Ratio Mathematica Volume 49, 2023

Separation Axioms on Soft S$\mathcal{I}_{g\delta s}$-Closed Sets

Y. Rosemathy*
K. Alli†

Abstract

In this paper, we have introduce and study weak separation axioms such as soft $\mathcal{I}_{g\delta s}$-T$_0$, soft $\mathcal{I}_{g\delta s}$-T$_1$, soft $\mathcal{I}_{g\delta s}$-T$_2$, δ-T$_{3/4}$, $\mathcal{I}_{g\delta s}$-regular space in soft ideal topological spaces and also investigate some of its characterizations and properties with the help of general topology concepts. Also give relationships between the above new regualr spaces. Then we give sufficient and necessity condition between soft S$\mathcal{I}_{g\delta s}$-closed sets and soft singleton sets in soft $\mathcal{I}_{g\delta s} - T_1$ space.
Keywords: soft ideal topological space, soft $\mathcal{I}_{g\delta s}$-T$_0$ space, soft $\mathcal{I}_{g\delta s}$-T$_1$ space, soft $\mathcal{I}_{g\delta s}$-T$_2$ space, δ-T$_{3/4}$ space, $\mathcal{I}_{g\delta s}$-regular space.
2020 AMS subject classifications: 54J99, 54A40, 54A99, 47H10, 06D72 [1]

*Research Scholar (Reg. No. 18111072092002), The M. D. T Hindu College, Tirunelveli-627010, Tamilnadu, India.(Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627012, Tamilnadu, India); ravimathy18@gmail.com.
†Assistant Professor, Department of Mathematics, The M. D. T Hindu College, Tirunelveli-627010, Tamilnadu, India.(Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627012, Tamilnadu, India); allimdt@gmail.com.
[1]Received on October 21, 2023. Accepted on November 30, 2023. Published on December 20, 2023. DOI: 10.23755/rm.v49i0.1440. ISSN: 1592-7415. eISSN: 2282-8214.

1 Introduction

Ideal is a key idea in topological space and has a significant impact on research into topological spaces. The notion of the ideal topological spaces introduced and studied by Kuratowski[11] and Vaidyanathaswamy[23]. In 1999, Molodtsov[15] introduced the new notion soft sets and Shabir and Naz[20] introduced soft topological spaces using soft sets in 2011. After that many researchers working with soft topological spaces. The properties and applications of soft set theory have been studied [3, 10, 16]. In recent years, many interesting applications of soft set theory have been expanded by incorporating fuzzy set concepts [1, 2, 4, 12, 13, 14, 16].

In this paper, we have introduce and study weak separation axioms such as soft $\mathcal{I}_{g\delta s}$-T_0, soft $\mathcal{I}_{g\delta s}$-T_1, soft $\mathcal{I}_{g\delta s}$-T_2, δ-$T_{3/4}$, $\mathcal{I}_{g\delta s}$-regular space in soft ideal topological spaces and also investigate some of its characterizations and properties with the help of general topology concepts. Also give relationships between the above new regualr spaces. Then we give sufficient and necessity condition between soft S$\mathcal{I}_{g\delta s}$-closed sets and soft singleton sets in soft $\mathcal{I}_{g\delta s}$-T_1 space.

2 Preliminaries

Definition 2.1. *[15] A soft set F_A over the universe X is defined by the set of ordered pairs*

F_A=$\{$(e, F_A(e)) : e$\in$E, F_A(e)$\in\wp(X)\}$

where F_A : E$\to\wp(X)$, such that F_A(e)$\neq\emptyset$, if e$\in$A$\subset$E and F_A(e)=$\emptyset$ if e$\notin$A. The family of all soft sets over X is denoted by SS(X).

Definition 2.2. *[14] The soft set $F_\emptyset$ over a common universe set X is said to be null soft set, denoted by $\emptyset$. Here $F_\emptyset$(e)=$\emptyset$, $\forall$e$\in$E.*

Definition 2.3. *[14] A soft set F_A over X is called an absolute soft set, denoted by $\tilde{A}$, if e$\in$A, F_A(e)=X.*

Definition 2.4. *[24] A soft set F_A in a soft topological space (X, τ, E) is said to be a soft regular open (resp. soft regular closed) if F_A=int(cl(F_A)) (resp. F_A=cl(int(F_A))).*

Definition 2.5. *Let $\mathcal{I}$ be a non-null collection of soft sets over a universe X with the same set of parameters E. Then $\mathcal{I}\subset$SS(X) is called a soft ideal on X with the same set E if*

(1) $F_A\in\mathcal{I}$ and $G_A\in\mathcal{I}\Rightarrow F_A\cup G_A\in\mathcal{I}$.

(2) $F_A\in\mathcal{I}$ and $G_A\subset F_A\Rightarrow G_A\in\mathcal{I}$

Definition 2.6. *Let (X, τ, E) be a soft topological space and $\mathcal{I}$ be a soft ideal over X with the same set of parameters E. Then F_A^*=$\cup\{x_e \in X : O_{x_e} \cap F_A \notin \mathcal{I}$, for all $O_{x_e} \in \tau\}$ is called the soft local function of F_A with respect to $\mathcal{I}$ and τ, where O_{x_e} is a τ-open set containing x_e.*

Definition 2.7. *[8] Let F_A be a soft subset of soft topological space (X, τ, E). Then*

1. *x_ε is called a soft δ-cluster point of F_A if $F_A \cap int(cl(U_A)) \neq \emptyset$ for every soft open set U_A containing x_ε.*
2. *The family of all soft δ-cluster point of F_A is called the soft δ-closure of F_A and is denoted by $cl_\delta(F_A)$.*
3. *A soft subset F_A is said to be soft δ-closed if $cl_\delta(F_A)$=F_A. The complement of a soft δ-closed set of X is said to be soft δ-open.*

Definition 2.8. *A soft subset F_A of a soft topological space (X, τ, E) is said to be*

1. *soft preopen if $F_A \subset int(cl(F_A))$,*
2. *soft semiopen if $F_A \subset cl(int(F_A))$,*
3. *soft α-open if $F_A \subset int(cl(int(F_A)))$.*

The complement of a soft preopen (resp. soft semiopen, soft α-open) set is called a soft preclosed (resp. soft semiclosed, soft α -closed) set. The set of all soft preopen (resp. soft semiopen, soft α-open, soft preclosed, soft semiclosed, soft α-closed) sets of $(X, \tau$, E) is denoted by SPO(X) (resp. SSO(X), SαO(X), SPC(X), SSC(X), SαC(X)).

int$_S(F_A)$=$\{\cup U_A : U_A \subset F_A$ and U_A is soft semiopen sets$\}$ and cl$_S(F_A)$=$\{\cap G_A : F_A \subset G_A$ and G_A is soft semiclosed$\}$.

Definition 2.9. *[8] A soft subset F_A of a soft ideal topological space (X, τ, E, $\mathcal{I}$) is said to be*

1. *soft pre-$\mathcal{I}$-open if $F_A \subset int(cl(F_A))$,*
2. *soft semi-$\mathcal{I}$-open if $F_A \subset cl(int(F_A))$,*
3. *soft α-$\mathcal{I}$-open if $F_A \subset int(cl(int(F_A)))$.*

The set of all soft semi-$\mathcal{I}$-open sets is denoted by SS$\mathcal{I}$O(X)

The complement of soft pre-$\mathcal{I}$-open (resp. soft semi-$\mathcal{I}$-open, soft α-$\mathcal{I}$-open) set is called a soft pre-$\mathcal{I}$-closed (resp. soft semi-$\mathcal{I}$-closed, soft α-$\mathcal{I}$-closed).

Definition 2.10. *The soft semi-$\mathcal{I}$-closure of F_A is defined by the intersection of all soft semi-$\mathcal{I}$-closed sets containing F_A and is denoted by $S\mathcal{I}_{scl}(F_A)$*

Definition 2.11. *[18] A soft set F_A of soft ideal topological space X is called soft generalized δ semi-closed (briefly soft $\mathcal{I}_{g\delta s}$-closed) set if $S\mathcal{I}_{scl}(F_A) \subset G_A$ whenever $F_A \subset G_A$ and G_A is soft δ-open over X.*

A soft set F_A of X is called soft generalized δ semi-open (briefly soft $S\mathcal{I}_{g\delta s}$-open) set if ${F_A}^c$ is soft $S\mathcal{I}_{g\delta s}$-closed.

