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Application of bipolar Pythagorean fuzzy regular alpha generalized closed soft matrices in decision-making problems

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Abstract

The main focus of this paper is to introduce bipolar Pythagorean fuzzy soft matrix topology and define bipolar Pythagorean fuzzy regular α generalized closed soft matrix and study some properties over the soft matrices. We propose the Moora strategy in bipolar Pythagorean fuzzy set environment to handle multiple criteria decision-making problems. Finally, one numerical example illustrates an application of bipolar Pythagorean fuzzy Moora method and analyses the results of different values of the decision-making weights of criteria on ranking order of the alternatives.

Keywords: Bipolar Pythagorean fuzzy soft set, bipolar Pythagorean fuzzy soft matrix, bipolar Pythagorean fuzzy soft matrix topology, bipolar Pythagorean fuzzy regular α generalized closed soft matrix, bipolar Pythagorean fuzzy Moora method.

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1. Introduction

Zadeh [3] was the first to introduce the theory of fuzzy sets. In 1999 Molodstov [1] introduced a novel mathematical tool for dealing with the uncertainties, called soft set, which is free from the inadequacy of parametrization and established various results based on this. Maji et al. [5] studied the notion of soft sets, fuzzy soft sets and intuitionistic fuzzy soft set. Yang and Chenli [16] initiated fuzzy soft matrix by introducing matrix representation of fuzzy soft sets and applied it to problems associated with decision making problem. Munir Abdul Khalik Al Khafaji and Majd Hamid Mahmood [11] introduced the fuzzy soft matrix topology. Decision making is one of the most significant and useful thing in our day-to-day life. But with the growing complexities of the systems it is difficult to the decision maker for making a correct decision under the environments.

In this paper, we propose a new hybrid bipolar Pythagorean fuzzy regular α generalized closed soft matrix. Further, we discuss some properties and definitions with relevant examples. Multi-objective optimization method on the basis of ratio analysis was introduced by Brauers and Zavadskas [9] in 2006. Here we apply Moora method in bipolar Pythagorean fuzzy environment. The basic idea of bipolar Pythagorean fuzzy Moora method is to calculate the overall performance of each alternative as the difference between the sums of its normalized performances which belongs to cost and benefit criteria.

2. Preliminaries

Definition 2.1. *Let $U = \{c_1, c_2, c_3, \ldots \ldots c_m\}$ be the universal set and E be the set of parameters given by $E = \{e_1, e_2, e_3, \ldots \ldots e_n\}$. Let $A \subseteq E$ and (F, A) be a fuzzy soft set in the fuzzy soft class (U, E). A fuzzy soft set (F, A) in matrix form is denoted by $A_{m\times n} = [a_{ij}]_{m\times n}$ or $A = [a_{ij}]$ i=1, 2,…m, j=1,2,3,…..n*

where $a_{ij} = \begin{cases} \mu_j(c_i) & if\ e_j \in A \\ 0 & if\ e_j \notin A \end{cases}$, $\mu_j(c_i)$ represents the membership of c_i in the fuzzy set.

Definition 2.2. *A Pythagorean fuzzy matrix (PFM) is a matrix of pairs $A = [(a_{ij}, a_{ij}')]$ of nonnegative real numbers, $a_{ij}, a_{ij}' \in [0, 1]$, satisfying $a_{ij}^2 + a_{ij}'^2 \le 1$ for all i, j.*

Definition 2.3. *A bipolar fuzzy matrix (BFM) $A = [(a_{ij})]_{l\times m} \in \mathcal{M}_{l\times m}$, then $a_{ij} = (-a_{ij}{}^n, a_{ij}{}^p)$, where $a_{ij}{}^n \in [-1,0]$ and $a_{ij}{}^p \in [0,1]$ are the negative and positive membership values of the element a_{ij} respectively.*

Definition 2.4. *A bipolar Pythagorean fuzzy matrix (BPFM) of order* $(l \times m)$ *is denoted by* $\mathcal{M}_{l\times m}$ *and that of order* $(m \times m)$*, that is square BPFM is denoted by* $\mathcal{M}_{m\times m}$. *We conclude that the matrix* $A = [((-a_{ij}{}^{n},\ a_{ij}{}^{p}\ -a_{ij}{}'^{n},\ a_{ij}{}'^{p})]$*, where* $a_{ij}{}^{n}$, $a_{ij}{}^{p}$, $a_{ij}{}'^{n}$, $a_{ij}{}'^{p} \in [-1,1]$ *are the negative and positive membership values of the element* a_{ij}, a'_{ij} *respectively.*

