	000000000010000001000 0	00001000000100
000000000111010000000 0	10000000101110	000000000010000000100 0	00010000000100
000000000111001000000 0	10000001001110	000000000010000000010 0	00100000000100
000000000111000100000 0	10000010001110	000000000010010100000 0	00000010100100
000000000111000010000 0	10000100001110	000000000010001010000 0	00000101000100
000000000111000001000 0	10001000001110	000000000010000101000 0	00001010000100
000000000111000000100 0	10010000001110	000000000010000010100 0	00010100000100
000000000111000000010 0	10100000001110	000000000010000001010 0	00101000000100
000000000111101000000 0	10000001011110	000000000010000000101 0	01010000000100
000000000111010100000 0	10000010101110	000000000010000000010 1	10100000000100
000000000111001010000 0	10000101001110	000000000010011000000 0	00000001100100
000000000111000101000 0	10001010001110	000000000010001100000 0	00000011000100

Table 1 contd. . . .
Error Patterns - Syndrome vectors

Error Vector	Syndrome	Error vector	Syndrome
000000000100001100000	0000110000100	000000000101000111000	0001110001010 0
000000000100000110000	0000110000010 0	000000000101000011100	0011100001010 0
000000000100000011000	0001100000010 0	000000000101000001110	0111000001010 0
000000000100000001100	0011000000010 0	000000000101000000111	1110000001010 0
000000000100000000110	0110000000010 0	000000000110100000000	0000000010110 0
000000000100111000000	0000001110010 0	000000000110010000000	0000000100110 0
000000000100011100000	0000011100010 0	000000000110001000000	0000001000110 0
000000000100001110000	0000111000010 0	000000000110000100000	0000010000110 0
000000000100000111000	0001110000010 0	000000000110000010000	0000100000110 0
000000000100000011100	0011100000010 0	000000000110000001000	0001000000110 0
000000000100000001110	0111000000010 0	000000000110000000100	0010000000110 0
000000000100000000111	1110000000010 0	000000000110101000000	0000001010110 0
000000000101100000000	0000000011010 0	000000000110010100000	0000010100110 0
000000000101010000000	0000000101010 0	000000000110001010000	0000101000110 0
000000000101001000000	0000001001010 0	000000000110000101000	0001010000110 0
000000000101000100000	0000010001010 0	000000000110000010100	0010100000110 0
000000000101000010000	0000100001010 0	000000000110000001010	0101000000110 0
000000000101000001000	0001000001010 0	000000000110000000101	1010000000110 0
000000000101000000100	0010000001010 0	000000000110110000000	0000000110110 0
000000000101101000000	0000001011010 0	000000000110011000000	0000001100110 0
000000000101010100000	0000010101010 0	000000000110001100000	0000011000110 0
000000000101001010000	0000101001010 0	000000000110000110000	0000110000110 0
000000000101000101000	0001010001010 0	000000000110000011000	0001100000110 0
000000000101000010100	0010100001010 0	000000000110000001100	0011000000110 0
000000000101000001010	0101000001010 0	000000000110000000110	0110000000110 0
000000000101000000101	1010000001010 0	000000000110111000000	0000001110110 0
000000000101110000000	0000000111010 0	000000000110011100000	0000011100110 0
000000000101011000000	0000001101010 0	000000000110001110000	0000111000110 0
000000000101001100000	0000011001010 0	000000000110000111000	0001110000110 0
000000000101000110000	0000110001010 0	000000000110000011100	0011100000110 0
000000000101000011000	0001100001010 0	000000000110000001110	0111000000110 0
000000000101000001100	0011000001010 0	000000000110000000111	1110000000110 0
000000000101000000110	0110000001010 0	000000000111100000000	0000000011110 0
000000000101111000000	0000001111010 0	000000000111010000000	0000000101110 0
000000000101011100000	0000011101010 0	000000000111001000000	0000001001110 0
000000000101001110000	0000111001010 0	000000000111000100000	0000010001110 0