The family of all soft $S\mathcal{I}_{g\delta s}$-closed subsets of the space X is denoted by $S\mathcal{I}_{g\delta s}$-C(X) and soft $S\mathcal{I}_{g\delta s}$-open subsets of the space X is denoted by $S\mathcal{I}_{g\delta s}$-O(X).

Definition 2.12. *[18] Let X be a soft ideal topological space and F_A be a soft subset over X. Then soft $S\mathcal{I}_{g\delta s}$-closure of F_A denoted by $S\mathcal{I}_{g\delta s}$-$cl(F_A)$ and defined as the intersection of all soft $S\mathcal{I}_{g\delta s}$-closed sets over X containing F_A.*

Definition 2.13. *[9] Let (X, τ, E) and (Y, σ, K) be soft topological spaces and soft function f:(X, τ, E)$\to$(Y, σ, K) is called*

1. *soft continuous if $f^{-1}(F_B)$ is soft open over X for every soft open set F_B over Y.*
2. *soft open (resp. soft closed) if f(F_A) is soft open over Y for every soft open set F_A over X (resp. f(F_A) is soft closed over Y for every soft closed set F_A over X).*
3. *soft semiopen (resp. soft semiclosed) if f(F_A) is soft semiopen over Y for every soft open set F_A over X (resp. f(F_A) is soft semiclosed over Y for every soft closed set F_A over X).*
4. *soft semicontinuous if $f^{-1}(F_B)$ is soft semiclosed over X for every soft closed set F_B over Y.*
5. *soft irresolute if $f^{-1}(F_B)$ is soft semiopen over X for every soft semiopen set F_B over Y.*

Definition 2.14. *A function $f : (X, \tau, E) \to (Y, \sigma, K, \mathcal{I})$ is said to be soft $S\mathcal{I}_{pg\delta s}$ closed if $f(V_A)$ is soft $S\mathcal{I}_{g\delta s}$-closed over Y for every soft semiclosed set V_A over X.*

Definition 2.15.
1. *A function $f : (X, \tau, E, \mathcal{I}) \to (Y, \sigma, K, \mathcal{J})$ is soft $S\mathcal{I}_{g\delta s}$-irresolute if $f^{-1}(V_B)$ is soft $S\mathcal{I}_{g\delta s}$-closed over X for every soft $S\mathcal{I}_{g\delta s}$-closed set V_B over Y.*
2. *A function $f : (X, \tau, E, \mathcal{I}) \to (Y, K, \sigma)$ is soft $S\mathcal{I}_{g\delta s}$-continuous if $f^{-1}(V_B)$ is soft $S\mathcal{I}_{g\delta s}$-closed over X for every soft closed set V_B over Y.*

Definition 2.16. *[7]*

1. *Two soft subsets G_A, H_A over X are said to be soft distinct, written $G_A \cap H_A = \emptyset$ if $G(e) \cap H(e) = \emptyset$, for all $e \in A$.*

2. *Two soft points x_e, y_e over X are distinct, written $x_e \neq y_e$, if there soft sets G_A and H_A are disjoint.*

3. *A soft ideal topological space X is said to be soft T_0 space if for each pair of distinct soft points x_e and y_e over X, there exists a soft open set containing one soft point but not the other.*

4. *A soft ideal topological space X is said to be soft T_1 space if for any pair of distinct soft points x_e and y_e, there exist soft open sets G_A and H_A such that $x_e \in G_A$, $y_e \notin G_A$ and $x_e \notin H_A$, $y_e \in H_A$.*

5. *A soft ideal topological space X is said to be soft T_2 space if for any pair of distinct soft points x_e and y_e, there exist disjoint soft open sets G_A and H_A such that $x_e \in G_A$ and $y_e \in H_A$.*

Definition 2.17. *[25] A soft topological space (X, τ, E) is said to be soft regular if for all soft closed sets F_A over X and each soft point x_e such that $x_e \notin F_A$, there exists soft open sets U_A and V_A over X such that $x_e \in U_A$, $F_A \subset V_A$ and $U_A \cap V_A = \emptyset$.*

Definition 2.18. *A soft ideal topological space X is said to be soft $T\mathcal{I}_{g\delta s}$-space if every soft $S\mathcal{I}_{g\delta s}$-closed set is soft closed over X.*

Definition 2.19.

1. *A function $f : (X, \tau, E) \to (Y, \sigma, K)$ is said to be perfectly soft continuous, if $f^{-1}(V_B)$ is soft clopen over X for every soft open set V_B over Y.*

2. *A function $f : (X, \tau, E) \to (Y, \sigma, K, \mathcal{I})$ is said to be perfectly soft $S\mathcal{I}_{g\delta s}$-continuous (resp. totally soft semicontinuous), if $f^{-1}(V_B)$ is soft clopen over X for every soft $S\mathcal{I}_{g\delta s}$-open (resp. soft semiopen) set V_B over Y.*

Definition 2.20. *A function $f : (X, \tau, E) \to (Y, \sigma, K, \mathcal{I})$ is said to be strongly soft $S\mathcal{I}_{g\delta s}$-closed (resp. strongly soft $S\mathcal{I}_{g\delta s}$-open), if $f(F_A)$ is soft $S\mathcal{I}_{g\delta s}$-closed (resp. soft $S\mathcal{I}_{g\delta s}$-open) set over Y for every soft $S\mathcal{I}_{g\delta s}$-closed (resp. soft $S\mathcal{I}_{g\delta s}$-open) set F_A over X.*

Definition 2.21. *A function $f : (X, \tau, E) \to (Y, \sigma, K, \mathcal{I})$ is said to be quasi soft $S\mathcal{I}_{g\delta s}$-closed (resp. quasi soft $S\mathcal{I}_{g\delta s}$-open) if for each soft $S\mathcal{I}_{g\delta s}$-closed (resp. soft $S\mathcal{I}_{g\delta s}$-open) set F_A over X, f(F_A) is soft closed (resp. open) set over Y.*

3 soft Separation Axioms

Definition 3.1. *A soft ideal topological space X is said to be soft $\mathcal{I}_{g\delta s}$-T_0 space if for each pair of distinct soft points x_e and y_e over X, there exists a soft $S\mathcal{I}_{g\delta s}$-open set containing one soft point but not the other.*

Theorem 3.1. *A soft ideal topological space X is a soft $\mathcal{I}_{g\delta s}$-T_0 space if and only if soft $S\mathcal{I}_{g\delta s}$-closures of distinct soft points are distinct.*

Proof: Let x_e and y_e be distinct soft points over X. Since X is soft $\mathcal{I}_{g\delta s}$-T_0 space, there exists a soft S$\mathcal{I}_{g\delta s}$-open set G_A such that $x_e \in G_A$ and $y_e \notin G_A$. Consequently, $X - G_A$ is a soft S$\mathcal{I}_{g\delta s}$-closed set containing y_e but not x_e. But S$\mathcal{I}_{g\delta s}$-$cl\{y_e\}$ is the intersection of all soft S$\mathcal{I}_{g\delta s}$-closed sets containing y_e. Hence $y_e \in S\mathcal{I}_{g\delta s}$-$cl\{y_e\}$ but $x_e \notin S\mathcal{I}_{g\delta s}$-$cl\{y_e\}$ as $x_e \notin X - G_A$. Therefore, S$\mathcal{I}_{g\delta s}$-$cl\{x_e\} \neq S\mathcal{I}_{g\delta s}$-$cl\{y_e\}$.
Conversely, let S$\mathcal{I}_{g\delta s}$-$cl\{x_e\} \neq S\mathcal{I}_{g\delta s}$-$cl\{y_e\}$ for $x_e \neq y_e$. Then there exists at least one soft point $z_e over X$ such that $z_e \in S\mathcal{I}_{g\delta s}$-$cl\{x_e\}$ but $z_e \notin S\mathcal{I}_{g\delta s}$-$cl\{y_e\}$. We claim $x_e \notin S\mathcal{I}_{g\delta s}$-$cl\{y_e\}$, because if $x_e \in S\mathcal{I}_{g\delta s}$-$cl\{y_e\}$ then $\{x_e\} \subset S\mathcal{I}_{g\delta s}$-$cl\{y_e\}$ implies S$\mathcal{I}_{g\delta s}$-$cl\{x_e\} \subset S\mathcal{I}_{g\delta s}$-$cl\{y_e\}$. So $z_e \in S\mathcal{I}_{g\delta s}$-$cl\{y_e\}$, which is a contradiction. Hence $x_e \notin S\mathcal{I}_{g\delta s}$-$cl\{y_e\}$, which implies $x_e \in X - S\mathcal{I}_{g\delta s}$-$cl\{y_e\}$, which is a soft S$\mathcal{I}_{g\delta s}$-open set containing x_e but not y_e. Hence X is soft $\mathcal{I}_{g\delta s}$-T_0 space.

