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Weighted Norlund-Euler Statistical Convergence In Neutrosophic Normed Linear Spaces

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Abstract

This paper explores the neutrosophic Norlund $\mathfrak{I}$-statistically convergent sequence space. In Norlund convergent spaces, we introduce some neutrosophic normed spaces ($\mathfrak{NNS}$s). The topological and algebraic characteristics of these convergent sequence spaces are also examined. Results are gathered from many angles, and fresh instances are created to support the equivalents and show the validity of the new ideas. The findings of this study provide a thorough foundation for $\mathfrak{NNS}$ and significantly advance the literature's theoretical understanding of the phenomenon. This study's distinguishing characteristic is the first comprehensive analysis of the properties and use of neutrosophic Norlund $\mathfrak{I}$-statistically convergent sequences in $\mathfrak{NNS}$, based on the accepted definition.

Keywords: Statistical convergence, weighted statistical convergence, neutrosophic normed linear space.
2020 AMS subject classifications: 46S40, 54H25 [1]

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1 Introduction

Fuzzy theory has significantly advanced the mathematical foundations of fuzzy set ($\mathfrak{FS}$) theory, which was developed by Zadeh [15] in 1965. According to Zadeh [15], a $\mathfrak{FS}$ gives each component of a given crisp universe set from [0,1] a membership value. Lack of membership degree knowledge is a problem that $\mathfrak{FS}$s frequently struggle to solve. As a result, Atanassov [1] investigated the intuitionistic $\mathfrak{FS}$ ($\mathfrak{IFS}$), a development of $\mathfrak{FS}$.
Neutosophy is the idea of objective knowledge of thinking, and neutral describes the fundamental distinction between neutral, fuzzy, intuitive $\mathfrak{FS}$s and logic. Following the development of the neutrosophic set ($\mathfrak{NS}$), which is a generalisation of the classical set, $\mathfrak{FS}$, and $\mathfrak{IFS}$ by Smarandache [14]. Smarandache implemented the $\mathfrak{IFS}$ theory by defining a new component, called the indeterminacy membership function, after taking everything into consideration.
Every element of the universe has a degree of $\mathfrak{T}$, $\mathfrak{F}$, and $\mathfrak{I}$ in $\mathfrak{NS}$, which is defined as a set. The degree of non-belongingness in $\mathfrak{IFS}$s is dependent on the degree of belongingness rather than being independent of it. The degree of non-belongingness of an element in $\mathfrak{FS}$s is exactly equivalent to one degree of belongingness, which makes it a notable example of an $\mathfrak{IFS}$. In $\mathfrak{IFS}$s, uncertainty is predicated on the degree of belongingness, whereas in $\mathfrak{NS}$, uncertainty is taken into account independently of $\mathfrak{T}$ and $\mathfrak{F}$ values. $\mathfrak{NS}$s are really more general than $\mathfrak{IFS}$ because there are no limitations on the degree of $\mathfrak{T}$, $\mathfrak{F}$, and $\mathfrak{I}$. This has led to a variety of mathematicians producing their research in various mathematical forms.
For weak contractions on neutrosophic normed spaces, Jeyaraman, Ramachandran and Shakila [6] demonstrated approximate fixed point theorems in 2022. In this paper, we study the neutrosophic deferred statistical convergence of double sequences in the neutrosophic normed space. Along with being compared to itself under various limitations, this novel method is also contrasted with neutrosophic statistical convergence of double sequences. The fundamental theory of mathematics makes use of the notion of sequence convergence. Summability theory contains a large number of convergence 3 concepts, including classical measure theory, fuzzy theory, approximation theory, and probability theory. The connections between these concepts are examined.

2 Preliminary

Definition 2.1. *[2] Let* $\mathfrak{K} \subseteq \mathbb{N}$. *The number* $\delta^{\mathfrak{q}}_{\mathfrak{NE}}(\mathfrak{K}) = \lim_{\mathfrak{n}\to\infty} \frac{1}{\mathfrak{R}_\mathfrak{n}} |\{\mathfrak{k} \leq \mathfrak{R}_\mathfrak{n} : \mathfrak{k} \in \mathfrak{K}\}|$ *is said to be weighted Norlund-Euler density of* $\mathfrak{K}$. *A sequence* $x = (\mathfrak{x}_\mathfrak{k})$ *is* *said to be generalized weighted Norlund-Euler statistically convergent (or $\mathfrak{S}^{\mathfrak{q}}_{\mathfrak{N}\mathfrak{E}}$-convergent) to $\mathfrak{L}$ if, for every $\varepsilon > 0$,*

$$\lim_{\mathfrak{n}\to\infty} \frac{1}{\mathfrak{R}_{\mathfrak{n}}} \left| \left\{ \mathfrak{k} \leq \mathfrak{R}_{\mathfrak{n}} : \mathfrak{p}_{\mathfrak{n}-\mathfrak{k}} \mathfrak{q}_{\mathfrak{k}} \frac{1}{(1+\mathfrak{q})^{\mathfrak{k}}} \sum_{\upsilon=0}^{\mathfrak{k}} \binom{\mathfrak{k}}{\upsilon} \mathfrak{q}_{\mathfrak{k}-\upsilon} |x_\upsilon - \mathfrak{L}| \geq \varepsilon \right\} \right| = 0.$$

In this case, we write $\mathfrak{S}^{\mathfrak{q}}_{\mathfrak{N}\mathfrak{E}} - \lim x = \mathfrak{L}$.

Definition 2.2. *[10] The 7-tuple $(\mathfrak{Y}, \zeta, \eta, \vartheta, *, \diamond, \otimes)$ is said to be Neutrosophic Normed Linear Space ($\mathfrak{N}\mathfrak{N}\mathfrak{L}\mathfrak{S}$) where $\mathfrak{Y}$ is a linear space over a field $\mathfrak{F}$, $*$ is a continuous t-norm, $\diamond$ and $\otimes$ are continuous t-conorm, ζ, η, ϑ are fuzzy sets on $\mathfrak{Y} \times (0, \infty)$, ζ denotes the degree of membership and η, ϑ denotes the degree of nonmembership of $(x, \lambda) \in \mathfrak{Y} \times (0, \infty)$ satisfying the following conditions for every $x, z \in \mathfrak{Y}$ and $s, \lambda > 0$:*

(i) $0 \leq \zeta(x,\lambda) \leq 1, 0 \leq \eta(x,\lambda) \leq 1, 0 \leq \vartheta(x,\lambda) \leq 1$

(ii) $\zeta(x,\lambda) + \eta(x,\lambda) + \vartheta(x,\lambda) \leq 3$,

(iii) $\zeta(x,\lambda) > 0$,

(iv) $\zeta(x,\lambda) = 1$ *if and only if* $x = 0$,

(v) $\zeta(\alpha x,\lambda) = \zeta\left(x, \frac{t}{|\alpha|}\right)$ *if* $\alpha \neq 0$,