Also satisfies $0 \le (a_{ij}{}^{n})^2 + (a_{ij}{}^{p})^2 \le 1$ *and* $0 \le (a_{ij}{}'^{n})^2 + (a_{ij}{}'^{p})^2 \le 1$ *for all i, j.*

Definition 2.5. *Let* $U = \{c_1, c_2, c_3, \dots\dots c_m\}$ *be the universal set and* E *be the set of parameters given by* $E = \{e_1, e_2, e_3, \dots\dots e_n\}$. *Let* $A \subseteq E$ *and* (F, A) *be a bipolar Pythagorean fuzzy soft set in the fuzzy soft class* (U, E). *Then bipolar Pythagorean fuzzy soft set* (F, A) *in a matrix form as* $A_{m\times n} = [a_{ij}]_{m\times n}$ *or* $A = [a_{ij}]$ *i=1, 2,…m, j=1,2,3,…..n.*

$$\text{Where } a_{ij} = \begin{cases} \mu_j^{+}(c_i), \vartheta_j^{+}(c_i), \mu_j^{-}(c_i), \vartheta_j^{-}(c_i) & if\ e_j \in A \\ (0, 1, 0, -1) & if\ e_j \notin A \end{cases}$$

$\mu_j^{+}(c_i)$ *represents the positive membership of* c_i *in the bipolar Pythagorean fuzzy set* $F(e_i)$.

$\vartheta_j^{+}(c_i)$ *represents the non-membership of* c_i *in the bipolar Pythagorean fuzzy set* $F(e_i)$.

$\mu_j^{-}(c_i)$ *represents the negative membership of* c_i *in the bipolar Pythagorean fuzzy set* $F(e_i)$.

$\vartheta_j^{-}(c_i)$ *represents the negative non-membership of* c_i *in the bipolar Pythagorean fuzzy set* $F(e_i)$.

Definition 2.6. Some operations on BPF soft matrix: *Various standard operations over two bipolar Pythagorean fuzzy soft matrices* $A = [\mu_j^{A+}(c_i), \vartheta_j^{A+}(c_i), \mu_j^{A-}(c_i), \vartheta_j^{A-}(c_i)]$ *and*
$B = [\mu_j^{B+}(c_i), \vartheta_j^{B+}(c_i), \mu_j^{B-}(c_i), \vartheta_j^{B-}(c_i)] \in BPFSM_{m\times n}$ *can be defined as follows:*

(i) $A^c = [(\vartheta_j^{A+}(c_i), \mu_j^{A+}(c_i), \vartheta_j^{A-}(c_i), \mu_j^{A-}(c_i))]$ *for all i and j.*

(ii) $A \cup B = [(max\left(\mu_{ij}^{A+}, \mu_{ij}^{B+}\right), min\left(\vartheta_{ij}^{A+}, \vartheta_{ij}^{B+}\right), min\left(\mu_{ij}^{A-}, \mu_{ij}^{B-}\right), max\left(\vartheta_{ij}^{A-}, \vartheta_{ij}^{B-}\right)]$

(iii) $A \cap B = [(min\left(\mu_{ij}^{A+}, \mu_{ij}^{B+}\right), max\left(\vartheta_{ij}^{A+}, \vartheta_{ij}^{B+}\right), max\left(\mu_{ij}^{A-}, \mu_{ij}^{B-}\right), min\left(\vartheta_{ij}^{A-}, \vartheta_{ij}^{B-}\right)]$

(iv) $A * B$
$= \left(max\, min\left(\mu_{ij}^{A+}, \mu_{ij}^{B+}\right), min\, max\left(\vartheta_{ij}^{A+}, \vartheta_{ij}^{B-}\right), minmax\left(\mu_{ij}^{A-}, \mu_{ij}^{B-}\right), maxmin(\vartheta_{ij}^{A-}, \vartheta_{ij}^{B-})\right)$
for all i, j.

