

Some topological properties of revised fuzzy cone metric spaces

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Abstract

In this paper, we introduced Revised fuzzy cone Metric space with its topological properties. Likewise A necessary and sufficient condition for a Revised fuzzy cone metric space to be precompact is given. We additionally show that each distinct Revised fuzzy cone metric space is second countable and that a subspace of a separable Revised fuzzy cone metric space is separable.

Keywords: Revised fuzzy metric space; Revised fuzzy cone metric space; separable; second countable.

2020 AMS subject classifications: 54A40, 54E35, 54E15, 54H25.¹

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1 Introduction

After Zadeh [1965] introduced the idea of fuzzy sets, several authors have introduced and studied many notions of metric indistinctness [Huang and Zhang, 2007, Kramosil and Michalek, 1975, Ahmad and Mesiarová-Zemánková, 2007, Navara, 2007, Muraliraj and Thangathamizh, 2021a,b,d, Oner and Tanay, 2015, Grigorenko et al., 2020] and metric cone indistinctness. By modifying the idea of metric indistinctness introduced by George and Veeramani [1994], Zadeh [1965] studied the notion of fuzzy cone metric areas. Especially, they evidenced that every fuzzy cone topological space generates a Hausdorff first-countable topology. Here we tend to study additional topological properties of these areas whose fuzzy metric version are usually found in George and Veeramani [1994], Ghareeb and Al-Omeri [2018], Grabiec [1988], Gregori et al. [2011].

Sostak [2018] additionally represented the idea of “George–Veeramani Fuzzy Metrics Revised”. Presently Olga Grigorenko, Juan Jose Minana, Alexander Sostak, Muraliraj and Thangathamizh [2021c] have introduced “On t -conorm primarily based Fuzzy (Pseudo) metrics”. Recently Muraliraj and Thangathamizh [2021a,c,d] proved various fixed point theorems in revised fuzzy metric spaces. Muraliraj and Thangathamizh [2021b] introduce the concept of Revised fuzzy modular metric space. Moreover, we tend to prove that a Revised fuzzy cone topological space is precompact if and providing each sequence in it’s a Cauchy subsequence. Further, we tend to show that $X_1 \times X_2$ may be a complete Revised fuzzy cone topological space if and providing X_1 and X_2 are complete Revised fuzzy cone metric areas. Finally it’s tried that each divisible Revised fuzzy cone topological space is second calculable and a mathematical space of a separable Revised fuzzy cone topological space is separable.

2 Preliminaries

Definition 2.1 (Gregori et al. [2011]). *Let E be a real Banach space, θ the zero of E and P a subset of E . Then P is called a cone if and only if*

- (i) P is closed, nonempty, and $P \neq \{\theta\}$,
- (ii) if $ab \in R, ab \geq 0$ and $xy \in P$, then $ax + by \in P$,
- (iii) if both $x \in P$ and $-x \in P$, then $x = \theta$.

Given a cone P , a partial ordering $\preceq$ on E with respect to P is defined by $x \preceq y$ if only if $y - x \in P$. The notation $x \prec y$ will stand for $x \preceq y$ and $x \neq y$, while $x \ll y$ will stand for $y - x \in \text{int}(P)$. Throughout this paper, we assume that all the cones have nonempty interiors.

There are two kinds of cones: normal and non-normal ones. A cone P is called normal if there exists a constant $K \geq 1$ such that for all $t, s \in E$, $\theta \lesssim t \lesssim s$ implies $\|t\| \leq K \|s\|$, and the least positive number K having this property is called normal constant of P Gregori et al. [2011]. It is clear that $K \geq 1$.

Definition 2.2 (Sostak [2018]). A binary operation $\oplus : [0, 1] \times [0, 1] \rightarrow [0, 1]$ is a t -conorm if it satisfies the following conditions:

- (i) $\oplus$ is associative and commutative,
- (ii) $\oplus$ is continuous,
- (iii) $a \oplus 0 = a$ for all $a \in [0, 1]$,
- (iv) $a \oplus b \leq c \oplus d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0, 1]$.

