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Some fixed point results in fuzzy metric space using intimate mappings

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Abstract

The aim of this research paper is to prove the existence and uniqueness of common fixed point theorems for four self-mappings in fuzzy metric space using the notion of Intimate mappings.We also provide appropriate illustrations to justify the key points mentioned in the main results.

Keywords: Fuzzy metric space, Intimate mappings, E.A property, Common E.A property.

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1. Introduction

L.A Zadeh [1] initiated the new concept, as extension an of classical set namely Fuzzy set. Lateron the notion of fuzzy metric space was introduced by Kramosil and Mechalek in [2]. Further this was altered by George and veeramani [4] in order to obtain Harsdorff topology for the class of fuzzy metric spaces. Thereafter many fixed point theorems came into light under various conditions like ([5],[6],[9],[10],[11],[13],[16],[19]) in fuzzy metric space.

Under other conditions, Sahu and others [12] developed the notion of generalized compatible mappings of type $(\mathcal{A})$ called Intimate mappings. These were further extended by Chugh and Madhu Aggarwal [13] which resulted in the formation of some results in Hausdorff uniform spaces. Further some more results can be witnessed like [14] using intimate mappings in complex valued metric space. Apart from this Praveenkumar and others [15] proved some theorems in multiplicative metric space (MMS) using the notion of intimate mappings and subsequently many results came into existence on this platform like ([17],[18]).

The concept of non-compatible mappings extended as the E. A property was introduced in metric space by Aamri and Matouwakil [20]. Consequently, the concept of improved E.A property resulted in the formation of common property E.A was introduced by Yicheng liu et al. [21].

The important note of this article is to extend the notion of intimate mappings in fuzzy metric space using recent concepts like the different forms of E.A properties.In this process we prove three unique common fixed point theorems using these concepts. Cocequently these results stand as generalizations of some of the existing results like [16] [19]. Furthermore, some illustrations are provided to support our findings.

2. Definitions and Preliminaries

Definition 2.1 (B.Schweizer and A.Sklar [7]):A binary operation $*$:[0,1] × [0, 1]→[0,1] is said to be continuous triangular norm (i.e continuous $t-$norm) if the following assertions hold: (CT-i) $*$ is continuous;(CT-ii)$a * b \leq c * d$ where $a \leq b, c \leq d$ and $a, b, c, d \in [0,1]$;(CT-iii)$a * 1 = a$ for $a \in [0,1]$; (CT-iv) $*$ is associative and commutative.

Definition 2.2 (Kramosil and Mechalek [2]**)**: A triplet $(\mathbb{X}, M_{KM}, *)$ is fuzzy metric space (i.e., FMS) if $\mathbb{X}$ is a arbitrary set, $*$ is continuous $t-$norm and M_{KM} is fuzzy set on $\mathbb{X}^2 \times (0, \infty)$ satisfying the following conditions for all $x, y, z \in \mathbb{X}$ such that $t, s \in (0, \infty)$:

(KMFM-i) $\mathrm{M_{KM}}(x, y, 0) = 0$

(KMFM-ii) $\mathrm{M_{KM}}(x, y, t) = 1 \ \ \forall t > 0 \iff x = y$

(KMFM-iii) $\mathrm{M_{KM}}(y, x, t) = \mathrm{M_{KM}}(x, y, t)$

(KMFM-iv) $\mathrm{M_{KM}}(x, z, t + s) \geq \mathrm{M_{KM}}(x, y, t) * \mathrm{M_{KM}}(y, z, s)$

(KMFM-v) $\mathrm{M_{KM}}(x, y, .)$: [0.1]→[0,1] left continuous.

Example 2.3 (George &Veeramani [4])**:** Consider$(\mathbb{X}, d_u)$ is a metric space and define $M_{KM}(x, y, t) = \dfrac{t}{t + d_u(x, y)}$ then $(\mathbb{X}, \mathrm{M_{KM}}, *)$ is FMS where $\forall x, y \in \mathbb{X}$, $t > 0$ and $*$ is continuous $t-$norm with $a * b = \min\{a, b\}$.

In the entire paper, $(\mathbb{X}, M_{KM}, *)$ is to be assumed FMS with the condition (KFFM-6) : $\lim_{t\to\infty} \mathrm{M_{KM}}(x, y, t) = 1$ for all $x, y, \in \mathbb{X}$.

Definition 2.4 (Grabiec [3]): Let $\langle x_n \rangle$ be sequence in FMS $(\mathbb{X}, \mathrm{M_{KM}}, *)$, $\langle x_n \rangle$ then converges to a point $\ell \in \mathbb{X}$ if $\lim_{n\longrightarrow\infty} \mathrm{M_{KM}}(x_n, \ell, t) = 1, \forall t > 0$.

Definition 2.5 (Garbaic [3]): Let $\langle x_n \rangle$ be a sequence in FMS $(\mathbb{X}, \mathrm{M_{KM}}, *)$, this sequence $< x_n >$ in $\mathbb{X}$ is said to be Cauchy sequence in FMS if $\lim_{n\to\infty} \mathrm{M_{KM}}\left(x_{n+p}, x_n, t\right) = 1$, $\forall t > 0$ and $p > 0$.