Definition 2.7. Fuzzy soft matrix topology: *Let X be a nonempty set, E be a set of parameters, let* $\tau_{\mathcal{M}}$ *be a collection of FSM-sets,* $A \subset U$, *if* $\tau_{\mathcal{M}}$ *satisfies the following axioms:*

(i) $\phi_{\mathcal{M}}, U_{\mathcal{M}}$ *belongs to* $\tau_{\mathcal{M}}$

(ii) *Arbitrary union of any members of FSM-sets in* $\tau_{\mathcal{M}}$ *belongs to* $\tau_{\mathcal{M}}$

(iii) *Finite intersection of any two FSM -sets in* $\tau_{\mathcal{M}}$ *belongs to* $\tau_{\mathcal{M}}$.

Then $\tau_{\mathcal{M}}$ is said to be fuzzy soft matrix topology. The triplet (X, $\tau_{\mathcal{M}}, E$) is called FSM topological space over X, the members of $\tau_{\mathcal{M}}$ are called fuzzy soft open matrices and their complements are called fuzzy soft closed matrices.

Definition 2.8. *Let (X, $\tau_{\mathcal{M}}, E$) be a FSM topological space and P = {x, $\mu_A^+(x)$, $\mu_A^-(x)$, $\vartheta_A^+(x)$, $\vartheta_A^-(x)$:x ∈ X} be a fuzzy soft set over X. A fuzzy soft interior and fuzzy soft closure of P is defined by:*

(i) $Fint(P_{\mathcal{M}}) = \cup\{G_{\mathcal{M}} / G_{\mathcal{M}}$ *is a* FOM *in* $(X, \tau_{\mathcal{M}})$ *and* $G_{\mathcal{M}} \subseteq P_{\mathcal{M}}\}$

(ii) $Fcl(P_{\mathcal{M}}) = \cap\{K_{\mathcal{M}} / K_{\mathcal{M}}$ *is a* FCM *in* $(X, \tau_{\mathcal{M}})$ *and* $P_{\mathcal{M}} \subseteq K_{\mathcal{M}}\}$

It is clear that

a) $Fint(P_{\mathcal{M}})$ is the biggest fuzzy soft open matrix contained in $P_{\mathcal{M}}$.

b) $Fcl(P_{\mathcal{M}})$ is the smallest fuzzy soft closed matrix containing $P_{\mathcal{M}}$.

In this paper we denote,
BPF- Bipolar Pythagorean Fuzzy
BPFSM- Bipolar Pythagorean fuzzy soft matrix
BPFSMT- Bipolar Pythagorean fuzzy soft matrix topology
BPFRαGCSM- Bipolar Pythagorean fuzzy regular α generalized closed soft matrix.

3. Bipolar Pythagorean fuzzy soft matrix topology

Definition 3.1. *Let X be a nonempty set, E be a set of parameters, let $\tau_{\mathcal{M}}$ be a collection of BPFSM-sets, $A \subset U$, if $\tau_{\mathcal{M}}$ satisfies the following axioms:*

(i) $\phi_{\mathcal{M}}, U_{\mathcal{M}}$ *belongs to* $\tau_{\mathcal{M}}$.

(ii) *Arbitrary union of any members of BPFSM-sets in $\tau_{\mathcal{M}}$ belongs to $\tau_{\mathcal{M}}$.*

(iii) *Finite intersection of any two BPFSM sets in $\tau_{\mathcal{M}}$ belongs to $\tau_{\mathcal{M}}$.*

Then $\tau_{\mathcal{M}}$ is said to be BPF soft matrix topology. The triplet $(X, \tau_{\mathcal{M}}, E)$ is called BPFSM topological space over X, the members of $\tau_{\mathcal{M}}$ are called BPF soft open matrices and their complements are called BPF soft closed matrices.