(vi) $\zeta(x,\lambda) * \zeta(z,s) \leq \zeta(x+z, \lambda+s)$,

(vii) $\zeta(x,.) : (0,\infty) \to [0,1]$ *is continuous,*

(viii) $\lim_{\lambda\to\infty} \zeta(x,\lambda) = 1$ *and* $\lim_{t\to 0} \zeta(x,\lambda) = 0$,

(ix) $\eta(x,\lambda) < 1$,

(x) $\eta(x,\lambda) = 0$ *if and only if* $x = 0$,

(xi) $\eta(\alpha x,\lambda) = \eta\left(x, \frac{\lambda}{|\alpha|}\right)$ *if* $\alpha \neq 0$,

(xii) $\eta(x,\lambda) \diamond \eta(z,s) \geq \eta(x+z, s+\lambda)$,

(xiii) $\eta(x,.) : (0,\infty) \to [0,1]$ *is continuous,*

(xiv) $\lim_{\lambda\to\infty} \eta(x,\lambda) = 0$ *and* $\lim_{t\to 0} \eta(x,\lambda) = 1$.

(xv) $\vartheta(x,\lambda) < 1$,

(xvi) $\vartheta(x, \lambda) = 0$ *if and only if* $x = 0$,

(xvii) $\vartheta(\alpha x, \lambda) = \vartheta\left(x, \frac{\lambda}{|\alpha|}\right)$ *if* $\alpha \neq 0$,

(xviii) $\vartheta(x, \lambda) \otimes \omega(z, s) \geq \vartheta(x + z, s + \lambda)$,

(xix) $\vartheta(x, .) : (0, \infty) \to [0, 1]$ *is continuous,*

(xx) $\lim\limits_{t\to\infty} \vartheta(x, \lambda) = 0$ *and* $\lim\limits_{\lambda\to 0} \vartheta(x, \lambda) = 1$.

In this case (ζ, η, ϑ) *is called Neutrosophic fuzzy linear norm.*

Example 2.1. *[10] Let* $(\mathfrak{Y}, ||.||)$ *be a normed linear space, and let* $p * q = pq, p\diamond q = \min\{p + q, 1\}$ *and* $p \otimes q = \max\{p + q, 1\}$ *for all* $p, q \in [0, 1]$. *For all* $x \in \mathfrak{Y}$ *and every* $\lambda > 0$, *consider* $\zeta(x, \lambda) := \frac{\lambda}{\lambda+||x||}, \eta(x, \lambda) := \frac{||x||}{\lambda+||x||}$ *and* $\vartheta(x, \lambda) := \frac{||x||}{||\lambda+x||}$. *Then* $(\mathfrak{Y}, \zeta, \eta, \vartheta, *, \diamond, \otimes)$ *is an* $\mathfrak{NNLS}$.

Definition 2.3. *[10] Let* $(\mathfrak{Y}, \zeta, \eta, \vartheta, *, \diamond, \otimes)$ *be an* $\mathfrak{NNLS}$. *A sequence* $x = (\mathfrak{x}_\mathfrak{k})$ *in* $\mathfrak{Y}$ *is convergent to* $\mathfrak{L} \in \mathfrak{Y}$ *with respect to the Neutrosophic Normed linear norm* (ζ, η, ϑ) *if, for every* $\varepsilon > 0$ *and* $\lambda > 0$, *there exists* $\mathfrak{k}_0 \in \mathbb{N}$ *such that* $\zeta(\mathfrak{x}_\mathfrak{k} - \mathfrak{L}, \lambda) > 1 - \varepsilon, \eta(\mathfrak{x}_\mathfrak{k} - \mathfrak{L}, \lambda) < \varepsilon$ *and* $\vartheta(\mathfrak{x}_\mathfrak{k} - \mathfrak{L}, \lambda) < \varepsilon$ *for all* $\mathfrak{k} \geq \mathfrak{k}_0$ *where* $\mathfrak{k} \in \mathbb{N}$. *It is denoted by* $(\zeta, \eta, \vartheta) - \lim x = \mathfrak{L}$.

Theorem 2.1. *[11] Let* $(\mathfrak{Y}, \zeta, \eta, \vartheta, *, \diamond, \otimes)$ *be an* $\mathfrak{NNLS}$. *Then, a sequence* $x = (\mathfrak{x}_\mathfrak{k})$ *in* $\mathfrak{Y}$ *is convergent to* $\mathfrak{L} \in \mathfrak{Y}$ *if and only if* $\lim\limits_{\mathfrak{k}\to\infty} \zeta(\mathfrak{x}_\mathfrak{k} - \mathfrak{L}, \lambda) = 1, \lim\limits_{\mathfrak{k}\to\infty} \eta(\mathfrak{x}_\mathfrak{k} - \mathfrak{L}, \lambda) = 0$ *and* $\lim\limits_{\mathfrak{k}\to\infty} \vartheta(\mathfrak{x}_\mathfrak{k} - \mathfrak{L}, \lambda) = 0$.

3 Main results

In this section, we introduce a new summability method named as $(\mathfrak{N}, \mathfrak{p}, \mathfrak{q})(\mathfrak{E}, \mathfrak{q})^{(\zeta,\eta,\vartheta)}$-summability. We introduce a new concept of statistical convergence called weighted Norlund-Euler statistical convergence with regard to the Neutrosophic fuzzy norm (ζ, η, ϑ) as a result of this definition. We also look into some relationships between these ideas.

Definition 3.1. *Let* $(\mathfrak{Y}, \zeta, \eta, \vartheta, *, \diamond, \otimes)$ *be an* $\mathfrak{NNLS}$. *A sequence* $x = (\mathfrak{x}_\mathfrak{k})$ *in* $\mathfrak{Y}$ *is said to be* $(\mathfrak{N}, \mathfrak{p}, \mathfrak{q})(\mathfrak{E}, \mathfrak{q})$*-summable to* $\mathfrak{L} \in \mathfrak{Y}$ *with respect to the Neutrosophic fuzzy norm* (ζ, η, ω) *(or* $(\mathfrak{N}, \mathfrak{p}, \mathfrak{q})(\mathfrak{E}, \mathfrak{q})^{(\zeta,\eta,\vartheta)}$*- summable to* $\mathfrak{L} \in \mathfrak{Y}$*) if, for every*