Theorem 3.2. *If $f : X \to Y$ is a bijection, strongly soft $S\mathcal{I}_{g\delta s}$-open and X is soft $\mathcal{I}_{g\delta s}$-T_0 space, then Y is also soft $\mathcal{I}_{g\delta s}$-T_0 space.*

Proof: Let y_e and z_e be two distinct soft points over Y . Since f is bijective there exist distinct soft points o_e and x_e over X such that $f(o_e) = y_e$ and $f(x_e) = z_e$. Since X is soft $\mathcal{I}_{g\delta s}$-T_0 space there exists a soft S$\mathcal{I}_{g\delta s}$-open set G_A such that $o_e \in G_A$ and $x_e \notin G_A$. Therefore $y_e = f(o_e) \in f(G_A)$ and $z_e = f(x_e) \notin f(G_A)$. Since f being strongly soft S$\mathcal{I}_{g\delta s}$-open function, $f(G_A)$ is soft S$\mathcal{I}_{g\delta s}$-open over Y . Thus, there exists a soft S$\mathcal{I}_{g\delta s}$-open set $f(G_A)$ over Y such that $y_e \in f(G_A)$ and $z_e \notin f(G_A)$. Therefore Y is soft $\mathcal{I}_{g\delta s}$-T_0 space.

Definition 3.2. *A soft ideal topological space X is said to be soft $\mathcal{I}_{g\delta s}$-T_1 space if for any pair of distinct soft points x_e and y_e, there exist soft $S\mathcal{I}_{g\delta s}$-open sets G_A and H_A such that $x_e \in G_A$, $y_e \notin G_A$ and $x_e \notin H_A$, $y_e \in H_A$.*

Theorem 3.3. *A soft ideal topological space X is soft $\mathcal{I}_{g\delta s}$-T_1 space if and only if soft singleton are soft $S\mathcal{I}_{g\delta s}$-closed sets.*

Proof: Let X be a soft $\mathcal{I}_{g\delta s}$-T_1 space and $x_e \in K_A$. Let $y_e \in K_A - \{x_e\}$. Then for $x_e \neq y_e$, there exists soft S$\mathcal{I}_{g\delta s}$-open set U_A such that $y_e \in U_A$ and $x_e \notin U_A$.

Consequently, $y_e \in U_A \subset X-\{x_e\}$. That is $X-\{x_e\} = \cup\{U_A : y_e \in K_A-\{x_e\}\}$, which is soft S$\mathcal{I}_{g\delta s}$-open set. Hence $\{x_e\}$ is soft S$\mathcal{I}_{g\delta s}$-closed set.
Conversely, suppose $\{x_e\}$ is soft S$\mathcal{I}_{g\delta s}$-closed set for every soft point $x_e over X$. Let x_e and y_e soft point over X with $x_e \neq y_e$. Now $x_e \neq y_e$ implies $y_e \in X-\{x_e\}$. Hence $X-\{x_e\}$ is soft S$\mathcal{I}_{g\delta s}$-open set containing y_e but not x_e. Similarly, $X-\{y_e\}$ is soft S$\mathcal{I}_{g\delta s}$-open set containing x_e but not y_e. Therefore X is soft $\mathcal{I}_{g\delta s}$-T_1 space.

Theorem 3.4. *The property being soft $\mathcal{I}_{g\delta s}$-T_1 space is preserved under bijection and strongly soft S$\mathcal{I}_{g\delta s}$-open function.*

Proof: Let $f : X \to Y$ be bijective and strongly soft S$\mathcal{I}_{g\delta s}$-open function. Let X be a soft $\mathcal{I}_{g\delta s}$-T_1 space and y$_e$, z$_e$ be any two distinct soft points over Y . Since f is bijective there exist distinct soft points o_e, x_e over X such that $y_e = f(o_e)$ and $z_e = f(x_e)$. Now X being a soft $\mathcal{I}_{g\delta s}$-T_1 space, there exist soft S$\mathcal{I}_{g\delta s}$-open sets G_A and H_A such that $o_e \in G_A$, $x_e \notin G_A$ and $o_e \notin H_A$, $x_e \in H_A$. Therefore $y_e = f(o_e) \in f(G_A)$ but $z_e = f(x_e) \notin f(G_A)$ and $z_e = f(x_e) \in f(H_A)$ and $y_e = f(o_e) \notin f(H_A)$. Now f being strongly soft S$\mathcal{I}_{g\delta s}$-open, $f(G_A)$ and $f(H_A)$ are soft S$\mathcal{I}_{g\delta s}$-open subsets over Y such that $y_e \in f(G_A)$ but $z_e \notin f(G_A)$ and $z_e \in f(H_A)$ and $y_e \notin f(H_A)$. Hence Y is soft $\mathcal{I}_{g\delta s}$-T_1 space.

Theorem 3.5. *Let $f : X \to Y$ be bijective and soft S$\mathcal{I}_{g\delta s}$-open function. If X is soft $\mathcal{I}_{g\delta s}$-T_1 and soft T$\mathcal{I}_{g\delta s}$-space, then Y is soft $\mathcal{I}_{g\delta s}$-T_1 space.*

Proof: Let y$_e$, z$_e$ be any two distinct soft points over Y. Since f is bijective there exist distinct soft points o_e, x_e over X such that $y_e = f(o_e)$ and $z_e = f(x_e)$. Now X being a soft $\mathcal{I}_{g\delta s}$-T_1 space, there exist soft S$\mathcal{I}_{g\delta s}$-open sets G_A and H_A such that $o_e \in G_A$, $x_e \notin G_A$ and $o_e \notin H_A$, $x_e \in H_A$. Therefore $y_e = f(o_e) \in f(G_A)$ but $z_e = f(x_e) \notin f(G_A)$ and $z_e = f(x_e) \in f(H_A)$ and $y_e = f(o_e) \notin f(H_A)$. Now X is soft T$\mathcal{I}_{g\delta s}$-space which implies G_A and H_A are soft open sets over X and f is soft S$\mathcal{I}_{g\delta s}$-open function, $f(G_A)$ and $f(H_A)$ are soft S$\mathcal{I}_{g\delta s}$-open subsets over Y. Thus there exist soft S$\mathcal{I}_{g\delta s}$-open sets such that $y_e \in f(G_A)$ but $z_e \notin f(G_A)$ and $z_e \in f(H_A)$ and $y_e \notin f(H_A)$. Hence Y is soft $\mathcal{I}_{g\delta s}$-T_1 space.