Definition 3.2. *Let (X, $\tau_{\mathcal{M}}, E$) be a BPFSM topological space and P = {x, $\mu_A^+(x)$, $\mu_A^-(x)$, $\vartheta_A^+(x)$, $\vartheta_A^-(x)$:x ∈ X} be a BPF soft set over X. Then the bipolar Pythagorean fuzzy soft interior, bipolar Pythagorean fuzzy soft closure of P is defined by:*

(i) $BPFint(P_{\mathcal{M}}) = \cup\{G_{\mathcal{M}} / G_{\mathcal{M}}$ *is a* $BPFOM$ *in* $(X, \tau_{\mathcal{M}})$ *and* $G_{\mathcal{M}} \subseteq P_{\mathcal{M}}\}$

(ii) $BPFcl(P_{\mathcal{M}}) = \cap\{K_{\mathcal{M}} / K_{\mathcal{M}}$ *is a* $BPFCM$ *in* $(X, \tau_{\mathcal{M}})$ *and* $P_{\mathcal{M}} \subseteq K_{\mathcal{M}}\}$

It is clear that

a) *$BPFint(P_{\mathcal{M}})$ is the biggest bipolar Pythagorean fuzzy soft open matrix contained in $P_{\mathcal{M}}$.*

b) *$BPFcl(P_{\mathcal{M}})$ is the smallest bipolar Pythagorean fuzzy soft closed matrix containing $P_{\mathcal{M}}$.*

Example 3.1. *Let $X = \{a, b\}$ and $\tau_{\mathcal{M}} = \{\phi_{\mathcal{M}}, A_{\mathcal{M}}, B_{\mathcal{M}}, C_{\mathcal{M}}, U_{\mathcal{M}}\}$ is a BPF soft matrix topology on X, where*

$$A_{\mathcal{M}} = \begin{bmatrix} (a, 0.3, 0.8, -0.4, -0.9) & (b, 0.2, 0.7, -0.5, -0.8) \\ (a, 0.5, 0.6, -0.5, -0.7) & (b, 0.4, 0.5, -0.6, -0.8) \end{bmatrix}$$
$$B_{\mathcal{M}} = \begin{bmatrix} (a, 0.6, 0.8, -0.4, -0.9) & (b, 0.2, 0.3, -0.3, -0.8) \\ (a, 0.5, 0.7, -0.1, -0.7) & (b, 0.4, 0.6, -0.9, -0.8) \end{bmatrix}$$
$$C_{\mathcal{M}} = \begin{bmatrix} (a, 0.3, 0.8, -0.1, -0.9) & (b, 0.7, 0.4, -0.5, -0.8) \\ (a, 0.5, 0.6, -0.6, -0.7) & (b, 0.4, 0.1, -0.9, -0.8) \end{bmatrix}$$

Definition 3.3. *Let* $(X, \tau_{\mathcal{M}}, E)$ *be a BPFSMTS and* $A_{\mathcal{M}}$, $B_{\mathcal{M}}$ *be two bipolar Pythagorean fuzzy soft matrices in* X. *Then bipolar Pythagorean fuzzy soft interior holds the following properties:*

a) $int(A_{\mathcal{M}}) \subseteq A_{\mathcal{M}}$
b) $A_{\mathcal{M}} \subseteq B_{\mathcal{M}} \Rightarrow int(A_{\mathcal{M}}) \subseteq int(B_{\mathcal{M}})$
c) $int\big(int(A_{\mathcal{M}})\big) = int(A_{\mathcal{M}})$
d) $int(A_{\mathcal{M}} \cap B_{\mathcal{M}}) = int(A_{\mathcal{M}}) \cap int(B_{\mathcal{M}})$
e) $int(\phi_{\mathcal{M}}) = \phi_{\mathcal{M}}$
f) $int(U_{\mathcal{M}}) = U_{\mathcal{M}}$
g) $int(A_{\mathcal{M}} \cup B_{\mathcal{M}}) \supseteq int(A_{\mathcal{M}}) \cup int(B_{\mathcal{M}})$

Definition 3.4. *Let* $(X, \tau_{\mathcal{M}}, E)$ *be a BPFTS and* $A_{\mathcal{M}}, B_{\mathcal{M}}$ *be two bipolar pythagorean fuzzy soft matrices in* X. *Then bipolar pythagorean fuzzy soft closure holds the following properties:*

a) $A_{\mathcal{M}} \subseteq cl(A_{\mathcal{M}})$
b) $A_{\mathcal{M}} \subseteq B_{\mathcal{M}} \Rightarrow cl(A_{\mathcal{M}}) \subseteq cl(B_{\mathcal{M}})$
c) $cl(cl(A_{\mathcal{M}})) = cl(A_{\mathcal{M}})$
d) $cl(A_{\mathcal{M}} \cup B_{\mathcal{M}}) = cl(A_{\mathcal{M}}) \cup cl(B_{\mathcal{M}})$
e) $cl(\phi_{\mathcal{M}}) = \phi_{\mathcal{M}}$
f) $cl(U_{\mathcal{M}}) = U_{\mathcal{M}}$
g) $cl(A_{\mathcal{M}} \cap B_{\mathcal{M}}) \subseteq cl(A_{\mathcal{M}}) \cap cl(B_{\mathcal{M}})$.