$\varepsilon > 0$ *and* $\lambda > 0$*, there exists* $n_0 \in \mathbb{N}$ *such that*

$$\frac{1}{\mathfrak{R}_\mathfrak{n}} \sum_{\mathfrak{k}=0}^{\mathfrak{n}} \zeta \left(\mathfrak{p}_{\mathfrak{n}-\mathfrak{k}} \mathfrak{q}_\mathfrak{k} \frac{1}{(1+\mathfrak{q})^\mathfrak{k}} \sum_{\upsilon=0}^{\mathfrak{k}} \binom{\mathfrak{k}}{\upsilon} \mathfrak{q}^{\mathfrak{k}-\upsilon} (x_\upsilon - \mathfrak{L}), \lambda \right) > 1 - \varepsilon,$$
$$\frac{1}{\mathfrak{R}_\mathfrak{n}} \sum_{\mathfrak{k}=0}^{\mathfrak{n}} \eta \left(\mathfrak{p}_{\mathfrak{n}-\mathfrak{k}} \mathfrak{q}_\mathfrak{k} \frac{1}{(1+\mathfrak{q})^\mathfrak{k}} \sum_{\upsilon=0}^{\mathfrak{k}} \binom{\mathfrak{k}}{\upsilon} \mathfrak{q}^{\mathfrak{k}-\upsilon} (x_\upsilon - \mathfrak{L}), \lambda \right) < \varepsilon \text{ and}$$
$$\frac{1}{\mathfrak{R}_\mathfrak{n}} \sum_{\mathfrak{k}=0}^{\mathfrak{n}} \vartheta \left(\mathfrak{p}_{\mathfrak{n}-\mathfrak{k}} \mathfrak{q}_\mathfrak{k} \frac{1}{(1+\mathfrak{q})^\mathfrak{k}} \sum_{\upsilon=0}^{\mathfrak{k}} \binom{\mathfrak{k}}{\upsilon} \mathfrak{q}^{\mathfrak{k}-\upsilon} (x_\upsilon - \mathfrak{L}), \lambda \right) < \varepsilon,$$

for all $n \geq n_0$*. In this case, we write* $(\mathfrak{N}, \mathfrak{p}, \mathfrak{q})(\mathfrak{E}, \mathfrak{q})^{(\zeta,\eta,\vartheta)} - \lim x = \mathfrak{L}$*.*

Definition 3.2. *Let* $(\mathfrak{Y}, \zeta, \eta, \vartheta, *, \diamond, \otimes)$ *be an* $\mathfrak{NNLS}$*. A sequence* $x = (\mathfrak{x}_\mathfrak{k})$ *in* $\mathfrak{Y}$ *is said to be weighted Norlund-Euler statistical convergent to* $\mathfrak{L} \in \mathfrak{Y}$ *with respect to the Neutrosophic fuzzy norm* (ζ, η, ϑ) *(or* $\mathfrak{S}_{\mathfrak{NE}}^{\mathfrak{q}(\zeta,\eta,\vartheta)}$ *- convergent to* $\mathfrak{L} \in \mathfrak{Y}$*) if, for every* $\varepsilon > 0$ *and* $\lambda > 0$*,*

$$\lim_{\mathfrak{n}\to\infty} \frac{1}{\mathfrak{R}_\mathfrak{n}} \left| \left\{ \begin{array}{c} \zeta \left(\mathfrak{p}_{\mathfrak{n}-\mathfrak{k}} \mathfrak{q}_\mathfrak{k} \frac{1}{(1+\mathfrak{q})^\mathfrak{k}} \sum_{\upsilon=0}^{\mathfrak{k}} \binom{\mathfrak{k}}{\upsilon} \mathfrak{q}^{\mathfrak{k}-\upsilon} (x_\upsilon - \mathfrak{L}), \lambda \right) \leq 1 - \varepsilon \text{ or} \\ \eta \left(\mathfrak{p}_{\mathfrak{n}-\mathfrak{k}} \mathfrak{q}_\mathfrak{k} \frac{1}{(1+\mathfrak{q})^\mathfrak{k}} \sum_{\upsilon=0}^{\mathfrak{k}} \binom{\mathfrak{k}}{\upsilon} \mathfrak{q}^{\mathfrak{k}-\upsilon} (x_\upsilon - \mathfrak{L}), \lambda \right) \geq \varepsilon \text{ and} \\ \vartheta \left(\mathfrak{p}_{\mathfrak{n}-\mathfrak{k}} \mathfrak{q}_\mathfrak{k} \frac{1}{(1+\mathfrak{q})^\mathfrak{k}} \sum_{\upsilon=0}^{\mathfrak{k}} \binom{\mathfrak{k}}{\upsilon} \mathfrak{q}^{\mathfrak{k}-\upsilon} (x_\upsilon - \mathfrak{L}), \lambda \right) \geq \varepsilon \end{array} \right\} \right| = 0.$$

In this case, we write $\mathfrak{S}_{\mathfrak{NE}}^{\mathfrak{q}(\zeta,\eta,\vartheta)} - \lim x = \mathfrak{L}$*.*

In the following theorems, we establish the relation between weighted Norlund Euler statistical convergence and $(\mathfrak{N}, \mathfrak{p}, \mathfrak{q})(\mathfrak{E}, \mathfrak{q})$- summability in Neutrosophic Normed Linear spaces.

Theorem 3.1. *Let* $(\mathfrak{Y}, \zeta, \eta, \omega, *, \diamond, \otimes)$ *be an* $\mathfrak{NNLS}$*,* $x = (\mathfrak{x}_\mathfrak{k})$ *be a sequence in* $\mathfrak{Y}$ *and* $\frac{\mathfrak{R}_\mathfrak{n}}{\mathfrak{n}} \geq 1$ *for all* $n \in \mathbb{N}$*. If* $\mathfrak{S}_{\mathfrak{NE}}^{\mathfrak{q}(\zeta,\eta,\omega)} - \lim x = \mathfrak{L}$*,*

$$\zeta \left(\mathfrak{p}_{\mathfrak{n}-\mathfrak{k}} \mathfrak{q}_\mathfrak{k} \frac{1}{(1+\mathfrak{q})^\mathfrak{k}} \sum_{\upsilon=0}^{\mathfrak{k}} \binom{\mathfrak{k}}{\upsilon} \mathfrak{q}^{\mathfrak{k}-\upsilon} (x_\upsilon - \mathfrak{L}), \lambda \right) \geq 1 - M,$$
$$\eta \left(\mathfrak{p}_{\mathfrak{n}-\mathfrak{k}} \mathfrak{q}_\mathfrak{k} \frac{1}{(1+\mathfrak{q})^\mathfrak{k}} \sum_{\upsilon=0}^{\mathfrak{k}} \binom{\mathfrak{k}}{\upsilon} \mathfrak{q}^{\mathfrak{k}-\upsilon} (x_\upsilon - \mathfrak{L}), \lambda \right) \leq M \text{ and}$$
$$\vartheta \left(\mathfrak{p}_{\mathfrak{n}-\mathfrak{k}} \mathfrak{q}_\mathfrak{k} \frac{1}{(1+\mathfrak{q})^\mathfrak{k}} \sum_{\upsilon=0}^{\mathfrak{k}} \binom{\mathfrak{k}}{\upsilon} \mathfrak{q}^{\mathfrak{k}-\upsilon} (x_\upsilon - \mathfrak{L}), \lambda \right) \leq M,$$

for all $\mathfrak{k} \in \mathbb{N}$*, then* $(\mathfrak{N}, \mathfrak{p}, \mathfrak{q})(\mathfrak{E}, \mathfrak{q})^{(\zeta,\eta,\vartheta)} - \lim x = \mathfrak{L}$*.*