Theorem 3.6. *If $f : X \to Y$ is soft S$\mathcal{I}_{g\delta s}$-continuous injection and Y is soft T_1 space then X is soft $\mathcal{I}_{g\delta s}$-T_1 space.*

Proof: Let $f : X \to Y$ be soft S$\mathcal{I}_{g\delta s}$-continuous injection and Y be soft T_1. For any two distinct soft points o_e, x_e over X there exist distinct soft points y$_e$, z$_e$ over Y such that $y_e = f(o_e)$ and $z_e = f(x_e)$. Since Y is soft T_1 space there exist soft open sets G_A and H_A over Y such that $y_e \in G_A$, $z_e \notin G_A$ and $y_e \notin H_A$, $z_e \in H_A$. That is $o_e \in f^{-1}(G_A)$, $o_e \notin f^{-1}(H_A)$ and $x_e \in f^{-1}(H_A)$, $x_e \notin f^{-1}(G_A)$. Since f is soft S$\mathcal{I}_{g\delta s}$-continuous $f^{-1}(G_A)$, $f^{-1}(H_A)$ are soft S$\mathcal{I}_{g\delta s}$-open sets over X. Thus, for two distinct soft points o_e, x_e over X there exist soft

S$\mathcal{I}_{g\delta s}$-open sets $f^{-1}(G_A)$ and $f^{-1}(H_A)$ such that $o_e \in f^{-1}(G_A)$, $o_e \notin f^{-1}(H_A)$ and $x_e \in f^{-1}(H_A)$, $x_e \notin f^{-1}(G_A)$. Therefore X is soft $\mathcal{I}_{g\delta s}$-T_1 space.

Theorem 3.7. *If $f : X \to Y$ is soft S$\mathcal{I}_{g\delta s}$-irresolute injective function and Y is soft $\mathcal{I}_{g\delta s}$-T_1 space then X is soft $\mathcal{I}_{g\delta s}$-T_1 space.*

Proof: Let o_e, x_e be pair of distinct soft points over X. Since f is injective there exist distinct soft points y$_e$, z$_e$ over Y such that $y_e = f(o_e)$ and $z_e = f(x_e)$. Since Y is soft $\mathcal{I}_{g\delta s}$-T_1 space there exist soft S$\mathcal{I}_{g\delta s}$-open sets G_A and H_A over Y such that $y_e \in G_A$, $z_e \notin G_A$ and $y_e \notin H_A$, $z_e \in H_A$. That is $o_e \in f^{-1}(G_A)$, $o_e \notin f^{-1}(H_A)$ and $x_e \in f^{-1}(H_A)$, $x_e \notin f^{-1}(G_A)$. Since f is soft S$\mathcal{I}_{g\delta s}$-irresolute $f^{-1}(G_A)$, $f^{-1}(H_A)$ are soft S$\mathcal{I}_{g\delta s}$-open sets over X. Thus, for two distinct soft points o_e, x_e over X there exist soft S$\mathcal{I}_{g\delta s}$-open sets $f^{-1}(G_A)$ and $f^{-1}(H_A)$ such that $o_e \in f^{-1}(G_A)$, $o_e \notin f^{-1}(H_A)$ and $x_e \in f^{-1}(H_A)$, $x_e \notin f^{-1}(G_A)$. Therefore X is soft $\mathcal{I}_{g\delta s}$-T_1 space.

Definition 3.3. *A soft ideal topological space X is said to be soft $\mathcal{I}_{g\delta s}$-T_2 space if for any pair of distinct soft points x_e and y_e, there exist disjoint soft S$\mathcal{I}_{g\delta s}$-open sets G_A and H_A such that $x_e \in G_A$ and $y_e \in H_A$.*

Theorem 3.8. *If $f : X \to Y$ is soft S$\mathcal{I}_{g\delta s}$-continuous injection and Y is soft T_2 then X is soft $\mathcal{I}_{g\delta s}$-T_2 space.*

Proof: Let $f : X \to Y$ be soft S$\mathcal{I}_{g\delta s}$-continuous injection and Y be soft T_2. For any two distinct soft points o$_e$, x$_e$ over X there exist distinct soft points y_e, z_e over Y such that $y_e = f(o_e)$ and $z_e = f(x_e)$. Since Y is soft T_2 space there exist disjoint soft open sets G_A and H_A over Y such that $y_e \in G_A$ and $z_e \in H_A$. That is $o_e \in f^{-1}(G_A)$ and $x_e \in f^{-1}(H_A)$. Since f is soft S$\mathcal{I}_{g\delta s}$-continuous $f^{-1}(G_A)$, $f^{-1}(H_A)$ are soft S$\mathcal{I}_{g\delta s}$-open sets over X. Further f is injective, $f^{-1}(G_A) \cap f^{-1}(H_A) = f^{-1}(G_A \cap H_A) = f^{-1}(\emptyset) = \emptyset$. Thus, for two disjoint soft points o_e, x_e over X there exist disjoint soft S$\mathcal{I}_{g\delta s}$-open sets $f^{-1}(G_A)$ and $f^{-1}(H_A)$ such that $o_e \in f^{-1}(G_A)$ and $x_e \in f^{-1}(H_A)$. Therefore X is soft $\mathcal{I}_{g\delta s}$-T_2 space.

Theorem 3.9. *If $f : X \to Y$ is soft S$\mathcal{I}_{g\delta s}$-irresolute injective function and Y is soft $\mathcal{I}_{g\delta s}$-T_2 space then X is soft $\mathcal{I}_{g\delta s}$-T_2 space.*

Proof: Let o$_e$, x$_e$ be pair of distinct soft points over X. Since f is injective there exist distinct soft points y$_e$, z$_e$ over Y such that $y_e = f(o_e)$ and $z_e = f(x_e)$. Since Y is soft $\mathcal{I}_{g\delta s}$-T_2 space there exist disjoint soft S$\mathcal{I}_{g\delta s}$-open sets G_A and H_A over Y such that $y_e \in G_A$ and $z_e \in H_A$. That is $o_e \in f^{-1}(G_A)$ and $x_e \in f^{-1}(H_A)$. Since f is soft S$\mathcal{I}_{g\delta s}$-irresolute injective $f^{-1}(G_A)$, $f^{-1}(H_A)$ are distinct soft S$\mathcal{I}_{g\delta s}$-open sets over X. Thus, for two disjoint soft points o_e, x_e over X there exist disjoint soft S$\mathcal{I}_{g\delta s}$-open sets $f^{-1}(G_A)$ and $f^{-1}(H_A)$ such that $o_e \in f^{-1}(G_A)$ and $x_e \in f^{-1}(H_A)$. Therefore X is soft $\mathcal{I}_{g\delta s}$-T_2 space.

Definition 3.4. *A soft ideal topological space X is called soft δ-$T_{3/4}$ if every soft $S\mathcal{I}_{g\delta s}$-closed set is soft δ-$\mathcal{I}$-closed.*

Theorem 3.10. *For a soft ideal topological space X, if X is a soft δ-$T_{3/4}$- space, then every singleton soft set $\{x_e\}$ is soft δ-$\mathcal{I}$-open or soft δ-$\mathcal{I}$-closed.*

Proof: Suppose X is a soft δ-$T_{3/4}$ space. If $\{x_e\}$ is not soft δ-$\mathcal{I}$-closed, then $X - \{x_e\}$ is not soft δ-$\mathcal{I}$-open. Then the only soft δ-$\mathcal{I}$-open set containing $X - \{x_e\}$ is X. Therefore $X - \{x_e\}$ is soft S$\mathcal{I}_{g\delta s}$-closed set over X. Since X is a soft δ-$T_{3/4}$ space, $X - \{x_e\}$ is soft δ-$\mathcal{I}$-closed, which implies $\{x_e\}$ is soft δ-$\mathcal{I}$-open.

Definition 3.5. *A soft subset F_A over X is soft δ-$\mathcal{I}$-nowhere dense if $int(cl^*_\delta(F_A)) = \emptyset$.*

Lemma 3.1. *Let X be a soft ideal topological space then, every singleton soft set is soft δ-$\mathcal{I}$-nowhere dense or soft δ-pre-$\mathcal{I}$-open over X.*

Proof: Suppose $\{x_e\}$ is not a soft δ-$\mathcal{I}$-nowhere dense over X. Therfore $int(cl^*_\delta(F_A)) \neq \emptyset$ which implies that $\{x_e\} \subset int(cl^*_\delta(\{x_e\}))$. Hence $\{x_e\}$ is a soft δ-pre-$\mathcal{I}$-open over X.