Definition 2.6 (Garbiec [3]): If every Cauchy sequence is convergent in $(\mathbb{X}, \mathrm{M_{KM}}, *)$ then we say that it is complete.

Lemma 2.7 (S.N. Mishra et al [5]): Let$(\mathbb{X}, \mathrm{M_{KM}}, *)$ be a FMS if there exists $k \in (0,1)$ such that $\mathrm{M_{KM}}(x, y, kt) \geq \mathrm{M_{KM}}(x, y, t)$ then $x = y$.

Definition 2.8 ([5],[10]): Let $\mathfrak{S}$ and $\mathfrak{T}$ be two self mappings of a FMS$(\mathbb{X}, \mathrm{M_{KM}}, *)$. Then $\mathfrak{S}$ and $\mathfrak{T}$ are

(1) compatible if $\lim_{n\to\infty} \mathrm{M}_{KM}(\mathfrak{S}\mathfrak{T}x_n, \mathfrak{T}\mathfrak{S}x_n, t) = 1$ whenever a sequence $\langle x_n \rangle$ in $\mathbb{X}$ provided $\lim_{n\to\infty} \mathfrak{S}x_n = \lim_{n\to\infty} \mathfrak{T}x_n = t$ for some $t \in \mathbb{X}$

(2) compatible of type $(\mathcal{A})$ if

$\lim_{n\to\infty} \mathrm{M}_{KM}(\mathfrak{S}\mathfrak{T}x_n, \mathfrak{T}\mathfrak{T}x_n, t) = 1$ $\lim_{n\to\infty} \mathrm{M}_{KM}(\mathfrak{T}\mathfrak{S}x_n, \mathfrak{S}\mathfrak{S}x_n, t) = 1$ whenever $\langle x_n \rangle$ in $\mathbb{X}$ such that $\lim_{\mathrm{n}\to\infty} \mathfrak{S}x_n = \lim_{\mathrm{n}\to\infty} \mathfrak{T}x_n = t$ for some $t \in \mathbb{X}$.

Now we discuss some definitions related to intimate mappings in FMS.

Definition 2.9: Let $\mathfrak{A}$ *and* $\mathfrak{S}$ be two mappings of a FMS $(\mathbb{X}, \mathrm{M_{KM}}, *)$ into itself. Then $\mathfrak{A}$ and $\mathfrak{S}$ are said to be

(1). $\mathcal{A}$-Intimate mappings if $\alpha\, \mathrm{M_{KM}}(\mathfrak{A}\mathfrak{S}x_n, \mathfrak{A}x_n, t) \geq \alpha\, \mathrm{M_{KM}}(\mathfrak{S}\mathfrak{S}x_n, \mathfrak{S}x_n, t)$ where $\alpha = \lim_{n\longrightarrow\infty}$ Sup or $\lim_{n\longrightarrow\infty}$ Inf and $\langle x_n \rangle$ is a sequence in $\mathbb{X} \ni \lim_{n\to\infty} \mathfrak{A}x_n = \lim_{n\to\infty} \mathfrak{S}\mathrm{S}x_n = t$ for some $t \in \mathbb{X}$.

(2). $\mathcal{S}$-Intimate mapping if $\alpha\, \mathrm{M}_{KM}(\mathfrak{S}\mathfrak{A}x_n, \mathfrak{S}x_n, t) \geq \alpha\, \mathrm{M}_{KM}(\mathfrak{A}\mathfrak{A}x_n, \mathfrak{A}x_n, t)$ where $\alpha = \lim_{n\longrightarrow\infty}$ Sup or $\lim_{n\longrightarrow\infty}$ Inf and a sequence $\langle x_n \rangle$ in $\mathbb{X} \ni \lim_{n\to\infty} \mathfrak{A}x_n = \lim_{n\to\infty} \mathfrak{S}x_n = t$ for some $t \in \mathbb{X}$.

Proposition 2.10: Let $\mathfrak{A}$ *and* $\mathfrak{S}$ be two self mappings of a FMS $(\mathbb{X}, M_{KM}, *)$. Suppose $\mathfrak{A}$ and $\mathfrak{S}$ are compatible mappings of type $(\mathcal{A})$ then the pair of mappings $\mathfrak{A}$ *and* $\mathfrak{S}$ are $\mathcal{A}-$ intimate mappings and $\mathcal{S}$-intimate mappings.

Proof: Since $\mathfrak{A}$ *and* $\mathfrak{S}$are compatible of type $(\mathcal{A})$, we have $\lim_{n\to\infty} \mathrm{M}_{KM}(\mathfrak{A}\mathfrak{S}x_n, \mathfrak{S}\mathfrak{S}x_n, t) = 1$ and $\lim_{\mathrm{n}\to\infty} \mathrm{M}_{KM}(\mathfrak{S}\mathfrak{A}x_n, \mathfrak{A}\mathfrak{A}x_n, t) = 1$ whenever $\langle x_n \rangle$ in $\mathbb{X} \ni \lim_{\mathrm{n}\to\infty} \mathfrak{A}\, x_n = \lim_{n\to\infty} \mathfrak{S}x_n = t$ for some $t \in \mathbb{X}$.