Proof. Suppose that $\mathfrak{S}_{\mathfrak{N}\mathfrak{E}}^{\mathfrak{q}(\zeta,\eta,\vartheta)} - \lim x = \mathfrak{L}$. Then, for every $\varepsilon > 0$ and $\lambda > 0$, we define the sets $\mathfrak{K}_{\mathfrak{R}_\mathfrak{n}}(\varepsilon)$ and $\mathfrak{K}^c_{\mathfrak{R}_\mathfrak{n}}(\varepsilon)$ such that

$$\mathfrak{K}_{\mathfrak{R}_\mathfrak{n}}(\varepsilon) = \left\{ \begin{array}{c} \zeta\left(\mathfrak{p}_{\mathfrak{n}-\mathfrak{k}}\mathfrak{q}_\mathfrak{k}\frac{1}{(1+\mathfrak{q})^\mathfrak{k}}\sum_{\upsilon=0}^{\mathfrak{k}}\binom{\mathfrak{k}}{\upsilon}\mathfrak{q}^{\mathfrak{k}-\upsilon}(x_\upsilon - \mathfrak{L}), \lambda\right) \leq 1-\varepsilon \text{ or} \\ \eta\left(\mathfrak{p}_{\mathfrak{n}-\mathfrak{k}}\mathfrak{q}_\mathfrak{k}\frac{1}{(1+\mathfrak{q})^\mathfrak{k}}\sum_{\upsilon=0}^{\mathfrak{k}}\binom{\mathfrak{k}}{\upsilon}\mathfrak{q}^{\mathfrak{k}-\upsilon}(x_\upsilon - \mathfrak{L}), \lambda\right) \geq \varepsilon \text{ and} \\ \vartheta\left(\mathfrak{p}_{\mathfrak{n}-\mathfrak{k}}\mathfrak{q}_\mathfrak{k}\frac{1}{(1+\mathfrak{q})^\mathfrak{k}}\sum_{\upsilon=0}^{\mathfrak{k}}\binom{\mathfrak{k}}{\upsilon}\mathfrak{q}^{\mathfrak{k}-\upsilon}(x_\upsilon - \mathfrak{L}), \lambda\right) \geq \varepsilon \end{array} \right\}$$

and

$$\mathfrak{K}^c_{\mathfrak{R}_\mathfrak{n}}(\varepsilon) = \left\{ \begin{array}{c} \zeta\left(\mathfrak{p}_{\mathfrak{n}-\mathfrak{k}}\mathfrak{q}_\mathfrak{k}\frac{1}{(1+\mathfrak{q})^\mathfrak{k}}\sum_{\upsilon=0}^{\mathfrak{k}}\binom{\mathfrak{k}}{\upsilon}\mathfrak{q}^{\mathfrak{k}-\upsilon}(x_\upsilon - \mathfrak{L}), \lambda\right) > 1-\varepsilon \text{ or} \\ \eta\left(\mathfrak{p}_{\mathfrak{n}-\mathfrak{k}}\mathfrak{q}_\mathfrak{k}\frac{1}{(1+\mathfrak{q})^\mathfrak{k}}\sum_{\upsilon=0}^{\mathfrak{k}}\binom{\mathfrak{k}}{\upsilon}\mathfrak{q}^{\mathfrak{k}-\upsilon}(x_\upsilon - \mathfrak{L}), \lambda\right) < \varepsilon \text{ and} \\ \vartheta\left(\mathfrak{p}_{\mathfrak{n}-\mathfrak{k}}\mathfrak{q}_\mathfrak{k}\frac{1}{(1+\mathfrak{q})^\mathfrak{k}}\sum_{\upsilon=0}^{\mathfrak{k}}\binom{\mathfrak{k}}{\upsilon}\mathfrak{q}^{\mathfrak{k}-\upsilon}(x_\upsilon - \mathfrak{L}), \lambda\right) < \varepsilon \end{array} \right\}.$$

Hence

$$\begin{aligned} &\frac{1}{\mathfrak{R}_\mathfrak{n}}\sum_{\mathfrak{k}=0}^{\mathfrak{n}}\zeta\left(\mathfrak{p}_{\mathfrak{n}-\mathfrak{k}}\mathfrak{q}_\mathfrak{k}\frac{1}{(1+\mathfrak{q})^\mathfrak{k}}\sum_{\upsilon=0}^{\mathfrak{k}}\binom{\mathfrak{k}}{\upsilon}\mathfrak{q}^{\mathfrak{k}-\upsilon}(x_\upsilon - \mathfrak{L}), \lambda\right) \\ &= \frac{1}{\mathfrak{R}_\mathfrak{n}}\sum_{\substack{\mathfrak{k}=0 \\ \mathfrak{k}\in\mathfrak{K}_{\mathfrak{R}_\mathfrak{n}}(\varepsilon)}}^{\mathfrak{n}}\zeta\left(\mathfrak{p}_{\mathfrak{n}-\mathfrak{k}}\mathfrak{q}_\mathfrak{k}\frac{1}{(1+\mathfrak{q})^\mathfrak{k}}\sum_{\upsilon=0}^{\mathfrak{k}}\binom{\mathfrak{k}}{\upsilon}\mathfrak{q}^{\mathfrak{k}-\upsilon}(x_\mathfrak{v} - \mathfrak{L}), \lambda\right) \\ &+ \frac{1}{\mathfrak{R}_\mathfrak{n}}\sum_{\substack{\mathfrak{k}=0 \\ \mathfrak{k}\in\mathfrak{K}^c_{\mathfrak{R}_\mathfrak{n}(\varepsilon)}}}^{\mathfrak{n}}\zeta\left(\mathfrak{p}_{\mathfrak{n}-\mathfrak{k}}\mathfrak{q}_\mathfrak{k}\frac{1}{(1+\mathfrak{q})^\mathfrak{k}}\sum_{\upsilon=0}^{\mathfrak{k}}\binom{\mathfrak{k}}{\upsilon}\mathfrak{q}^{\mathfrak{k}-\upsilon}(x_\upsilon - \mathfrak{L}), \lambda\right) \\ &= \mathfrak{S}_1(\mathfrak{n}) + \mathfrak{S}_2(\mathfrak{n}) \end{aligned}$$