Definition 3.6. *A soft ideal topological space X is called soft δ-$T_{1/2}$ space if every soft $S\mathcal{I}_{g\delta s}$-closed set is soft semi-$\mathcal{I}$-closed.*

Theorem 3.11. *For a soft ideal topological space X the following conditions are equivalent*

1. *X is soft δ-$T_{1/2}$ space.*
2. *Every soft singleton set is either soft δ-$\mathcal{I}$-closed or soft semi-$\mathcal{I}$-open.*

Proof: $(1) \Rightarrow (2)$ If $\{x_e\}$ is not soft δ-$\mathcal{I}$closed, then $X - \{x_e\}$ is not soft δ-$\mathcal{I}$-open. Then the only soft δ-$\mathcal{I}$-open set containing $X - \{x_e\}$ is $\tilde{X}$. Therefore $X - \{x_e\}$ is soft S$\mathcal{I}_{g\delta s}$-closed set over X. By (1), $X - \{x_e\}$ is soft semi-$\mathcal{I}$-closed, which implies $\{x_e\}$ is soft semi-$\mathcal{I}$-open.
$(2) \Rightarrow (1)$ Let F_A be soft subest over X and $x_e \in S\mathcal{I}_{scl}$(F$_A$). Then consider the following cases

Case (1): Let $\{x_e\}$ be soft semi-$\mathcal{I}$-open. Since $x_e \in S\mathcal{I}_{scl}(F_A)$, then $\{x_e\} \cap F_A \neq \emptyset$. This implies $x_e \in F_A$.

Case (2): Let $\{x_e\}$ be soft δ-$\mathcal{I}$-closed. Assume that $x_e \notin F_A$, then $x_e \in S\mathcal{I}_{scl}(F_A) - F_A$, which implies $\{x_e\} \subset S\mathcal{I}_{scl}(F_A) - F_A$. This is not possible according to Theorem **??**. This shows that, $x_e \in F_A$.

So in both cases, S$\mathcal{I}_{scl}(F_A) \subset F_A$. Since the reverse inclusion is trivial, implies S$\mathcal{I}_{scl}(F_A) = F_A$. Therefore F_A is soft semi-$\mathcal{I}$-closed.

Theorem 3.12. *Every soft δ-$T_{3/4}$ space is soft δ-$T_{1/2}$ space.*

Proof: Let X be a soft δ-$T_{3/4}$ space. Then by Theorem 3.10, every soft singleton set over X is soft δ-$\mathcal{I}$-open or soft δ-$\mathcal{I}$-closed. But every soft δ-$\mathcal{I}$-open set is soft semi-$\mathcal{I}$-open set. Thus every soft singleton set over X is soft semi-$\mathcal{I}$-open or soft δ-$\mathcal{I}$-closed. By theorem 3.11, X is soft δ-$T_{1/2}$ space.

Theorem 3.13. *For any soft ideal topological space X*

1. *SS$\mathcal{I}$O(X) $\subset$ $S\mathcal{I}_{g\delta s}$-O(X).*

2. *A space X is soft δ-$T_{1/2}$-space if and only if SS$\mathcal{I}$O(X) = $S\mathcal{I}_{g\delta s}$-O(X).*

Proof: (1) If F_A is soft semi-$\mathcal{I}$-open, then $X - F_A$ is soft semi-$\mathcal{I}$-closed. So $X - F_A$ is soft S$\mathcal{I}_{g\delta s}$-closed, this implies F_A is soft S$\mathcal{I}_{g\delta s}$-open. Hence SS$\mathcal{I}$O(X) $\subset S\mathcal{I}_{g\delta s}$-O(X).
(2) Let F_A be a soft δ-$T_{1/2}$ space and $F_A \in S\mathcal{I}_{g\delta s}$-O(X). Then $X - F_A$ is soft S$\mathcal{I}_{g\delta s}$-closed set. By hypothesis, $X - F_A$ is soft semi-$\mathcal{I}$-closed and hence $F_A \in SS\mathcal{I}O(X)$. Therefore S$\mathcal{I}_{g\delta s}$-O(X) $\subset SS\mathcal{I}O(X)$. By (1), SS$\mathcal{I}$O(X) $\subset S\mathcal{I}_{g\delta s}$-O(X). Therefore SS$\mathcal{I}$O(X) = S$\mathcal{I}_{g\delta s}$-O(X).
Conversely, let SS$\mathcal{I}$O(X) = S$\mathcal{I}_{g\delta s}$-O(X) and F_A be a soft S$\mathcal{I}_{g\delta s}$-closed set. Then $X - F_A$ is soft S$\mathcal{I}_{g\delta s}$-open. Hence $X - F_A$ is soft semi-$\mathcal{I}$-open, which implies F_A is soft semi-$\mathcal{I}$-closed. Thus every soft S$\mathcal{I}_{g\delta s}$-closed set is soft semi-$\mathcal{I}$-closed. Therefore X is soft δ-$T_{1/2}$-space.

Lemma 3.2. *For a soft ideal topological space X the following are equivalent*

1. *Every soft δ-pre-$\mathcal{I}$-open singleton set is soft δ-$\mathcal{I}$-closed.*

2. *Every soft singleton set is soft δ-$\mathcal{I}$-nowhere dense or soft δ-$\mathcal{I}$-closed.*

Proof: $(1) \Rightarrow (2)$ By Lemma 3.1, every soft singleton set is either soft δ-$\mathcal{I}$-nowhere dense or soft δ-pre-$\mathcal{I}$-open. In the first case we are done and in the second case soft δ-$\mathcal{I}$-closedness follows from the assumption.
$(2) \Rightarrow (1)$ Let $\{x_e\}$ be soft δ-pre-$\mathcal{I}$-open. Assume that $\{x_e\}$ is not soft δ-$\mathcal{I}$-closed. Then by (2), it is soft δ-$\mathcal{I}$-nowhere dense. Thus $\{x_e\} \subset int(cl^*_\delta(\{x_e\})) = \emptyset$, which is not possible. Hence $\{x_e\}$ is soft δ-$\mathcal{I}$-closed.

Theorem 3.14. *For a soft ideal topological space X the following are equivalent*

1. *X is soft δ-T_1 space.*

2. *X is soft δ-$T_{3/4}$ space and every soft singleton set is soft δ-$\mathcal{I}$-nowhere dense or soft δ-$\mathcal{I}$-closed.*

3. *X is soft δ-$T_{3/4}$ space and every soft δ-pre-$\mathcal{I}$-open singleton set is soft δ-$\mathcal{I}$-closed.*

Proof: $(1) \Rightarrow (2)$ Obvious.
$(2) \Rightarrow (1)$ If soft singleton set is not soft δ-$\mathcal{I}$-closed, then it must be soft δ-$\mathcal{I}$-open, since X is soft δ-$T_{3/4}$-space. But soft singleton set is soft δ-$\mathcal{I}$-open if and only if it is soft regular-$\mathcal{I}$-open, which implies soft singleton set is soft regular-$\mathcal{I}$-open. Moreover, by rest of assumption X is soft δ-$\mathcal{I}$-nowhere dense at the same time, X must be soft δ-T_1-space.
$(2) \Leftrightarrow (3)$ Follows from lemma 3.2

Theorem 3.15. *In any soft ideal topological space the following are equivalent*

1. *X is soft $\mathcal{I}_{g\delta s}$-T_2 space.*
2. *For each $x_e \neq y_e$, there exists a soft $S\mathcal{I}_{g\delta s}$-open set G_A such that $x_e \in G_A$ and $y_e \notin S\mathcal{I}_{g\delta s}$-$cl(G_A)$.*
3. *For each $x_e \in G_A$, $\{x_e\} = \cap\{S\mathcal{I}_{g\delta s}$-$cl(G_A) : G_A$ is a soft $S\mathcal{I}_{g\delta s}$-open set over X and $x_e \in G_A\}$.*