Table 1 contd. ...
Error Patterns - Syndrome vectors

Error Vector	Syndrome	Error vector	Syndrome
000000000111000010000	0000100011100	000000000010011000000	0000011001000
000000000111000001000	0001000011100	000000000010001100000	0000110001000
000000000111000000100	0010000011100	000000000010000110000	0001100001000
000000000111101000000	0000010111100	000000000010000011000	0011000001000
000000000111010100000	0000101011100	000000000010000001100	0110000001000
000000000111001010000	0001010011100	000000000010000000110	1100000001000
000000000111000101000	0010100011100	000000000010011100000	0000111001000
000000000111000010100	0101000011100	000000000010001110000	0001110001000
000000000111000001010	1010000011100	000000000010000111000	0011100001000
000000000111000000101	10100000011100	000000000010000011100	0111000001000
000000000111110000000	0000001111100	000000000010000001110	1110000001000
000000000111011000000	0000011011100	000000000010000000111	11100000001000
000000000111001100000	0000110011100	000000000010110000000	0000001101000
000000000111000110000	0001100011100	000000000010101000000	0000010101000
000000000111000011000	0011000011100	000000000010100100000	0000100101000
000000000111000001100	0110000011100	000000000010100010000	0001000101000
000000000111000000110	1100000011100	000000000010100001000	0010000101000
000000000111111000000	0000011111100	000000000010100000100	0100000101000
000000000111011100000	0000111011100	000000000010110100000	0000101101000
000000000111001110000	0001110011100	000000000010101010000	0001010101000
000000000111000111000	0011100011100	000000000010100101000	0010100101000
000000000111000011100	0111000011100	000000000010100010100	0101000101000
000000000111000001110	1110000011100	000000000010100001010	1010000101000
000000000111000000111	11100000011100	000000000010100000101	10100000101000
000000000010010000000	0000001001000	000000000010111000000	0000011101000
000000000010001000000	0000010001000	000000000010101100000	0000110101000
000000000010000100000	0000100001000	000000000010100110000	0001100101000
000000000010000010000	0001000001000	000000000010100011000	0011000101000
000000000010000001000	0010000001000	000000000010100001100	0110000101000
000000000010000000100	0100000001000	000000000010100000110	1100000101000
000000000010010100000	0000101001000	000000000010111100000	0000111101000
000000000010001010000	0001010001000	000000000010101110000	0001110101000
000000000010000101000	0010100001000	000000000010100111000	0011100101000
000000000010000010100	0101000001000	000000000010100011100	0111000101000
000000000010000001010	1010000001000	000000000010100001110	1110000101000
000000000010000000101	10100000001000	000000000010100000111	11100000101000

Table 1 contd. . . .
Error Patterns - Syndrome vectors

Error Vector	Syndrome	Error vector	Syndrome
0000000000011010000000	00000001011000	0000000000011111000000	00000011111000
0000000000011001000000	00000010011000	0000000000011101100000	00000110111000
0000000000011000100000	00000100011000	0000000000011100110000	00001100111000
0000000000011000010000	00001000011000	0000000000011100011000	00011000111000
0000000000011000001000	00010000011000	0000000000011100001100	00110000111000
0000000000011000000100	00100000011000	0000000000011100000110	01100000111000
0000000000011010100000	00000101011000	0000000000011111100000	00000111111000
0000000000011001010000	00001010011000	0000000000011101110000	00001110111000
0000000000011000101000	00010100011000	0000000000011100111000	00011100111000
0000000000011000010100	00101000011000	0000000000011100011100	00111000111000
0000000000011000001010	01010000011000	0000000000011100001110	01110000111000
0000000000011000000101	10100000011000	0000000000011100000111	11100000111000
0000000000011011000000	00000011011000	0000000000001001000000	00000010010000
0000000000011001100000	00000110011000	0000000000001000100000	00000100010000
0000000000011000110000	00001100011000	0000000000001000010000	00001000010000
0000000000011000011000	00011000011000	0000000000001000001000	00010000010000
0000000000011000001100	00110000011000	0000000000001000000100	00100000010000
0000000000011000000110	01100000011000	0000000000001001010000	00001010010000
0000000000011011100000	00000111011000	0000000000001000101000	00010100010000
0000000000011001110000	00001110011000	0000000000001000010100	00101000010000
0000000000011000111000	00011100011000	0000000000001000001010	01010000010000
0000000000011000011100	00111000011000	0000000000001000000101	10100000010000
0000000000011000001110	01110000011000	0000000000001001100000	00000110010000
0000000000011000000111	11100000011000	0000000000001000110000	00001100010000
0000000000011110000000	00000001111000	0000000000001000011000	00011000010000
0000000000011101000000	00000010111000	0000000000001000001100	00110000010000
0000000000011100100000	00000100111000	0000000000001000000110	01100000010000
0000000000011100010000	00001000111000	0000000000001001110000	00001110010000
0000000000011100001000	00010000111000	0000000000001000111000	00011100010000
0000000000011100000100	00100000111000	0000000000001000011100	00111000010000
0000000000011110100000	00000101111000	0000000000001000001110	01110000010000
0000000000011101010000	00001010111000	0000000000001000000111	11100000010000
0000000000011100101000	00010100111000	0000000000001011000000	00000011010000
0000000000011100010100	00101000111000	0000000000001010100000	00000101010000
0000000000011100001010	01010000111000	0000000000001010010000	00001001010000
0000000000011100000101	10100000111000	0000000000001010001000	00010001010000