where

$$\mathfrak{S}_1(\mathfrak{n}) = \frac{1}{\mathfrak{R}_\mathfrak{n}}\sum_{\substack{\mathfrak{k}=0 \\ \mathfrak{k}\in\mathfrak{K}_{\mathfrak{R}_\mathfrak{n}}(\varepsilon)}}^{\mathfrak{n}}\zeta\left(\mathfrak{p}_{\mathfrak{n}-\mathfrak{k}}\mathfrak{q}_\mathfrak{k}\frac{1}{(1+\mathfrak{q})^\mathfrak{k}}\sum_{\upsilon=0}^{\mathfrak{k}}\binom{\mathfrak{k}}{\mathfrak{v}}\mathfrak{q}^{\mathfrak{k}-\upsilon}(x_\upsilon - \mathfrak{L}), \lambda\right) \text{ and} \tag{1}$$

$$\mathfrak{S}_2(\mathfrak{n}) = \frac{1}{\mathfrak{R}_\mathfrak{n}}\sum_{\substack{\mathfrak{k}=0 \\ \mathfrak{k}\in\mathfrak{K}^c_{\mathfrak{R}_\mathfrak{n}}(\varepsilon)}}^{\mathfrak{n}}\zeta\left(\mathfrak{p}_{\mathfrak{n}-\mathfrak{k}}\mathfrak{q}_\mathfrak{k}\frac{1}{(1+\mathfrak{q})^\mathfrak{k}}\sum_{\upsilon=0}^{\mathfrak{k}}\binom{\mathfrak{k}}{\upsilon}\mathfrak{q}^{\mathfrak{k}-\upsilon}(x_\upsilon - \mathfrak{L}), \lambda\right). \tag{2}$$

If $\mathfrak{k} \in \mathfrak{K}_{\mathfrak{R}_\mathfrak{n}}(\varepsilon)$, then

$$\mathfrak{S}_1(\mathfrak{n}) = \frac{1}{\mathfrak{R}_\mathfrak{n}}\sum_{\substack{\mathfrak{k}=0 \\ \mathfrak{k}\in\mathfrak{K}_{\mathfrak{R}_\mathfrak{n}}(\varepsilon)}}^{\mathfrak{n}}\zeta\left(\mathfrak{p}_{\mathfrak{n}-\mathfrak{k}}\mathfrak{q}_\mathfrak{k}\frac{1}{(1+\mathfrak{q})^\mathfrak{k}}\sum_{\upsilon=0}^{\mathfrak{k}}\binom{\mathfrak{k}}{\upsilon}\mathfrak{q}^{\mathfrak{k}-\upsilon}(x_\upsilon - \mathfrak{L}), \lambda\right) \geq \frac{|\mathfrak{K}_{\mathfrak{R}_\mathfrak{n}}(\varepsilon)|}{\mathfrak{R}_\mathfrak{n}}(1-M). \tag{3}$$

Since $\mathfrak{S}_{\mathfrak{N}\mathfrak{E}}^{\mathfrak{q}(\zeta,\eta,\vartheta)} - \lim x = \mathfrak{L}$.

$$\lim_{\mathfrak{n}\to\infty} \mathfrak{S}_1(\mathfrak{n}) \geq 0. \tag{4}$$

If $\mathfrak{k} \in \mathfrak{K}^c_{\mathfrak{R}_\mathfrak{n}}(\varepsilon)$, then we have

$$\mathfrak{S}_2(\mathfrak{n}) = \frac{1}{\mathfrak{R}_\mathfrak{n}} \sum_{\substack{\mathfrak{k}=0 \\ \mathfrak{k}\in\mathfrak{K}^c_{\mathfrak{R}_\mathfrak{n}}(\varepsilon)}}^{\mathfrak{n}} \zeta\left(\mathfrak{p}_{\mathfrak{n}-\mathfrak{k}}\mathfrak{q}_\mathfrak{k}\frac{1}{(1+\mathfrak{q})^\mathfrak{k}}\sum_{\upsilon=0}^{\mathfrak{k}}\binom{\mathfrak{k}}{\upsilon}\mathfrak{q}^{\mathfrak{k}-\upsilon}(x_\upsilon-\mathfrak{L}),\lambda\right) > \frac{|\mathfrak{K}^c_{\mathfrak{R}_\mathfrak{n}}(\varepsilon)|}{\mathfrak{R}_\mathfrak{n}}(1-\varepsilon). \tag{5}$$

which yields that

$$\lim_{\mathfrak{n}\to\infty} \mathfrak{S}_2(\mathfrak{n}) > (1-\varepsilon). \tag{6}$$

Using equalities (1)-(2) and inequalities (3)-(6), we get

$$\lim_{\mathfrak{n}\to\infty}\frac{1}{\mathfrak{R}_\mathfrak{n}}\sum_{\mathfrak{k}=0}^{\mathfrak{n}} \zeta\left(\mathfrak{p}_{\mathfrak{n}-\mathfrak{k}}\mathfrak{q}_\mathfrak{k}\frac{1}{(1+\mathfrak{q})^\mathfrak{k}}\sum_{\upsilon=0}^{\mathfrak{k}}\binom{\mathfrak{k}}{\upsilon}\mathfrak{q}^{\mathfrak{k}-\upsilon}(x_\upsilon-\mathfrak{L}),\lambda\right) = 1. \tag{7}$$

Similarly, we obtain

$$\lim_{\mathfrak{n}\to\infty}\frac{1}{\mathfrak{R}_\mathfrak{n}}\sum_{\mathfrak{k}=0}^{\mathfrak{n}} \eta\left(\mathfrak{p}_{\mathfrak{n}-\mathfrak{k}}\mathfrak{q}_\mathfrak{k}\frac{1}{(1+\mathfrak{q})^\mathfrak{k}}\sum_{\upsilon=0}^{\mathfrak{k}}\binom{\mathfrak{k}}{\upsilon}\mathfrak{q}^{\mathfrak{k}-\upsilon}(x_\upsilon-\mathfrak{L}),\lambda\right) = 0. \tag{8}$$

$$\lim_{\mathfrak{n}\to\infty}\frac{1}{\mathfrak{R}_\mathfrak{n}}\sum_{\mathfrak{k}=0}^{\mathfrak{n}} \vartheta\left(\mathfrak{p}_{\mathfrak{n}-\mathfrak{k}}\mathfrak{q}_\mathfrak{k}\frac{1}{(1+\mathfrak{q})^\mathfrak{k}}\sum_{\upsilon=0}^{\mathfrak{k}}\binom{\mathfrak{k}}{\upsilon}\mathfrak{q}^{\mathfrak{k}-\upsilon}(x_\upsilon-\mathfrak{L}),\lambda\right) = 0. \tag{9}$$