Proof: $(1) \Rightarrow (2)$ Assume (1) holds. Let $x_e \in G_A$ and $x_e \neq y_e$, then there exist disjoint soft S$\mathcal{I}_{g\delta s}$-open sets G_A and H_A such that $x_e \in G_A$ and $y_e \in H_A$. Clearly, $X - G_A$ is soft S$\mathcal{I}_{g\delta s}$-closed set. Since $G_A \cap H_A = \emptyset$, $G_A \subset X - H_A$. Therefore S$\mathcal{I}_{g\delta s}$-$cl(G_A) \subset S\mathcal{I}_{g\delta s}$-$cl(X - H_A) = X - H_A$. Now $y_e \notin X - H_A$ implies $y_e \notin S\mathcal{I}_{g\delta s}$-$cl(G_A)$.
$(2) \Rightarrow (3)$ For each $x_e \neq y_e$, there exists a soft S$\mathcal{I}_{g\delta s}$-open set G_A such that $x_e \in G_A$ and $y_e \notin S\mathcal{I}_{g\delta s}$-$cl(G_A)$. So $y_e \notin \cap\{S\mathcal{I}_{g\delta s}$-$cl(G_A) : G_A$ is a soft S$\mathcal{I}_{g\delta s}$-open set over X and $x_e \in G_A\} = \{x_e\}$.
$(3) \Rightarrow (1)$ Let x_e, y_e be soft sets over X and $x_e \neq y_e$. By hpothesis there exists a soft S$\mathcal{I}_{g\delta s}$-open set G_A such that $x_e \in G_A$ and $y_e \notin S\mathcal{I}_{g\delta s}$-$cl(G_A)$. This implies there exists a soft S$\mathcal{I}_{g\delta s}$-closed set H_A such that $y_e \notin H_A$. Therefore $y_e \in X - H_A$ and $X - H_A$ is soft S$\mathcal{I}_{g\delta s}$-open set. Thus, there exist two disjoint soft S$\mathcal{I}_{g\delta s}$-open sets G_A and $X - H_A$ such that $x_e \in G_A$ and $y_e \in X - H_A$. Therefore X is soft $\mathcal{I}_{g\delta s}$-T_2 space.

4 soft $\mathcal{I}_{g\delta s}$-regular spaces

Definition 4.1. *A soft ideal topological space X is said to be soft $\mathcal{I}_{g\delta s}$-regular if for each soft closed set F_A and each soft point $x_e \in X - F_A$, there exist disjoint soft $S\mathcal{I}_{g\delta s}$-open sets U_A and W_A such that $x_e \in U_A$ and $F_A \subset W_A$.*

Remark 4.1. *Every soft regular space is soft $\mathcal{I}_{g\delta s}$-regular space.*

Theorem 4.1. *Every soft $\mathcal{I}_{g\delta s}$-regular and soft T_0 space is soft $\mathcal{I}_{g\delta s}$-T_2.*

Proof: Let X be a soft $\mathcal{I}_{g\delta s}$-regular space and x_e, y_e be soft point over X such that $x_e \neq y_e$. Since X being soft T_0 space, there exists a soft open set U_A over X such that $x_e \in U_A$ and $y_e \notin U_A$. Then $X - U_A$ is a soft closed set containing y_e but not x_e. Since X is soft $\mathcal{I}_{g\delta s}$-regular, there exist disjoint soft S$\mathcal{I}_{g\delta s}$-open sets V_A and W_A such that $X - U_A \subset W_A$ and $x_e \in V_A$. Now $y_e \in X - U_A$ implies $y_e \in W_A$. Thus for x_e, y_e soft set over X such that $x_e \neq y_e$, there exist disjoint soft open sets V_A and W_A such that $x_e \in V_A$ and $y_e \in W_A$. Therefore X is soft $\mathcal{I}_{g\delta s}$-T_2 space.

Theorem 4.2. *For a soft ideal topological space X, the following conditions are equivalent.*

1. *X is soft $\mathcal{I}_{g\delta s}$-regular.*
2. *For every soft point x_e over X and soft open set U_A containing x_e there exists a soft $S\mathcal{I}_{g\delta s}$-open set W_A such that $x_e \in W_A \subset S\mathcal{I}_{g\delta s}\text{-}cl(W_A) \subset U_A$.*
3. *For every soft closed set F_A, $F_A = \cap\{S\mathcal{I}_{g\delta s}\text{-}cl(U_A) : F_A \subset U_A$ and U_A is soft $S\mathcal{I}_{g\delta s}$-open set over $X\}$.*
4. *For every soft set F_A and a soft open set H_A such that $F_A \cap H_A \neq \emptyset$, there exists soft $S\mathcal{I}_{g\delta s}$-open set O_A such that $F_A \cap O_A \neq \emptyset$ and $S\mathcal{I}_{g\delta s}\text{-}cl(O_A) \subset H_A$.*
5. *For every non empty soft set F_A and soft closed set H_A such that $F_A \cap H_A \neq \emptyset$, there exist disjoint soft $S\mathcal{I}_{g\delta s}$-open sets L_A and M_A such that $F_A \cap L_A \neq \emptyset$ and $H_A \subset M_A$.*

Proof: $(1) \Rightarrow (2)$ Let U_A be a soft open set containing x_e. Then $X - U_A$ is soft closed set not containing x_e. Since X is soft $\mathcal{I}_{g\delta s}$-regular, there exist soft S$\mathcal{I}_{g\delta s}$-open sets L_A and W_A such that $x_e \in W_A$, $X - U_A \subset L_A$ and $W_A \cap L_A = \emptyset$. This implies $W_A \subset X - L_A$. Therefore, S$\mathcal{I}_{g\delta s}$-$cl(W_A) \subset S\mathcal{I}_{g\delta s}\text{-}cl(X - L_A) = X - L_A$, because $X - L_A$ is soft S$\mathcal{I}_{g\delta s}$-closed. Hence $x_e \in W_A \subset S\mathcal{I}_{g\delta s}\text{-}cl(W_A) \subset X - L_A \subset U_A$. That is $x_e \in W_A \subset \mathcal{I}_{g\delta s}\text{-}cl(W_A) \subset U_A$.

$(2) \Rightarrow (3)$ Let F_A be a soft closed set and $x_e \notin F_A$. Then $X - F_A$ is a soft open set containing x_e. By (2), there is a soft S$\mathcal{I}_{g\delta s}$-open set W_A such that $x_e \in W_A \subset S\mathcal{I}_{g\delta s}\text{-}cl(W_A) \subset X - F_A$. And so, $F_A \subset X - S\mathcal{I}_{g\delta s}\text{-}cl(W_A) \subset X - W_A$. Consequently $X - W_A$ is soft S$\mathcal{I}_{g\delta s}$-closed set not containing x_e. Put $U_A = X - S\mathcal{I}_{g\delta s}\text{-}cl(W_A)$. This implies $F_A \subset U_A$ and U_A is soft S$\mathcal{I}_{g\delta s}$-open set over X and $x_e \notin S\mathcal{I}_{g\delta s}\text{-}cl(U_A)$, implies $\cap\{S\mathcal{I}_{g\delta s}\text{-}cl(U_A) : F_A \subset U_A$ and U_A is soft S$\mathcal{I}_{g\delta s}$-open set over $X\} \subset F_A$.......(1) But F_A is soft closed and every soft closed set is soft S$\mathcal{I}_{g\delta s}$-closed. Therefore $F_A \subset \cap\{S\mathcal{I}_{g\delta s}\text{-}cl(U_A) : F_A \subset U_A$ and U_A is soft S$\mathcal{I}_{g\delta s}$-open set over $X\}$........ (2) is always true. From (1) and (2), $F_A = \cap\{S\mathcal{I}_{g\delta s}\text{-}cl(U_A) : F_A \subset U_A$ and U_A is soft S$\mathcal{I}_{g\delta s}$-open set over $X\}$.

$(3) \Rightarrow (4)$ Let $F_A \cap H_A \neq \emptyset$ and H_A is soft open. Let $x_e \in F_A \cap H_A$. Then $X - H_A$ is a soft closed set not containing x_e. By (3), there exists a soft S$\mathcal{I}_{g\delta s}$-open set U_A over X such that $X - H_A \subset U_A$ and $x_e \notin S\mathcal{I}_{g\delta s}\text{-}cl(U_A)$. Put $O_A = X - S\mathcal{I}_{g\delta s}\text{-}cl(U_A)$, then O_A is soft S$\mathcal{I}_{g\delta s}$-open set over X, $x_e \in F_A \cap O_A$ and S$\mathcal{I}_{g\delta s}$-$cl(O_A) \subset S\mathcal{I}_{g\delta s}\text{-}cl(X - U_A) = X - U_A \subset H_A$. Hence S$\mathcal{I}_{g\delta s}$-$cl(O_A) \subset H_A$.