Table 1 contd. . . .
Error Patterns - Syndrome vectors

Error Vector	Syndrome	Error vector	Syndrome
0000000000001010000100	00100001010000	0000000000001111000000	00000011110000
0000000000001011010000	00001011010000	0000000000001110100000	00000101110000
0000000000001010101000	00010101010000	0000000000001110010000	00001001110000
0000000000001010010100	00101001010000	0000000000001110001000	00010001110000
0000000000001010001010	01010001010000	0000000000001110000100	00100001110000
0000000000001010000101	10100001010000	0000000000001111010000	00001011110000
0000000000001011100000	00000111010000	0000000000001110101000	00010101110000
0000000000001010110000	00001101010000	0000000000001110010100	00101001110000
0000000000001010011000	00011001010000	0000000000001110001010	01010001110000
0000000000001010001100	00110001010000	0000000000001110000101	10100001110000
0000000000001010000110	01100001010000	0000000000001111100000	00000111110000
0000000000001011110000	00001111010000	0000000000001110110000	00001101110000
0000000000001010111000	00011101010000	0000000000001110011000	00011001110000
0000000000001010011100	00111001010000	0000000000001110001100	00110001110000
0000000000001010001110	01110001010000	0000000000001110000110	01100001110000
0000000000001010000111	11100001010000	0000000000001111110000	00001111110000
0000000000001101000000	00000010110000	0000000000001110111000	00011101110000
0000000000001100100000	00000100110000	0000000000001110011100	00111001110000
0000000000001100010000	00001000110000	0000000000001110001110	01110001110000
0000000000001100001000	00010000110000	0000000000001110000111	11100001110000
0000000000001100000100	00100000110000	0000000000000100100000	00000100100000
0000000000001101010000	00001010110000	0000000000000100010000	00001000100000
0000000000001100101000	00010100110000	0000000000000100001000	00010000100000
0000000000001100010100	00101000110000	0000000000000100000100	00100000100000
0000000000001100001010	01010000110000	0000000000000100101000	00010100100000
0000000000001100000101	10100000110000	0000000000000100010100	00101000100000
0000000000001101100000	00000110110000	0000000000000100001010	01010000100000
0000000000001100110000	00001100110000	0000000000000100000101	10100000100000
0000000000001100011000	00011000110000	0000000000000100110000	00001100100000
0000000000001100001100	00110000110000	0000000000000100011000	00011000100000
0000000000001100000110	01100000110000	0000000000000100001100	00110000100000
0000000000001101110000	00001110110000	0000000000000100000110	01100000100000
0000000000001100111000	00011100110000	0000000000000100111000	00011100100000
0000000000001100011100	00111000110000	0000000000000100011100	00111000100000
0000000000001100001110	01110000110000	0000000000000100001110	01110000100000
0000000000001100000111	11100000110000	0000000000000100000111	11100000100000