As a result of equalities (8), (9) and (10), we have

$$(\mathfrak{N},\mathfrak{p},\mathfrak{q})(\mathfrak{E},\mathfrak{q})^{(\zeta,\eta,\vartheta)} - \lim x = \mathfrak{L}. \tag{10}$$

□

Theorem 3.2. *Let $(\mathfrak{Y},\zeta,\eta,\vartheta,*,\diamond,\otimes)$ be an $\mathfrak{N}\mathfrak{N}\mathfrak{L}\mathfrak{S}$ and $\frac{\mathfrak{R}_\mathfrak{n}}{\mathfrak{n}} \geq 1$ for all $n \in \mathbb{N}$. If a sequence $x = (\mathfrak{x}_\mathfrak{k})$ in $\mathfrak{Y}$ is $(\mathfrak{N},\mathfrak{p},\mathfrak{q})(\mathfrak{E},\mathfrak{q})^{(\zeta,\eta,\vartheta)}$- summable to $\mathfrak{L} \in \mathfrak{Y}$, then a sequence $x = (\mathfrak{x}_\mathfrak{k})$ is $\mathfrak{S}_{\mathfrak{N}\mathfrak{E}}^{\mathfrak{q}(\zeta,\eta,\vartheta)}$- convergent to $\mathfrak{L} \in \mathfrak{Y}$.*

Proof. Suppose that $x = (\mathfrak{x}_\mathfrak{k})$ is $(\mathfrak{N},\mathfrak{p},\mathfrak{q})(\mathfrak{E},\mathfrak{q})^{(\zeta,\eta,\vartheta)}$-summable to $\mathfrak{L} \in \mathfrak{Y}$. For every $\varepsilon > 0$ and $\lambda > 0$, define the sets $\mathfrak{K}_{\mathfrak{R}_\mathfrak{n}}(\varepsilon)$ and $\mathfrak{K}^c_{\mathfrak{R}_\mathfrak{n}}(\varepsilon)$ such that

$$\mathfrak{K}_{\mathfrak{R}_\mathfrak{n}}(\varepsilon) = \left\{ \begin{array}{c} \zeta\left(\mathfrak{p}_{\mathfrak{n}-\mathfrak{k}}\mathfrak{q}_\mathfrak{k}\frac{1}{(1+\mathfrak{q})^\mathfrak{k}}\sum_{\upsilon=0}^{\mathfrak{k}}\binom{\mathfrak{k}}{\upsilon}\mathfrak{q}^{\mathfrak{k}-\upsilon}(x_\upsilon-\mathfrak{L}),\lambda\right) \leq 1-\varepsilon \text{ or} \\ \eta\left(\mathfrak{p}_{\mathfrak{n}-\mathfrak{k}}\mathfrak{q}_\mathfrak{k}\frac{1}{(1+\mathfrak{q})^\mathfrak{k}}\sum_{\upsilon=0}^{\mathfrak{k}}\binom{\mathfrak{k}}{\upsilon}\mathfrak{q}^{\mathfrak{k}-\upsilon}(x_\upsilon-\mathfrak{L}),\lambda\right) \geq \varepsilon \text{ and} \\ \vartheta\left(\mathfrak{p}_{\mathfrak{n}-\mathfrak{k}}\mathfrak{q}_\mathfrak{k}\frac{1}{(1+\mathfrak{q})^\mathfrak{k}}\sum_{\upsilon=0}^{\mathfrak{k}}\binom{\mathfrak{k}}{\upsilon}\mathfrak{q}^{\mathfrak{k}-\upsilon}(x_\upsilon-\mathfrak{L}),\lambda\right) \geq \varepsilon \end{array} \right\}$$

and

$$\mathfrak{K}^c_{\mathfrak{R}_\mathfrak{n}}(\varepsilon) = \left\{ \begin{array}{c} \zeta\left(\mathfrak{p}_{\mathfrak{n}-\mathfrak{k}}\mathfrak{q}_\mathfrak{k}\frac{1}{(1+\mathfrak{q})^\mathfrak{k}}\sum_{\upsilon=0}^{\mathfrak{k}}\binom{\mathfrak{k}}{\upsilon}\mathfrak{q}^{\mathfrak{k}-\upsilon}(x_\upsilon - \mathfrak{L}), \lambda\right) > 1-\varepsilon \text{ or} \\ \eta\left(\mathfrak{p}_{\mathfrak{n}-\mathfrak{k}}\mathfrak{q}_\mathfrak{k}\frac{1}{(1+\mathfrak{q})^\mathfrak{k}}\sum_{\upsilon=0}^{\mathfrak{k}}\binom{\mathfrak{k}}{\upsilon}\mathfrak{q}^{\mathfrak{k}-\upsilon}(x_\upsilon - \mathfrak{L}), \lambda\right) < \varepsilon \text{ and} \\ \vartheta\left(\mathfrak{p}_{\mathfrak{n}-\mathfrak{k}}\mathfrak{q}_\mathfrak{k}\frac{1}{(1+\mathfrak{q})^\mathfrak{k}}\sum_{\upsilon=0}^{\mathfrak{k}}\binom{\mathfrak{k}}{\upsilon}\mathfrak{q}^{\mathfrak{k}-\upsilon}(x_\upsilon - \mathfrak{L}), \lambda\right) < \varepsilon \end{array} \right\}.$$