4. Bipolar Pythagorean fuzzy regular α generalized closed soft matrix

Definition 4.1 *Let* X *be a nonempty set,* E *be a set of parameters. A BPF soft matrix* $A_{\mathcal{M}}$ *of a BPF soft topological space* $(X, \tau_{\mathcal{M}}, E)$ *is called a BPFRαG closed soft matrix if* $\alpha cl(A_{\mathcal{M}}) \subseteq (U_R)_M$ *whenever* $A_{\mathcal{M}} \subseteq (U_R)_M$, $(U_R)_M$ *is a BPF regular open soft matrices in* $(X, \tau_{\mathcal{M}}, E)$.

Example 4.1. *Let* $X = \{a, b\}$ *and* $\tau_{\mathcal{M}} = \{\phi_{\mathcal{M}}, T_{\mathcal{M}}, U_{\mathcal{M}}\}$ *be a BPFSMT on* X, *where*
$$\phi_{\mathcal{M}} = \begin{bmatrix} (0,1,0,-1) & (0,1,0,-1) \\ (0,1,0,-1) & (0,1,0,-1) \end{bmatrix},$$

$T_{\mathcal{M}} = \begin{bmatrix} (0.3,0.8,-0.4,-0.9) & (0.2,0.7,-0.5,-0.8) \\ (0.5,0.6,-0.5,-0.7) & (0.4,0.5,-0.6,-0.8) \end{bmatrix}$,

$U_{\mathcal{M}} = \begin{bmatrix} (1,0,-1,0) & (1,0,-1,0) \\ (1,0,-1,0) & (1,0,-1,0) \end{bmatrix}$.

Consider the BPFSM $A_{\mathcal{M}}$ *where* $A_{\mathcal{M}} = \begin{bmatrix} (0.5,0.4,-0.6,-0.5) & (0.4,0.3,-0.7,-0.6) \\ (0.6,0.6,-0.7,-0.7) & (0.5,0.5,-0.8,-0.8) \end{bmatrix}$.

Now $\alpha cl(A_{\mathcal{M}}) = T_{\mathcal{M}}^{c} \subseteq (U_R)_M$ *whenever* $A_{\mathcal{M}} \subseteq (U_R)_M$ *and* $(U_R)_M$ *is BPF regular open soft matrix. Therefore,* $A_{\mathcal{M}}$ *is a BPFRαG closed soft matrix.*

Definition 4.2 *Let* X *be a nonempty set,* E *be a set of parameters. A BPF soft matrix* $A_{\mathcal{M}}$ *of a BPF soft topological space* $(X, \tau_{\mathcal{M}}, E)$ *is called BPFRαG open soft matrix if* $\alpha cl(A_{\mathcal{M}}) \supseteq (U_R)_M$ *whenever* $A_{\mathcal{M}} \supseteq (U_R)_M$, $(U_R)_M$ *is a BPF regular closed soft matrices in* $(X, \tau_{\mathcal{M}}, E)$.

5. Application of BPF Moora method in multi criterion decision making problem

In this section, we present a multi attribute decision making application of the BPFRαG open soft matrix and BPF soft topology using bipolar Pythagorean fuzzy Moora (BPF- Moora) method. In this paper, we present BPF- Moora method by solving MCDM problems that are equipped with bipolar Pythagorean fuzzy information. We illustrate our proposed method with a numerical example.

Algorithm for BPF-Moora method:

Step 1: Construct the BPF open soft matrix which are BPFRαG open soft matrix in BPF soft matrix topological space.

Step 2: Write the decision matrix for positive and negative information and provide weightage for positive and negative information where $W_P, W_N \in [0,1]$.