2.1 Examples Sostak and Öner [2020]

- (i) Lukasiewicz t -conorm: $a \oplus b = \max\{a, b\}$;
- (ii) Product t -conorm: $a \oplus b = a + b - ab$;
- (iii) Minimum t -conorm: $a \oplus b = \min(a + b, 1)$.

Definition 2.3 (Öner and Tanay [2015]). A 3-tuple $(U, M_0, *)$ is called a fuzzy cone metric space (FCM space) if C is a cone of E , U is an arbitrary set, $(*)$ is a continuous t -norm, and M_0 is a fuzzy set on $U^2 \times \text{int}(P)$ satisfying the following conditions:

- (i) $M_0(\lambda_1, \lambda_2, t) > 0$ and $M_0(\lambda_1, \lambda_2, t) = 1 \Leftrightarrow \lambda_1 = \lambda_2$,
- (ii) $M_0(\lambda_1, \lambda_2, t) = M_0(\lambda_2, \lambda_1, t)$
- (iii) $M_0(\lambda_1, \lambda_2, t) * M_0(\lambda_2, \lambda_3, s) \leq M_0(\lambda_1, \lambda_3, t + s)$,
- (iv) $M_0(\lambda_1, \lambda_2,) : \text{int}(P) \rightarrow [0, 1]$ is continuous $\forall \lambda_1, \lambda_2, \lambda_3 \in U$ and $t, s \in \text{int}(p)$.

Definition 2.4 (Sostak [2018]). A Revised fuzzy metric space is an ordered triple $(X, \mu, \oplus)$ such that X is a non empty set, $\oplus$ is a continuous t -conorm and μ is a Revised fuzzy set on $\mu : X^2 \times R^+ \rightarrow [0, 1]$ satisfies the following conditions:

- (RFM1) $\mu(x, y, t) < 1$;
- (RFM2) $\mu(x, y, t) = 0$ if and only if $x = y$;
- (RFM3) $\mu(x, y, t) = \mu(y, x, t)$;
- (RFM4) $\mu(x, z, t + s) \leq \mu(x, y, t) \oplus \mu(y, z, s)$;
- (RFM5) $\mu(x, y, -) : (0, \infty) \rightarrow [0, 1]$ is continuous $\forall x, y, z \in X$ and $t, s \in R^+$.
Then μ is called a Revised fuzzy metric on X .

3 Main Results

Definition 3.1. A Revised fuzzy cone metric space is an 3-triple $(X, \mu, \oplus)$ such that P is a cone of E , X is a non empty set, $\oplus$ is a continuous t -conorm and μ is a Revised fuzzy set on $X^2 \times \text{int}(P)$ satisfies the following conditions, $\forall x, y, z \in X$ and $s, t \in \text{int}(P)$ (that is $s \gg \theta, t \gg \theta$),

(RFCM 1) $\mu(x, y, t) < 1$,

(RFCM 2) $\mu(x, y, t) = 0$ if and only if $x = y$,

(RFCM 3) $\mu(x, y, t) = \mu(y, x, t)$,

(RFCM 4) $\mu(x, z, t + s) \leq \mu(x, y, t) \oplus \mu(y, z, s)$,

(RFCM 5) $\mu(x, y, -) : \text{int}(P) \rightarrow [0, 1]$ is continuous.

Then μ is called a Revised fuzzy cone metric on X .

If $(X, \mu, \oplus)$ is a Revised fuzzy cone metric space, we will say that μ is a Revised fuzzy cone metric on X .

Every revised fuzzy cone metric space $(X, \mu, \oplus)$ induces a Hausdorff first-countable topology τ_{fc} on X which has as a base the family of sets of the form $\{B(x, r, t) : x \in X; 0 < r < 1, t \gg \theta\}$,

where $\{B(x, r, t) : y \in X; \mu(x, y, t) < r\}$ for every r with $0 < r < 1$ and $t \gg \theta$.