Then

$$\begin{aligned} &\frac{1}{\mathfrak{R}_\mathfrak{n}}\sum_{\mathfrak{k}=0}^{\mathfrak{n}}\zeta\left(\mathfrak{p}_{\mathfrak{n}-\mathfrak{k}}\mathfrak{q}_\mathfrak{k}\frac{1}{(1+\mathfrak{q})^\mathfrak{k}}\sum_{\upsilon=0}^{\mathfrak{k}}\binom{\mathfrak{k}}{\upsilon}\mathfrak{q}^{\mathfrak{k}-\upsilon}(x_\upsilon - \mathfrak{L}), \lambda\right) \\ &= \frac{1}{\mathfrak{R}_\mathfrak{n}}\sum_{\substack{\mathfrak{k}=0 \\ \mathfrak{k}\in\mathfrak{K}_{\mathfrak{R}_\mathfrak{n}}(\varepsilon)}}^{\mathfrak{n}}\zeta\left(\mathfrak{p}_{\mathfrak{n}-\mathfrak{k}}\mathfrak{q}_\mathfrak{k}\frac{1}{(1+\mathfrak{q})^\mathfrak{k}}\sum_{\upsilon=0}^{\mathfrak{k}}\binom{\mathfrak{k}}{\upsilon}\mathfrak{q}^{\mathfrak{k}-\upsilon}(x_\upsilon - \mathfrak{L}), \lambda\right) \\ &+ \frac{1}{\mathfrak{R}_\mathfrak{n}}\sum_{\substack{\mathfrak{k}=0 \\ \mathfrak{k}\in\mathfrak{K}^c_{\mathfrak{R}_\mathfrak{n}(\varepsilon)}}}^{\mathfrak{n}}\zeta\left(\mathfrak{p}_{\mathfrak{n}-\mathfrak{k}}\mathfrak{q}_\mathfrak{k}\frac{1}{(1+\mathfrak{q})^\mathfrak{k}}\sum_{\upsilon=0}^{\mathfrak{k}}\binom{\mathfrak{k}}{\upsilon}\mathfrak{q}^{\mathfrak{k}-\upsilon}(x_\upsilon - \mathfrak{L}), \lambda\right) \\ &\geq \frac{1}{\mathfrak{R}_\mathfrak{n}}\sum_{\substack{\mathfrak{k}=0 \\ \mathfrak{k}\in\mathfrak{K}^c_{\mathfrak{R}_\mathfrak{n}(\varepsilon)}}}^{\mathfrak{n}}\zeta\left(\mathfrak{p}_{\mathfrak{n}-\mathfrak{k}}\mathfrak{q}_\mathfrak{k}\frac{1}{(1+\mathfrak{q})^\mathfrak{k}}\sum_{\upsilon=0}^{\mathfrak{k}}\binom{\mathfrak{k}}{\upsilon}\mathfrak{q}^{\mathfrak{k}-\upsilon}(x_\upsilon - \mathfrak{L}), \lambda\right) \\ &> \frac{1}{\mathfrak{R}_\mathfrak{n}}|\mathfrak{K}^c_{\mathfrak{R}_\mathfrak{n}}(\varepsilon)|(1-\varepsilon) \end{aligned} \tag{11}$$

As a consequence of inequality (11), we get $\lim_{\mathfrak{n}\to\infty}\frac{1}{\mathfrak{R}_\mathfrak{n}}\left|\mathfrak{K}^c_{\mathfrak{R}_\mathfrak{n}}(\varepsilon)\right| = 1$. Similarly, for every $\varepsilon > 0$ and $\lambda > 0$,

$$\begin{aligned}
&\frac{1}{\mathfrak{R}_\mathfrak{n}}\sum_{\mathfrak{k}=0}^{\mathfrak{n}}\eta\left(\mathfrak{p}_{\mathfrak{n}-\mathfrak{k}}\mathfrak{q}_\mathfrak{k}\frac{1}{(1+\mathfrak{q})^\mathfrak{k}}\sum_{\upsilon=0}^{\mathfrak{k}}\binom{\mathfrak{k}}{\upsilon}\mathfrak{q}^{\mathfrak{k}-\upsilon}(x_\upsilon-\mathfrak{L}),\lambda\right)\\
&=\frac{1}{\mathfrak{R}_\mathfrak{n}}\sum_{\substack{\mathfrak{k}=0\\ \mathfrak{k}\in\mathfrak{K}_{\mathfrak{R}_\mathfrak{n}}(\varepsilon)}}^{\mathfrak{n}}\eta\left(\mathfrak{p}_{\mathfrak{n}-\mathfrak{k}}\mathfrak{q}_\mathfrak{k}\frac{1}{(1+\mathfrak{q})^\mathfrak{k}}\sum_{\upsilon=0}^{\mathfrak{k}}\binom{\mathfrak{k}}{\upsilon}\mathfrak{q}^{\mathfrak{k}-\upsilon}(x_\upsilon-\mathfrak{L}),\lambda\right)\\
&+\frac{1}{\mathfrak{R}_\mathfrak{n}}\sum_{\substack{\mathfrak{k}=0\\ \mathfrak{k}\in\mathfrak{K}^c_{\mathfrak{R}_\mathfrak{n}(\varepsilon)}}}^{\mathfrak{n}}\eta\left(\mathfrak{p}_{\mathfrak{n}-\mathfrak{k}}\mathfrak{q}_\mathfrak{k}\frac{1}{(1+\mathfrak{q})^\mathfrak{k}}\sum_{\upsilon=0}^{\mathfrak{k}}\binom{\mathfrak{k}}{\upsilon}\mathfrak{q}^{\mathfrak{k}-\upsilon}(x_\upsilon-\mathfrak{L})\lambda\right)\\
&\leq\frac{1}{\mathfrak{R}_\mathfrak{n}}\sum_{\substack{\mathfrak{k}=0\\ \mathfrak{k}\in\mathfrak{K}_{\mathfrak{R}_\mathfrak{n}(\varepsilon)}}}^{\mathfrak{n}}\eta\left(\mathfrak{p}_{\mathfrak{n}-\mathfrak{k}}\mathfrak{q}_\mathfrak{k}\frac{1}{(1+\mathfrak{q})^\mathfrak{k}}\sum_{\upsilon=0}^{\mathfrak{k}}\binom{\mathfrak{k}}{\upsilon}\mathfrak{q}^{\mathfrak{k}-\upsilon}(x_\upsilon-\mathfrak{L}),\lambda\right)\\
&\leq\frac{1}{\mathfrak{R}_\mathfrak{n}}\sum_{\substack{\mathfrak{k}=0\\ \mathfrak{k}\in\mathfrak{K}_{\mathfrak{R}_\mathfrak{n}(\varepsilon)}}}^{\mathfrak{n}}\varepsilon=\frac{1}{\mathfrak{R}_\mathfrak{n}}|\mathfrak{K}_{\mathfrak{R}_\mathfrak{n}}(\varepsilon)|\varepsilon. \qquad (12)
\end{aligned}$$