Step 3: Normalize the decision matrix by using the formula $Nx_{ij} = \frac{x_{ij}}{\sqrt{\sum_{j=1}^{n} x_{ij}^2}}$ where i=1,2,3….m, j=1,2,3…….n.

Step 4: Calculate the combined weighted normalized decision matrix by using the following relation $WNM^+ = [NM^+][W_p]^T$ and $WNM^- = [NM^-][W_N]^T$ and the values of benefit and non-benefit criteria are used to compute the reference points R_p^*, R_p^{**}, R_N^* and R_N^{**} using the following equations

$$R_p^* = R_p^{**} = (\text{sum of benefit Criteria}) - (\text{sum of non benefit criteria})$$

$$R_N^* = R_N^{**} = (\text{sum of benefit Criteria}) - (\text{sum of non benefit criteria})$$

Step 5: Defuzzify the benefit and non- benefit criteria by using the reference points R_p^*, R_p^{**}, R_N^* and R_N^{**} by means of the following equations

$$M_R^+ = \left(R_p^*\right)^2 - \left(R_p^{**}\right)^2 \text{and } M_R^- = (R_N^*)^2 - (R_N^{**})^2$$

Step 6: Compute the value of M_R,where $M_R = M_R^+ - M_R^-$

Step 7: Obtain the ranking of alternatives with highest value as rank 1 and lowest as last rank.

Example: Tractors are available in a wide range of options to suit specific tasks and requirements. Tractors offer advantages on small farms as well as in regular lawn and garden work. Modern tractors are used for ploughing, tilling and planting fields in addition to routine lawn care, landscape maintenance, moving or spreading fertilizer and clearing bushes. We consider a problem to select agriculture machinery. Especially agriculture required a lot of machinery part, among that the major one is a tractor. The idea of bipolar Pythagorean fuzzy regular α generalized open soft matrix is to be applied in BPF Moora method and we select a best tractor.

Let $T = \{T_1, T_2, T_3, T_4\}$ be the tractors with the following criteria, S = $\{s_1$ = safety of operator when maneuvering, s_2 = customer service from supplier, s_3 = rated power, s_4 = initial cost of tractor$\}$. Out of these parameters, let s_1, s_3, s_4 be the beneficial criteria and $s_2$be the non- beneficial criteria. Our aim is to calculate the ranking of tractors by using bipolar Pythagorean fuzzy regular alpha generalized open soft matrix using Moora method.

Step 1: Construct the BPF soft matrix which is BPFRαG soft matrix in BPF soft matrix topological space.

$$DM = \begin{array}{c} T_1 \\ T_2 \\ T_3 \\ T_4 \end{array} \begin{bmatrix} (0.5 & 0.3 & -0.7 & -0.5) & (0.3 & 0.3 & -0.2 & -0.7) & (0.5 & 0.4 & -0.6 & -0.5) & (0.4 & 0.4 & -0.6 & -0.6) \\ (0.3 & 0.7 & -0.5 & -0.7) & (0.1 & 0.4 & -0.6 & -0.4) & (0.6 & 0.6 & -0.7 & -0.7) & (0.5 & 0.5 & -0.6 & -0.6) \\ (0.6 & 0.3 & -0.8 & -0.3) & (0.2 & 0.7 & -0.3 & -0.9) & (0.3 & 0.7 & -0.4 & -0.8) & (0.4 & 0.8 & -0.3 & -0.7) \\ (0.5 & 0.4 & -0.6 & -0.6) & (0.3 & 0.4 & -0.3 & -0.8) & (0.9 & 0.1 & -0.9 & -0.2) & (0.8 & 0.1 & -0.8 & -0.1) \end{bmatrix}$$

Step 2: Decision matrix for positive and negative information.