A Revised fuzzy cone metric space $(X, \mu, \oplus)$ is called complete if every Cauchy sequence in it is convergent, where a sequence $\{x_n\}$ is said to be a Cauchy sequence if for any $\varepsilon \in (0, 1)$ and any $t \gg \theta$ there exists a natural number n_0 such that $\mu(x_n, x_m, t) < \varepsilon$ for all $n, m \geq n_0$, and a sequence $\{x_n\}$ is said to converge to x if for any $t \gg \theta$ and any $r \in (0, 1)$ there exists a natural number n_0 such that $\mu(x_n, x, t) < r$ for all $n \geq n_0$. A sequence $\{x_n\}$ converges to x if and only if $\mu(x_n, x, t) \rightarrow 0$ for each $t \gg \theta$.

Definition 3.2. Let $(X, \mu, \oplus)$ be a Revised fuzzy cone metric space. For $t \gg \theta$, the closed ball $B[x, r, t]$ with center x and radius $r \in (0, 1)$ is defined by

$$B[x, r, t] = \{y \in X; \mu(x, y, t) < r\}.$$

Lemma 3.3. Every closed ball in a Revised fuzzy cone metric space $(X, \mu, \oplus)$ is a closed set.

Proof. Let $y \in \overline{B[x, r, t]}$. Since X is first countable, there exists a sequence $\{y_n\}$ in $B[x, r, t]$ converging to y . Therefore $\mu(y_n, y, t)$ converges to 0 for all $t \gg \theta$. For a given $\epsilon \gg 0$, we have, $\mu(x, y, t + \epsilon) \leq \mu(x, y_n, t) \oplus \mu(y_n, y, \epsilon)$

Hence, $\mu(x, y, t + \epsilon) \leq \mu(x, y_n, t) \oplus \mu(y_n, y, \epsilon) \leq 0 \oplus 0 = 0$.

(If $\mu(x, y_n, t)$ is bounded, then the sequence y_n has a subsequence, which we again denote by y_n , for which $\mu(x, y_n, t)$ exists.) In particular for $n \in N$, take $\epsilon = \frac{t}{n}$. Then, $\mu(x, y, t + \frac{t}{n}) < r$. Hence, $\mu(xyt) \leq \mu(x, y, t + \frac{t}{n}) < r$. Thus $y \in B[x, r, t]$. Therefore $B[x, r, t]$ is a closed set. $\square$

Definition 3.4. A Revised fuzzy cone metric space $(X, \mu, \oplus)$ is called precompact if for each r , with $0 < r < 1$, and each $t \gg \theta$, there is a finite subset A of X , such that $X = \bigcup_{a \in A} B(a, r, t)$. In this case, we say that μ is a precompact Revised fuzzy cone metric on X .

Lemma 3.5. A Revised fuzzy cone metric space is precompact if and only if every sequence has a Cauchy subsequence.

Proof. Suppose that $(X, \mu, \oplus)$ is a precompact Revised fuzzy cone metric space. Let x_n be a sequence in X . For each $m \in N$ there is a finite subset A_m of X such that $X = \bigcup_{a \in A_m} B(a, \frac{a}{m}, \frac{t_0}{m\|t_0\|})$ where $t_0, t \gg \theta$ is a constant. Hence, for $m = 1$, there exists an $a_1 \in A_1$ and a subsequence $x_{1(n)}$ of x_n such that $x_{1(n)} \in B(a_1, 1, \frac{t_0}{m\|t_0\|})$ for every $n \in N$. Similarly, there exist an $a_2 \in A_2$ and a subsequence $\{x_{2(n)}\}$ of $x_{1(n)}$ such that $x_{2(n)} \in B(a_2, \frac{1}{2}, \frac{t_0}{m\|t_0\|})$ for every $n \in N$. By continuing this process, we get that for $m \in N$, $m > 1$, there is an $a_m \in A_m$ and a subsequence $\{x_{m(n)}\}$ of $x_{m-1(n)}$ such that $x_{m(n)} \in B(a_m, \frac{1}{m}, \frac{t_0}{m\|t_0\|})$ for every $n \in N$. Now, consider the subsequence $x_{n(n)}$ of x_n . Given r with $0 < r < 1$ and $t \gg \theta$ there is an $n_0 \in N$ such that $\frac{1}{n_0} \oplus \frac{1}{n_0} < r$ and $\frac{2t_0}{n_0\|t_0\|} \ll t$. Then, for every $km \geq n_0$, we have

$$\begin{aligned} \mu(x_{k(n)}, x_{m(n)}, t) &\leq \mu\left(x_{k(n)}, x_{m(n)}, \frac{t_0}{n_0\|t_0\|}\right) \\ &\leq \mu\left(x_{k(n)}, a_{n_0}, \frac{t_0}{n_0\|t_0\|}\right) \oplus \mu\left(a_{n_0}, x_{m(n)}, \frac{t_0}{n_0\|t_0\|}\right) \\ &\leq \frac{1}{n_0} \oplus \frac{1}{n_0} < r \end{aligned}$$