$$\begin{aligned}
&\frac{1}{\mathfrak{R}_\mathfrak{n}}\sum_{\mathfrak{k}=0}^{\mathfrak{n}}\vartheta\left(\mathfrak{p}_{\mathfrak{n}-\mathfrak{k}}\mathfrak{q}_\mathfrak{k}\frac{1}{(1+\mathfrak{q})^\mathfrak{k}}\sum_{\upsilon=0}^{\mathfrak{k}}\binom{\mathfrak{k}}{\upsilon}\mathfrak{q}^{\mathfrak{k}-\upsilon}(x_\upsilon-\mathfrak{L}),\lambda\right)\\
&=\frac{1}{\mathfrak{R}_\mathfrak{n}}\sum_{\substack{\mathfrak{k}=0\\ \mathfrak{k}\in\mathfrak{K}_{\mathfrak{R}_\mathfrak{n}}(\varepsilon)}}^{\mathfrak{n}}\vartheta\left(\mathfrak{p}_{\mathfrak{n}-\mathfrak{k}}\mathfrak{q}_\mathfrak{k}\frac{1}{(1+\mathfrak{q})^\mathfrak{k}}\sum_{\upsilon=0}^{\mathfrak{k}}\binom{\mathfrak{k}}{\upsilon}\mathfrak{q}^{\mathfrak{k}-\upsilon}(x_\upsilon-\mathfrak{L}),\lambda\right)\\
&+\frac{1}{\mathfrak{R}_\mathfrak{n}}\sum_{\substack{\mathfrak{k}=0\\ \mathfrak{k}\in\mathfrak{K}^c_{\mathfrak{R}_\mathfrak{n}(\varepsilon)}}}^{\mathfrak{n}}\vartheta\left(\mathfrak{p}_{\mathfrak{n}-\mathfrak{k}}\mathfrak{q}_\mathfrak{k}\frac{1}{(1+\mathfrak{q})^\mathfrak{k}}\sum_{\upsilon=0}^{\mathfrak{k}}\binom{\mathfrak{k}}{\upsilon}\mathfrak{q}^{\mathfrak{k}-\upsilon}(x_\upsilon-\mathfrak{L}),\lambda\right)\\
&\leq\frac{1}{\mathfrak{R}_\mathfrak{n}}\sum_{\substack{\mathfrak{k}=0\\ \mathfrak{k}\in\mathfrak{K}_{\mathfrak{R}_\mathfrak{n}(\varepsilon)}}}^{\mathfrak{n}}\vartheta\left(\mathfrak{p}_{\mathfrak{n}-\mathfrak{k}}\mathfrak{q}_\mathfrak{k}\frac{1}{(1+\mathfrak{q})^\mathfrak{k}}\sum_{\upsilon=0}^{\mathfrak{k}}\binom{\mathfrak{k}}{\upsilon}\mathfrak{q}^{\mathfrak{k}-\upsilon}(x_\upsilon-\mathfrak{L}),\lambda\right)\\
&\leq\frac{1}{\mathfrak{R}_\mathfrak{n}}\sum_{\substack{\mathfrak{k}=0\\ \mathfrak{k}\in\mathfrak{K}_{\mathfrak{R}_\mathfrak{n}(\varepsilon)}}}^{\mathfrak{n}}\varepsilon=\frac{1}{\mathfrak{R}_\mathfrak{n}}|\mathfrak{K}_{\mathfrak{R}_\mathfrak{n}}(\varepsilon)|\varepsilon. \qquad (13)
\end{aligned}$$

By inequality (12) and (13), we have $\lim\limits_{\mathfrak{n}\to\infty}\frac{1}{\mathfrak{R}_\mathfrak{n}}|\mathfrak{K}_{\mathfrak{R}_\mathfrak{n}}(\varepsilon)|=0$. This completes the proof. □

4 Conclusion

Smarandache first presented $\mathfrak{NS}$ in 1998 [14]. Maji has created a unique concept for the $\mathfrak{NSS}$. The concept of $\mathfrak{NSNLS}$ was subsequently investigated, and some of its characteristics were proposed. Kirisi and Simsek used continuous t-norms and continuous t-conorms to study the $\mathfrak{NMS}$. Kirisci and Simsek proposed $\mathfrak{NNS}$ as well as statistical convergence in $\mathfrak{NNS}$. Although several aspects of neutrosophic Norlund convergent sequence spaces have been studied.

References

[1] Atanassov. K.T., Intuitionistic fuzzy sets, *Fuzzy Sets and Systems* **20(1)** (1986) 87-96. https://doi.org/10.1016/s0165-0114(86)80034-3

[2] Ekrem. A. Aljimi, Valdete Loku, Generalized Weighted Norlund-Euler Statistical Convergence, *Int. Journal of Math. Analysis* **8(7)** (2014) 345-354. https://doi.org/10.12988/ijma.2014.4012

[3] Jeyaraman M, Mangayarkkarasi A N, Jeyanthi V, and Pandiselvi R., Hyers Ulam-Rassias Stability for Functional Equation in Neutrosophic normed spaces, *International Journal of Neutrosophic Science (IJNS)* **18(1)** (2022) 127-143.https://doi.org/10.54216/ijns.180111

[4] Jeyaraman M, Ramachandran A, Shakila V B, Approximate fixed point Theorems for weak contractions on Neutrosophic normed space, *Journal of Computational Mathematica* **6(1)** (2022) 134-158.https://doi.org/10.26524/cm127

[5] Jeyaraman M, Satheesh Kanna. S, Silamparasan. B and Johnsy. J, Recent Developments regarding Lacunary $\mathfrak{I}$-Statistical Convergence in Neutrosophic n-Normed Linear Spaces, *Scientific Inquiry and Review(SIR)* **6(3)** (2022) 61-78 https://doi.org/10.32350/sir.63.04

[6] Karakaya.V,Chishti. T.A., Weighted statistical convergence, *Iranian Journal of Science and Technology Transaction A-Science* (2009) 119-223. https://doi.org/10.22099/ijsts.2009.2217

[7] Mursaleen. M., Mohiuddine. S.A., Statistical convergence of double sequences on intuitionistic fuzzy normed spaces, *Chaos, Solitons & Fractals* **41(5)** (2009) 2414-2421. https://doi.org/10.1016/j.chaos.2008.09.018

[8] Park. J.H., Intuitionistic fuzzy metric spaces, *Chaos, Solitons & Fractals* **22(5)**; (2004) 1039-1046. https://doi.org/10.1016/j.chaos.2004.02.051

[9] Saadati. R., Park. J.H., Intuitionistic Fuzzy Euclidean Normed Spaces, *Communications in Mathematical Analysis* **1(2)** (2006) 85-90.

[10] Saadati. R., Park. J.H., On the intuitionistic fuzzy topological spaces, *Chaos, Solitons & Fractals* **27(2)** (2006) 331-344. https://doi.org/10.1016/j.chaos.2005.03.019

[11] Samanta. T.K., Jebril,Iqbal H., Finite Dimensional Intuitionistic Fuzzy Normed Linear Space, *Int. J. Open Problems Compt. Math.* **2(4)** (2009) 574-591. https://doi.org/10.48550/arXiv.0804.1645

[12] Schweizer. B., Sklar. A., Statistical metric spaces, *Pacific J. Math* **10** (1960) 313-334. https://doi.org/10.2140/pjm.1960.10.313

[13] Smarandache F., Neutrosophic set-a generalization of the intuitionistic fuzzy set, *International Journal of Pure and Applied Mathematics* **24** (2005) 287-297. https://doi.org/10.1109/grc.2006.1635754

[14] Smarandache F., Neutrosophy: Neutrosophic probability, set, and logic: Analytic synthesis & synthetic analysis, *American Research Press, Michigan,* USA; 1998.

[15] Zadeh. L.A., Fuzzy Sets, *Information and control* **8(3)** (1965) 338-353. https://doi.org/10.1016/S0019-9958(65)90241-X