$$DM^+ = \begin{bmatrix} (0.5 & 0.3) & (0.3 & 0.3) & (0.5 & 0.4) & (0.4 & 0.4) \\ (0.3 & 0.7) & (0.1 & 0.4) & (0.6 & 0.6) & (0.5 & 0.5) \\ (0.6 & 0.3) & (0.2 & 0.7) & (0.3 & 0.7) & (0.4 & 0.8) \\ (0.5 & 0.4) & (0.3 & 0.4) & (0.9 & 0.1) & (0.8 & 0.1) \end{bmatrix}$$

$$DM^{-} = \begin{bmatrix} (-0.7 & -0.5) & (-0.2 & -0.7) & (-0.6 & -0.5) & (-0.6 & -0.6) \\ (-0.5 & -0.7) & (-0.6 & -0.4) & (-0.7 & -0.7) & (-0.6 & -0.6) \\ (-0.8 & -0.3) & (-0.3 & -0.9) & (-0.4 & -0.8) & (-0.3 & -0.7) \\ (-0.6 & -0.6) & (-0.3 & -0.8) & (-0.9 & -0.2) & (-0.8 & -0.1) \end{bmatrix}$$

Weightage for positive and negative information

$$W_p = [(0.3 \quad 0.2 \quad 0.3 \quad 0.2)] \qquad W_N = [(0.2 \quad 0.15 \quad 0.25 \quad 0.4)]$$

Step 3: Normalized decision matrix for positive and negative information.

$$NM^{+} = \begin{bmatrix} (0.513 & 0.329) & (0.626 & 0.316) & (0.407 & 0.396) & (0.363 & 0.388) \\ (0.308 & 0.768) & (0.208 & 0.421) & (0.488 & 0.594) & (0.454 & 0.485) \\ (0.616 & 0.329) & (0.417 & 0.738) & (0.244 & 0.693) & (0.363 & 0.777) \\ (0.513 & 0.439) & (0.626 & 0.421) & (0.732 & 0.099) & (0.727 & 0.097) \end{bmatrix}$$

$$NM^{-} = \begin{bmatrix} (-0.534 & -0.458) & (-0.262 & -0.483) & (-0.444 & -0.419) & (-0.498 & -0.543) \\ (-0.381 & -0.642) & (-0.788 & -0.276) & (-0.518 & -0.587) & (-0.498 & -0.543) \\ (-0.610 & -0.275) & (-0.394 & -0.621) & (-0.296 & -0.671) & (-0.249 & -0.634) \\ (-0.458 & -0.550) & (-0.394 & -0.552) & (-0.667 & -0.167) & (-0.664 & -0.0905) \end{bmatrix}$$

Step 4: Calculate the weighted normalized decision matrix for positive and negative information and in this decision-making problem s_1, s_3, s_4 are benefit criteria and s_2 is the non-benefit criteria. The reference points are calculated using the formula.

$$R_p^{*} = R_p^{**} = (sum\ of\ benefit\ criteria) - (sum\ of\ non\ benefit\ criteria)$$
$$= (S1 + S3 + S4) - (S2)$$

	S1	*S2*	*S3*	*S4*	(R_p^*, R_p^{**})
T_1	(0.153 0.098)	(0.125 0.063)	(0.122 0.118)	(0.072 0.077)	(0.222 0.230)
T_2	(0.092 0.230)	(0.041 0.084)	(0.146 0.178)	(0.090 0.097)	(0.287 0.421)
T_3	(0.184 0.098)	(0.083 0.147)	(0.073 0.207)	(0.072 0.155)	(0.246 0.313)
T_4	(0.153 0.131)	(0.125 0.084)	(0.219 0.029)	(0.145 0.019)	(0.392 0.095)

Table 1: Positive reference point

$$R_N^{*} = R_N^{**} = (sum\ of\ benefit\ crit\ eria) - (sum\ of\ non\ benefit\ criteria)$$
$$= (S1 + S3 + S4) - (S2)$$

	S1	*S2*	*S3*	*S4*	(R_N^*, R_N^{**})
T_1	(−0.106 −0.091)	(−0.039 −0.072)	(−0.111 −0.104)	(−0.199 −0.217)	(−0.377 −0.340)
T_2	(−0.076 −0.128)	(−0.118 −0.041)	(−0.129 −0.146)	(−0.199 −0.217)	(−0.286 −0.450)

T_3	$(-0.122 \quad -0.055)$	$(-0.059 \quad -0.093)$	$(-0.074 \quad -0.167)$	$(-0.099 \quad -0.253)$	$(-0.236 \quad -0.382)$
T_4	$(-0.091 \quad -0.11)$	$(-0.059 \quad -0.082)$	$(-0.166 \quad -0.041)$	$(-0.265 \quad -0.036)$	$(-0.463 \quad -0.105)$