$(4) \Rightarrow (5)$ If $F_A \cap H_A \neq \emptyset$, where F_A is non empty and H_A is soft closed, then $F_A \cap (X - H_A) \neq \emptyset$ and $(X - H_A)$ is soft open. Therefore by (4), there exists soft S$\mathcal{I}_{g\delta s}$-open set L_A such that $F_A \cap L_A \neq \emptyset$ and $L_A \subset S\mathcal{I}_{g\delta s}\text{-}cl(L_A) \subset X - H_A$.

$(5) \Rightarrow (1)$ Let F_A be a soft closed set such that $x_e \notin F_A$, then $\{x_e\} \cap F_A = \emptyset$. By (5), there exist disjoint soft open sets L_A and M_A such that $\{x_e\} \cap L_A \neq \emptyset$ and $F_A \subset M_A$, which implies $x_e \in L_A$ and $F_A \subset M_A$. Hence, X is soft $\mathcal{I}_{g\delta s}$-regular.

Theorem 4.3. *If $f : X \to Y$ is soft continuous bijective, soft $S\mathcal{I}_{g\delta s}$-open function and X is a soft regular space, then Y is soft $\mathcal{I}_{g\delta s}$-regular.*

Proof: Let F_A be a soft closed set over Y and $y_e \notin F_A$. Take $y_e = f(x_e)$ for some x_e over X. Since f is soft continuous $f^{-1}(F_A)$ is soft closed set over X such that $x_e \notin f^{-1}(F_A)$. Now X is soft regular, there exist disjoint soft open sets U_A and V_A such that $x_e \in U_A$ and $f^{-1}(F_A) \subset V_A$. That is, $y_e = f(x_e) \in f(U_A)$ and $F_A \subset f(V_A)$. Since f is soft S$\mathcal{I}_{g\delta s}$-open function $f(U_A)$ and $f(V_A)$ are soft S$\mathcal{I}_{g\delta s}$-open sets over Y and f is bijective $f(U_A) \cap f(V_A) = f(U_A \cap V_A) = f(\emptyset) = \emptyset$. Therefore, Y is soft $\mathcal{I}_{g\delta s}$-regular.

Theorem 4.4. *If $f : X \to Y$ is soft continuous surjective, strongly soft $S\mathcal{I}_{g\delta s}$-open (resp. quasi soft $S\mathcal{I}_{g\delta s}$-open) function and X is soft $\mathcal{I}_{g\delta s}$-regular space, then Y is soft $\mathcal{I}_{g\delta s}$-regular (resp. soft regular).*

Proof: Let F_A be a soft closed set over Y and $y_e \notin F_A$. Take $y_e = f(x_e)$ for some x_e over X. Since f is soft continuous surjective $f^{-1}(F_A)$ is soft closed set over X and $x_e \notin f^{-1}(F_A)$. Now since X is soft S$\mathcal{I}_{g\delta s}$-regular, there exist disjoint soft S$\mathcal{I}_{g\delta s}$-open sets U_A and V_A such that $x_e \in U_A$ and $f^{-1}(F_A) \subset V_A$. That is, $y_e = f(x_e) \in f(U_A)$ and $F_A \subset f(V_A)$. Since f is strongly soft S$\mathcal{I}_{g\delta s}$-open (resp. quasi soft S$\mathcal{I}_{g\delta s}$-open) and bijective, $f(U_A)$ and $f(V_A)$ are disjoint soft S$\mathcal{I}_{g\delta s}$-open (resp. soft open) sets over Y. Therefore, Y is soft $\mathcal{I}_{g\delta s}$-regular (resp. soft regular).

Theorem 4.5. *If $f : X \to Y$ is soft $S\mathcal{I}_{g\delta s}$-continuous, soft closed, injection and Y is soft regular, then X is soft $\mathcal{I}_{g\delta s}$-regular.*

Proof: Let F_A be a soft closed set over X and $x_e \notin F_A$. Since f is soft closed injection $f(F_A)$ is soft closed set over Y such that $f(x_e) \notin f(F_A)$. Now Y is soft regular, there exist disjoint soft open sets G_A and H_A such that $f(x_e) \in G_A$ and $f(F_A) \subset H_A$. This implies $x_e \in f^{-1}(G_A)$ and $F_A \subset f^{-1}(H_A)$. Since f is soft S$\mathcal{I}_{g\delta s}$-continuous, $f^{-1}(G_A)$ and $f^{-1}(H_A)$ are soft S$\mathcal{I}_{g\delta s}$-open sets over X. Further $f^{-1}(G_A) \cap f^{-1}(H_A) = \emptyset$. Hence X is soft $\mathcal{I}_{g\delta s}$-regular.

Theorem 4.6. *If $f : X \to Y$ is soft semi $\mathcal{I}_{g\delta s}$-continuous, soft semi-closed, injection and Y is soft semi-regular then X is soft $\mathcal{I}_{g\delta s}$-regular.*

Proof: Let F_A be a soft closed set over X and $x_e \notin F_A$. Since f is soft semi-closed injection $f(F_A)$ is soft semi-closed set over Y such that $f(x_e) \notin f(F_A)$. Now Y is soft semi-regular, there exist disjoint soft semi-open sets G_A and H_A such that $f(x_e) \in G_A$ and $f(F_A) \subset H_A$. This implies $x_e \in f^{-1}(G_A)$ and $F_A \subset f^{-1}(H_A)$. Since f is soft semi $\mathcal{I}_{g\delta s}$-continuous $f^{-1}(G_A)$ and $f^{-1}(H_A)$ are soft S$\mathcal{I}_{g\delta s}$-open sets over X. Further $f^{-1}(G_A) \cap f^{-1}(H_A) = \emptyset$. Hence X is soft $\mathcal{I}_{g\delta s}$-regular.

Theorem 4.7. *If $f : X \to Y$ is strongly soft S$\mathcal{I}_{g\delta s}$-continuous, soft closed, injection and Y is soft $\mathcal{I}_{g\delta s}$-regular then X is soft regular.*

Proof: Let F_A be a soft closed set over X and $x_e \notin F_A$. Since f is soft closed injection $f(F_A)$ is soft closed set over Y such that $f(x_e) \notin f(F_A)$. Now Y is soft $\mathcal{I}_{g\delta s}$-regular, there exist disjoint soft S$\mathcal{I}_{g\delta s}$-open sets G_A and H_A such that $f(x_e) \in G_A$ and $f(F_A) \subset H_A$. This implies $x_e \in f^{-1}(G_A)$ and $F_A \subset f^{-1}(H_A)$. Since f is strongly soft S$\mathcal{I}_{g\delta s}$-continuous $f^{-1}(G_A)$ and $f^{-1}(H_A)$ are soft open sets over X. Further $f^{-1}(G_A) \cap f^{-1}(H_A) = \emptyset$. Hence X is soft regular.

Theorem 4.8. *If $f : X \to Y$ is soft S$\mathcal{I}_{g\delta s}$-irresolute, soft closed, injection and Y is soft $\mathcal{I}_{g\delta s}$-regular then X is soft $\mathcal{I}_{g\delta s}$-regular.*

Proof: Let F_A be a soft closed set over X and $x_e \notin F_A$. Since f is soft closed injection $f(F_A)$ is soft closed set over Y such that $f(x_e) \notin f(F_A)$. Now Y is soft $\mathcal{I}_{g\delta s}$-regular, there exist disjoint soft $\mathcal{I}_{g\delta s}$-open sets G_A and H_A such that $f(x_e) \in G_A$ and $f(F_A) \subset H_A$. This implies $x_e \in f^{-1}(G_A)$ and $F_A \subset f^{-1}(H_A)$. Since f is soft S$\mathcal{I}_{g\delta s}$-irresolute, $f^{-1}(G_A)$ and $f^{-1}(H_A)$ are soft S$\mathcal{I}_{g\delta s}$-open sets over X. Further $f^{-1}(G_A) \cap f^{-1}(H_A) = \emptyset$. Hence X is soft $\mathcal{I}_{g\delta s}$-regular.