Hence $(x_{n(n)})$ is a Cauchy sequence in $(X, \mu, \oplus)$.

Conversely, suppose that $(X, \mu, \oplus)$ is a nonprecompact Revised fuzzy cone metric space. Then there exist an r with $0 < r < 1$ and $t \gg \theta$ such that for each finite subset A of X , we have $X \neq \bigcup_{a \in A} B(a, r, t)$ fix $x_1 \in X$. There is $x_2 \in X - B(x_1, r, t)$. Moreover, there is an $x_3 \in X - \bigcup_{k=1}^2 B(x_k, r, t)$. By continuing this process, we construct a sequence x_n of distinct points in X such that $x_{n+1} \notin \bigcup_{k=1}^n B(x_k, r, t)$ for every $n \in N$. Therefore x_n has no Cauchy subsequence. This completes the proof. $\square$

Lemma 3.6. Let $(X, \mu, \oplus)$ be a Revised fuzzy cone metric space. If a Cauchy sequence clusters around a point $x \in X$, then the sequence converges to x .

Proof. Let x_n be a Cauchy sequence in $(X, \mu, \oplus)$ having a cluster point $x \in X$. Then, there is a subsequence $\{x_{k(n)}\}$ of x_n that converges to x with respect to τf_c . Thus, given r with $0 < r < 1$ and $t \gg \theta$, there is an $n_0 \in N$ such that for each $n \geq n_0$, $\mu(x, x_{k(n)}, \frac{t}{2}) < s$ where $s > 0$ satisfies $s \oplus s < r$. On the other hand, there is $n_0 \geq k(n_0)$ such that for each $nm \geq n_1$, we have $\mu(x_n, x_m, \frac{t}{2}) < s$. Therefore, for each $n \geq n_1$, we have

$$\mu(x, x_n, t) \leq \mu(x, x_{k(n)}, \frac{t}{2}) \oplus \mu(x_{k(n)}, x, \frac{t}{2}) \leq s \oplus s < r.$$

We conclude that the Cauchy sequence x_n converges to x . $\square$

Proposition 3.7. Let $(X_1, \mu_1, \oplus)$ and $(X_2, \mu_2, \oplus)$ be Revised fuzzy cone metric spaces. For $(x_1, x_2), (y_1, y_2) \in X_1 \times X_2$, let $\mu((x_1, x_2), (y_1, y_2), t) = \mu_1(x_1, y_1, t) \oplus \mu_2(x_2, y_2, t)$. Then μ is a Revised fuzzy cone metric on $X_1 \times X_2$.

Proof. **RFCM 1:** Since $\mu_1(x_1, y_1, t) < 1$ and $\mu_2(x_2, y_2, t) < 1$, this implies that $\mu_1(x_1, y_1, t) \oplus \mu_2(x_2, y_2, t) < 1$. Therefore, $\mu((x_1, x_2), (y_1, y_2), t) < 1$.

RFCM 2: Suppose that for all $t \gg \theta$, $(x_1, y_1, t) = (x_2, y_2, t)$. This implies that $x_1 = y_1$ and $x_2 = y_2$ for all $t \gg \theta$. Hence, $\mu_1(x_1, y_1, t) = 0$ and $\mu_2(x_2, y_2, t) = 0$.

It follows that, $\mu((x_1, x_2), (y_1, y_2), t) = 0$.

Conversely, suppose that $\mu((x_1, x_2), (y_1, y_2), t) = 0$.

This implies that $\mu_1(x_1, y_1, t) \oplus \mu_2(x_2, y_2, t) = 0$.