Table 2: Negative reference point

Step 5: Compute M_R^+ and M_R^- by using the formulas follows:

$$M_R^+ = \left(R_p^*\right)^2 - \left(R_p^{**}\right)^2 \text{and} \qquad M_R^- = (R_N^*)^2 - (R_N^{**})^2$$

	$M_R^+ = \left(R_p^*\right)^2 - \left(R_p^{**}\right)^2$	$M_R^- = (R_N^*)^2 - (R_N^{**})^2$
M_{R1}^+	-0.0036	+0.0265
M_{R2}^+	-0.0949	-0.1207
M_{R3}^+	-0.0376	-0.0902
M_{R4}^+	+0.1446	+0.2033

Table 3: Defuzzification

Step 6: Calculate $M_R = M_R^+ - M_R^-$ for all alternatives and rank according to the descending order.

M_R	*Ranking*
-0.0301	3
+0.0258	2
+0.0527	1
-0.0587	4

Table 4: Ranking

Step 7: The ranking order is $T_3 > T_2 > T_1 > T_4$.

Hence T_3 is the first choice. The next choice will be T_2 followed by T_1 and T_4.

6. Conclusion

In this paper, we combine the concept of bipolar Pythagorean fuzzy soft set and matrix to introduce the concept of bipolar Pythagorean fuzzy soft matrix topology. Moreover, we discuss some properties of BPFS interior and BPFS closure. Finally, we apply BPFRαGCSM on a decision making problem by using BPF Moora method. In future work, it would be motivating to apply BPF Moora to device MCDM problems, for example robot selection, project selection.

References

[1] D. Molodtsov. Soft Set theory-first result. Computers and Mathematics with Applications,37, 19-31. 1999.

[2] K. T. Atanassov. Intuitionistic fuzzy sets. Fuzzy Sets Systems, 20, 87-96. 1986.

[3] L. A. Zadeh Fuzzy sets. Information and Control, 8, 338-353. 1965.

[4] R.R. Yager. Pythagorean membership grades in multicriteria decision making. IEEE Transactions on Fuzzy Systems, 22, 958-965. 2014.

[5] PK. Maji, R. Biswas, AR Roy. Intuitionistic fuzzy soft sets. Journal of Fuzzy Math 9, 677-692. 2001.

[6] PK Maji, R. Biswas, AR. Roy. An application of soft sets in a decision-making problem. Compute Math Appl 44,1077-1083. 2002.

[7] Md. Jalilul Islam Mondal, Tapan Kumar Roy. Intuitionistic fuzzy soft matrix theory. Mathematics and Statistics 1,43-49. 2013.

[8] W.K. Brauers, E.K. Zavadskas. The MOORA method and its application to privatization in a transition economy. Control and Cybernetics, 35, 445–46. 2006.

[9] D. Stanujkic, E.K. Zavadskas, F. Smarandache, W.K. Brauers, D. Karabasevic, A neutrosophic extension of the MULTIMOORA method. Informatica, 28(1),181–192.

[10] Munir Abdul Khalik Al Khafaji and Majd Hamid Mahmood. Fuzzy soft matrix topology and Fuzzy soft matrix topology on $\bar{A}$. American Academic & Scholarly Research Journal, 7, 116-125. 2015.

[11] AR Meenakshi and T. Gandhimathi.On regular intuitionistic fuzzy matrices. The journal of fuzzy mathematics 19, 1-7. 2011.

[12] Chiranjibe Jana, Madhumangal Pal. Application of Bipolar Intuitionistic fuzzy soft sets in decision making problem. International journal of fuzzy system applications, 7, 32-55. 2018.

[13] M. Palanikumar, K. Arulmozhi. Possibility Pythagorean fuzzy soft sets and its application. Open journal of discrete applied mathematics, 4, 17-29. 2021.

[14] S. Sandhiya, K.Selvakumari. Application of bipolar fuzzy soft set using Moora methods. European journal of molecular & clinical medicine, 7, 5110-5117. 2020.

[15] Y. Yang, J. Chenali, Fuzzy Soft Matrices and Their Applications. In Proceedings of International Conference on Artificial Intelligence and Computational Intelligence, Taiyuan, China, 618–627. 2011.