Corolary 4.1. *If $f : X \to Y$ is perfectly soft S$\mathcal{I}_{g\delta s}$-continuous injective, soft closed function and Y is soft $\mathcal{I}_{g\delta s}$-regular space, then X is soft regular.*

Proof: Obvious, because every perfectly soft S$\mathcal{I}_{g\delta s}$-continuous is strongly soft S$\mathcal{I}_{g\delta s}$-continuous.

Theorem 4.9. *If $f : X \to Y$ is perfectly soft S$\mathcal{I}_{g\delta s}$-continuous injective, soft closed function and Y is soft $\mathcal{I}_{g\delta s}$-regular space, then X is soft ultra regular.*

Proof: Let F_A be a soft closed set over X and $x_e \notin F_A$. Since f is soft closed injective, $f(F_A)$ is soft closed over Y such that $f(x_e) \notin f(F_A)$. By soft $\mathcal{I}_{g\delta s}$-regularity over Y, there exist disjoint soft S$\mathcal{I}_{g\delta s}$-open sets U_A and V_A such that $f(x_e) \in U_A$ and $f(F_A) \subset V_A$. That is $x_e \in f^{-1}(U_A)$ and $F_A \subset f^{-1}(V_A)$. Since f is perfectly soft S$\mathcal{I}_{g\delta s}$-continuous, $f^{-1}(U_A)$ and $f^{-1}(V_A)$ are disjoint soft clopen sets over X. Therefore, X is soft ultra regular.

Theorem 4.10. *If $f : X \to Y$ is totally soft continuous bijective, soft S$\mathcal{I}_{g\delta s}$-open function and X is soft clopen-regular space, then Y is soft $\mathcal{I}_{g\delta s}$-regular.*

Proof: Let F_A be a soft closed set over Y and $y_e \notin F_A$. Take $y_e = f(x_e)$ for some soft point x_e over X. Since f is totally soft continuous bijective, $f^{-1}(F_A)$ is soft clopen over X such that $x_e \notin f^{-1}(F_A)$. By soft clopen-regularity of X, there exist disjoint soft open sets U_A and V_A such that $x_e \in U_A$ and $f^{-1}(F_A) \subset V_A$. That is $y_e = f(x_e) \in f(U_A)$ and $F_A \subset f(V_A)$. Since f is soft S$\mathcal{I}_{g\delta s}$-open bijective $f(U_A)$ and $f(V_A)$ are disjoint soft S$\mathcal{I}_{g\delta s}$-open sets over Y. Therefore, Y is soft $\mathcal{I}_{g\delta s}$-regular.

Theorem 4.11. *If $f : X \to Y$ is completely soft continuous injective, soft S$\mathcal{I}_{g\delta s}$-open function and X is almost soft regular space, then Y is soft $\mathcal{I}_{g\delta s}$-regular.*

Proof: Let F_A be a soft closed set over Y and $y_e \notin F_A$. Take $y_e = f(x_e)$ for some soft point x_e over X. Since f is completely soft continuous, $f^{-1}(F_A)$ is soft regular open over X such that $x_e \notin f^{-1}(F_A)$. By almost soft regularity over X, there exist disjoint soft open sets U_A and V_A such that $x_e \in U_A$ and $f^{-1}(F_A) \subset V_A$. That is $y_e = f(x_e) \in f(U_A)$ and $F_A \subset f(V_A)$. Since f is soft S$\mathcal{I}_{g\delta s}$-open and injective, $f(U_A)$ and $f(V_A)$ are disjoint soft S$\mathcal{I}_{g\delta s}$-open sets over Y. Therefore, Y is soft $\mathcal{I}_{g\delta s}$-regular.

5 Conclusion

We concluded that using the above separation axioms concepts on soft S$\mathcal{I}_{g\delta s}$-closed sets in the ideal soft topological space may extended to Urysohn Theorem and then we will prove extension theorem. This findings would assist the budding researchers to engage in the future research in the real life applications.

References

[1] B. Ahmad and A. Kharal, On fuzzy soft sets, Advances in Fuzzy Systems, (2009), 1-6.

[2] H. Aktas and N. Cagman, Soft sets and soft groups, Information Sciences, 1(77)(2007), 2726-2735.

[3] M. I. Ali, F. Feng, X. Liu, W. K. Min and M. Shabir, On some new operations in soft set theory, Computers and Mathematics with Applications, 57(2009), 1547-1553.

[4] N. Cagman, F. Citak and S. Enginoglu, Fuzzy parameterized fuzzy soft set theory and its applications, Turkish Journal of Fuzzy Systems, 1(1)(2010), 21-35.

[5] N. Cagman and S. Enginoglu, Soft set theory and uni-int decision making, European Journal of Operational Research, 207(2010), 848-855.

[6] S. Hussain and B. Ahmad, Some properties of soft topological spaces, Comput. Math. Appl., 62(2011), 4058-4067.

[7] S. Hussain and B. Ahmad, Soft separation axioms in soft topological spaces, Hacettepe J. Maths. and Stat., 44(3)(2015), 559-565.

[8] S. Jeyashri, S. Tharmar and G. Ramkumar, International journal of mathematics trends and technology, 45(1)(2017), 28-34.

[9] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-latif, γ-operation and decompositions of some forms of soft continuity in soft topological spaces, Annals of Fuzzy Mathematics and Informatics, 7(2014), 181-196.

[10] D. V. Kovkov, V. M. Kolbanov and D. A. Molodtsov, Soft sets theory-based optimization, Journal of Computer and Systems Sciences International, 46(6)(2007), 872-880.

[11] K. Kuratowski, Topology Vol. I, Academic Press, New York, 1966.

[12] P. K. Maji, R. Biswas and A. R. Roy, Fuzzy soft sets, Journal of Fuzzy Mathematics, 9(3)(2001), 589-602.

[13] P. K. Maji, R. Biswas and A. R. Roy, Intuitionistic fuzzy soft sets, Journal of Fuzzy Mathematics, 9(3)(2001), 677-691.

[14] P. K. Maji, R. Biswas and A. R. Roy, Soft set theory, Computers and Mathematics with Applications, 45(2003), 555-562.

[15] D. A. Molodtsov, Soft set theory-first results, Computers and Mathematics with Applications, 37(1999), 19-31.

[16] D. Molodtsov, V. Y. Leonov and D. V. Kovkov, Soft sets technique and its application, Nechetkie Sistemy i Myagkie Vychisleniya, 1(1)(2006), 8-39.

[17] D. Pie, D. Miao, From soft sets to information systems, Granular computing, IEEE Int. Conf. 2(2005), 617-62.

[18] Y. Rosemathy and K. Alli, Soft $\mathcal{I}_{g\delta s}$-closed functions, Ratio Mathematica, 44(2022), 246-259.

[19] B. V. S. T Sai and V. Srinivasa Kumar, On soft semi-separability, International Journal of Mathematical Analysis, 7(84), 2013, 2663-2669.

[20] M. Shabir and M. Naz, On soft topological spaces, Comp. Math. Appl., 61(2011), 1786-1799.

[21] S. Tharmar and V. Pandiyarasu, Soft codense and completely soft codense ideals, International Journal of Applied Engineering Research, 11(1)(2016), 508-511.

[22] S. Tharmar, R. Veerasivaji and R. Senthil kumar, Theorytical Approach on Application of Generalized Closed Sets Environmental Lifestyle usinf r-neighbourhood Ideal Spaces, E3S Web of Conferences, 387, 04015 (2023).

[23] R. Vaidynathaswamy, The localization theory in set topology, Proc. Ind. Acad. of Sci. 20(1945), 515–61.

[24] S. Y$\ddot{u}$ksel, N. Tozlu and Z. G$\ddot{u}$zel Erg$\ddot{u}$l, Soft Regular Generalized closed sets in soft topological spaces, Int. J. Math. Analysis, 8(8)(2014), 355-367.

[25] I. Zorlutuna, M. Akdag, W.K. Min and S. Atmaca, Remarks on soft topological spaces, Annals of Fuzzy Mathematics and Informatics, 3(2012), 171-185.