Since, $0 < \mu_1(x_1, y_1, t) < 1$ and $0 < \mu_2(x_2, y_2, t) < 1$. It follows that, $\mu_1(x_1, y_1, t) = 0$ and $\mu_2(x_2, y_2, t) = 0$. Thus $x_1 = y_1$ and $x_2 = y_2$. Therefore $(x_1, x_2) = (y_1, y_2)$.

RFCM 3: To prove that $\mu((x_1, x_2), (y_1, y_2), t) = \mu((y_1, y_2), (x_1, x_2), t)$ we observe that $\mu_1(x_1, y_1, t) = \mu_1(y_1, x_1, t)$ and $\mu_2(x_2, y_2, t) = \mu_2(y_2, x_2, t)$. It follows that for all $(x_1, x_2)(y_1, y_2) \in X_1 \times X_2$ and $t \gg \theta$,

$$\mu((x_1, x_2), (y_1, y_2), t) = \mu((y_1, y_2), (x_1, x_2), t)$$

RFCM 4: Since $(X_1, \mu_1, \oplus)$ and $(X_2, \mu_2, \oplus)$ are Revised fuzzy cone metric spaces, we have that, $\mu_1(x_1, z_1, t + s) \leq \mu_1(x_1, y_1, t) \oplus \mu_1(y_1, z_1, s)$ and

$\mu_2(x_2, z_2, t + s) \leq \mu_2(x_2, y_2, t) \oplus \mu_2(y_2, z_2, s)$, for all

$(x_1, x_2)(y_1, y_2)(z_1, z_2) \in X_1 \times X_2$ and $t, s \gg \theta$. Therefore,

$$\begin{aligned} & \mu((x_1, x_2), (z_1, z_2), t + s) \\ &= \mu_1(x_1, z_1, t + s) \oplus \mu_2(x_2, z_2, t + s) \\ &\leq \mu_1(x_1, y_1, t) \oplus \mu_1(y_1, z_1, s) \oplus \mu_2(x_2, y_2, t) \oplus \mu_2(y_2, z_2, s) \\ &\leq \mu_1(x_1, y_1, t) \oplus \mu_2(x_2, y_2, t) \oplus \mu_1(y_1, z_1, s) \oplus \mu_2(y_2, z_2, s) \\ &\leq \mu_1(x_1, y_1, t) \oplus \mu_2(x_2, y_2, t) \oplus \mu_1(y_1, z_1, s) \oplus \mu_2(y_2, z_2, s) \\ &\leq \mu((x_1, x_2), (y_1, y_2), t) \oplus \mu((y_1, y_2), (z_1, z_2), t) \end{aligned}$$

RFCM 5: Note that $\mu_1(x_1, y_1, t)$ and $\mu_2(x_2, y_2, t)$ are continuous with respect to t and $\oplus$ is continuous too. It follows that,

$\mu((x_1, x_2), (y_1, y_2), t) = \mu_1(x_1, y_1, t) \oplus \mu_2(x_2, y_2, t)$ is also continuous. $\square$

Proposition 3.8. *Let $(X_1, \mu_1, \oplus)$ and $(X_2, \mu_2, \oplus)$ be Revised fuzzy cone metric spaces. We define, $\mu((x_1, x_2), (y_1, y_2), t) = \mu_1(x_1, y_1, t) \oplus \mu_2(x_2, y_2, t)$. Then μ is a complete Revised fuzzy cone metric on $X_1 \times X_2$ if and only if $(X_1, \mu_1, \oplus)$ and $(X_2, \mu_2, \oplus)$ are complete.*

Corollary 3.9. *Every separable Revised fuzzy cone metric space is second countable.*

Proof. Let $(X, \mu, \oplus)$ be the given separable Revised fuzzy cone metric space. Let $A = \{a_n : n \in N\}$ be a countable dense subset of X . Consider

$$B = \left\{ \left(a_j, \frac{1}{k}, \frac{t_1}{k \|t_1\|} \right) : j, k \in N \right\}$$

where $t_1 \gg \theta$ is constant. Then B is countable. We claim that B is a base for the family of all open sets in X . Let G be an open set in X . Let $x \in G$ then there exists r with $0 < r < 1$ and $t \gg \theta$ such that $B(x, rt) \subset G$.