Table 1 contd. . . .
Error Patterns - Syndrome vectors

Error Vector	Syndrome	Error vector	Syndrome
0000000000000101100000	0000110100000	0000000000000111101000	0001011100000
0000000000000101010000	0000101010000 0	0000000000000111010100	0010101100000 0
0000000000000101001000	0001001010000 0	0000000000000111001010	0101001110000 0
0000000000000101000100	0010001010000 0	0000000000000111000101	1010001110000 0
0000000000000101101000	0001011010000 0	0000000000000111110000	0000111110000 0
0000000000000101010100	0010101010000 0	0000000000000111011000	0001101110000 0
0000000000000101001010	0101001010000 0	0000000000000111001100	0011001110000 0
0000000000000101000101	1010001010000 0	0000000000000111000110	0110001110000 0
0000000000000101110000	0000111010000 0	0000000000000111111000	0001111110000 0
0000000000000101011000	0001101010000 0	0000000000000111011100	0011101110000 0
0000000000000101001100	0011001010000 0	0000000000000111001110	0111001110000 0
0000000000000101000110	0110001010000 0	0000000000000111000111	1110001110000 0
0000000000000101111000	0001111010000 0	0000000000000010010000	0000100100000 0
0000000000000101011100	0011101010000 0	0000000000000010001000	0001000100000 0
0000000000000101001110	0111001010000 0	0000000000000010000100	0010000100000 0
0000000000000101000111	1110001010000 0	0000000000000010010100	0010100100000 0
0000000000000110100000	0000010110000 0	0000000000000010001010	0101000100000 0
0000000000000110010000	0000100110000 0	0000000000000010000101	1010000100000 0
0000000000000110001000	0001000110000 0	0000000000000010011000	0001100100000 0
0000000000000110000100	0010000110000 0	0000000000000010001100	0011000100000 0
0000000000000110101000	0001010110000 0	0000000000000010000110	0110000100000 0
0000000000000110010100	0010100110000 0	0000000000000010011100	0011100100000 0
0000000000000110001010	0101000110000 0	0000000000000010001110	0111000100000 0
0000000000000110000101	1010000110000 0	0000000000000010000111	1110000100000 0
0000000000000110110000	0000110110000 0	0000000000000010110000	0000110100000 0
0000000000000110011000	0001100110000 0	0000000000000010101000	0001010100000 0
0000000000000110001100	0011000110000 0	0000000000000010100100	0010010100000 0
0000000000000110000110	0110000110000 0	0000000000000010110100	0010110100000 0
0000000000000110111000	0001110110000 0	0000000000000010101010	0101010100000 0
0000000000000110011100	0011100110000 0	0000000000000010100101	1010010100000 0
0000000000000110001110	0111000110000 0	0000000000000010111000	0001110100000 0
0000000000000110000111	1110000110000 0	0000000000000010101100	0011010100000 0
0000000000000111100000	0000011110000 0	0000000000000010100110	0110010100000 0
0000000000000111010000	0000101110000 0	0000000000000010111100	0011110100000 0
0000000000000111001000	0001001110000 0	0000000000000010101110	0111010100000 0
0000000000000111000100	0010001110000 0	0000000000000010100111	1110010100000 0

Table 1 contd. . . .
Error Patterns - Syndrome vectors

Error Vector	Syndrome	Error vector	Syndrome
0000000000000011010000	00001011000000	0000000000000001011100	00111010000000
0000000000000011001000	00010011000000	0000000000000001010110	01101010000000
0000000000000011000100	00100011000000	0000000000000001011110	01111010000000
0000000000000011010100	00101011000000	0000000000000001010111	11101010000000
0000000000000011001010	01010011000000	0000000000000001101000	00010110000000
0000000000000011000101	10100011000000	0000000000000001100100	00100110000000
0000000000000011011000	00011011000000	0000000000000001101010	01010110000000
0000000000000011001100	00110011000000	0000000000000001100101	10100110000000
0000000000000011000110	01100011000000	0000000000000001101100	00110110000000
0000000000000011011100	00111011000000	0000000000000001100110	01100110000000
0000000000000011001110	01110011000000	0000000000000001101110	01110110000000
0000000000000011000111	11100011000000	0000000000000001100111	11100110000000
0000000000000011110000	00001111000000	0000000000000001111000	00011110000000
0000000000000011101000	00010111000000	0000000000000001110100	00101110000000
0000000000000011100100	00100111000000	0000000000000001111010	01011110000000
0000000000000011110100	00101111000000	0000000000000001110101	10101110000000
0000000000000011101010	01010111000000	0000000000000001111100	00111110000000
0000000000000011100101	10100111000000	0000000000000001110110	01101110000000
0000000000000011111000	00011111000000	0000000000000001111110	01111110000000
0000000000000011101100	00110111000000	0000000000000001110111	11101110000000
0000000000000011100110	01100111000000	0000000000000000100100	00100100000000
0000000000000011111100	00111111000000	0000000000000000100101	10100100000000
0000000000000011101110	01110111000000	0000000000000000100110	01100100000000
0000000000000011100111	11100111000000	0000000000000000100111	11100100000000
0000000000000001001000	00010010000000	0000000000000000101100	00110100000000
0000000000000001000100	00100010000000	0000000000000000101101	10110100000000
0000000000000001001010	01010010000000	0000000000000000101110	01110100000000
0000000000000001000101	10100010000000	0000000000000000101111	11110100000000
0000000000000001001100	00110010000000	0000000000000000110100	00101100000000
0000000000000001000110	01100010000000	0000000000000000110101	10101100000000
0000000000000001001110	01110010000000	0000000000000000110110	01101100000000
0000000000000001000111	11100010000000	0000000000000000110111	11101100000000
0000000000000001011000	00011010000000	0000000000000000111100	00111100000000
0000000000000001010100	00101010000000	0000000000000000111101	10111100000000
0000000000000001011010	01011010000000	0000000000000000111110	01111100000000
0000000000000001010101	10101010000000	0000000000000000111111	11111100000000

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