Since $r \in (0, 1)$, we can find an $s \in (0, 1)$ such that $s \oplus s < r$. Choose $m \in N$ such that $\frac{1}{m} < s$ and $\frac{t_1}{m \|t_1\|} \ll \frac{t}{2}$. Since A is dense in X , there exists an $a_j \in A$ such that $a_j \in B\left(x, \frac{1}{m}, \frac{t_1}{m \|t_1\|}\right)$. Now if $y \in B\left(a_j, \frac{1}{m}, \frac{t_1}{m \|t_1\|}\right)$, then

$$\begin{aligned} \mu(x, y, t) &\leq \mu\left(x, a_j, \frac{t}{2}\right) \oplus \mu\left(y, a_j, \frac{t}{2}\right) \\ &\leq \mu\left(x, a_j, \frac{t_1}{k \|t_1\|}\right) \oplus \mu\left(x, a_j, \frac{t_1}{k \|t_1\|}\right) \\ &\leq \frac{1}{m} \oplus \frac{1}{m} \\ &\leq s \oplus s < r < r. \end{aligned}$$

Thus $y \in B(x, r, t)$ and hence B is a basis. $\square$

Proposition 3.10. *A subspace of a separable Revised fuzzy cone metric space is separable.*

Proof. Let X be a separable Revised fuzzy cone metric space and Y a subspace of X . Let $A = \{x_n : n \in N\}$ be a countable dense subset of X . For arbitrary but fixed $nk \in N$, if there are points $x \in X$ such that $\mu\left(x_n, x, \frac{t_1}{k \|t_1\|}\right) < \frac{1}{k}$ where $t_1 \gg \theta$ is constant, choose one of them and denote it by x_{n_k} .

Let $B = \{x_{n_k} : n, k \in N\}$ then B is countable. Now we claim that $Y \subset \bar{B}$. Let $x \in Y$. Given r with $0 < r < 1$ and $t \gg \theta$ we can find $k \in N$ such that $\frac{1}{k} \oplus \frac{1}{k} < r$ and $\frac{t_1}{k\|t_1\|} \ll \frac{t}{2}$.

Since A is dense in X , there exists an $m \in N$ such that $\mu\left(x_m, y, \frac{t_1}{k\|t_1\|}\right) < \frac{1}{k}$. But by definition of B , there exists an x_{m_k} such that $\mu\left(x_{m_k}, x_m, \frac{t_1}{k\|t_1\|}\right) < \frac{1}{k}$. Now

$$\begin{aligned} \mu\left(x_{m_k}, y, \frac{t_1}{k\|t_1\|}\right) &\leq \mu\left(x_{m_k}, x_m, \frac{t}{2}\right) \oplus \mu\left(x_m, y, \frac{t}{2}\right) \\ &\leq \mu\left(x_{m_k}, x_m, \frac{t_1}{k\|t_1\|}\right) \oplus \mu\left(x_m, y, \frac{t_1}{k\|t_1\|}\right) \\ &\leq \frac{1}{k} \oplus \frac{1}{k} < r. \end{aligned}$$

Thus $Y \subset \bar{B}$ and hence Y is separable. □

Corollary 3.11. *Let $(X, \mu, \oplus)$ be a Revised fuzzy cone metric space. Then $(X, \tau f_c)$ is Hausdorff.*

Corollary 3.12. *Let $(X, \mu, \oplus)$ be a Revised fuzzy cone metric space. Define $\tau f_c = A \subset X : x \in A$ if and only if there exist $r \in (0, 1)$, and $t \gg \theta$ such that $B(x, r, tt) \subset A$, then τf_c is a topology on X .*

Corollary 3.13. *In a Revised fuzzy cone metric space, every compact set is closed and RFC-bounded.*

4 Conclusion

In this paper we proved a necessary and sufficient condition for a revised fuzzy cone metric space to be precompact. We also show that every separable revised fuzzy cone metric space is second countable and that a subspace of a separable revised fuzzy cone metric space is separable.

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