Now $\mathrm{M}_{KM}(\mathfrak{A}\mathfrak{S}n_n, \mathfrak{A}x_n, (2-\beta)t) = \mathrm{M}_{KM}(\mathfrak{A}\mathfrak{S}x_n, \mathfrak{A}x_n, (1+k_1)t)$

$$\geq \mathrm{M}_{KM}(\mathfrak{A}\mathfrak{S}x_n, \mathfrak{S}\mathfrak{S}x_n, k_1 t) * \mathrm{M}_{KM}(\mathfrak{S}\mathfrak{S}x_n, \mathfrak{A}x_n, t).$$

By taking $k_1 = 1 - \beta$ and $0 < k_1 < 1$ and letting $n \to \infty$ and $\beta \to 1$ we obtain

$$\begin{aligned} \mathrm{M}_{KM}(\mathfrak{A}\mathfrak{S}x_n, \mathfrak{A}x_n, t) &\geq \mathrm{M}_{KM}(\mathfrak{A}\mathfrak{S}x_n, \mathfrak{S}\mathfrak{S}x_n, k_1 t) * \mathrm{M}_{KM}(\mathfrak{S}\mathfrak{S}x_n, \mathfrak{A}x_n, t) \\ &= \mathrm{M}_{KM}(\mathfrak{S}\mathfrak{S}x_n, \mathrm{A}x_n, t). \end{aligned}$$

By applying limit supremum on both sides,

$\alpha\, \mathrm{M}_{KM}(\mathfrak{A}\mathfrak{S}x_n, \mathfrak{A}x_n, t) \geq \alpha\, \mathrm{M}_{KM}(\mathfrak{S}\mathfrak{S}x_n, \mathfrak{A}x_n, t)$ this implies $\mathfrak{A}$ *and* $\mathfrak{S}$ are $\mathcal{A}$-intimate mappings whenever $\{x_n\}$ is a sequence in $\mathbb{X}$ such that $\lim_{n\to\infty} \mathfrak{A}x_n = \lim_{n\to\infty} \mathfrak{S}x_n = t$ for some $t \in \mathbb{X}$. Likewise, we can prove that the pair of these mappings is $\mathcal{S}$-intimate.

Proposition 2.11: Let $\mathfrak{A}$ and $\mathfrak{S}$ be two self mappings on FMS. $\mathfrak{A}$ and $\mathfrak{S}$ are $\mathcal{A}$-intimate mappings and $\mathfrak{A}t_1 = \mathfrak{S}t_1 = p, p \in \mathbb{X}$ then $\mathrm{M}_{KM}(\mathfrak{A}\mathrm{p}, \mathrm{p}, t) \geq \mathrm{M}_{KM}(\mathfrak{S}\mathrm{p}, \mathrm{p}, t)$.

Proof: Suppose that $\{x_n\} \in \mathbb{X}$ is a sequence such that $\mathfrak{A}\mathrm{x_n} = \mathfrak{S}\mathrm{x_n} \to \mathfrak{A}\mathrm{t}_1 = \mathfrak{S}\mathrm{t}_1 = p$ for some $p, t \in \mathbb{X}$.

Since the pair of mappings $\mathfrak{A}$ and $\mathfrak{S}$ are $\mathcal{A}-$ intimate, then we obtain

$$\begin{aligned} M_{KM}(\mathfrak{A}p, p, t) = \lim_{n\to\infty} \mathrm{M}_{KM}(\mathfrak{A}\mathfrak{S}x_n, \mathfrak{A}x_n, t) &\geq \lim_{n\to\infty} \mathrm{M}_{KM}(\mathfrak{S}\mathfrak{S}x_n, \mathfrak{S}x_n, t) \\ &= M_{KM}(\mathfrak{S}p, p, t). \end{aligned}$$

Thus $\mathrm{M}_{KM}(\mathfrak{A}\mathrm{p}, \mathrm{p}, t) \geq \mathrm{M}_{KM}(\mathfrak{S}\mathrm{p}, \mathrm{p}, t)$.

Remark 2.12: A pair of mappings $\mathfrak{A}$ and $\mathfrak{S}$ is $\mathcal{A}$-intimate or $\mathcal{S}$-intimate but not compatible mapping of type $(\mathcal{A})$.

The following example revels the relation between intimate mappings and compatible mappings of type $(\mathcal{A})$.

Example 2.13: Suppose $\mathbb{X} = [0,1]$. Define two self-mappings $\mathfrak{A}$ and $\mathfrak{S}$ as follows $\mathfrak{A}(x) = \frac{5}{x+5}$ $\mathfrak{S}(x) = \frac{1}{x+1}$ for every x in [0,1].

Consider a sequence $\langle x_n \rangle = \frac{1}{n}$ $n \in \mathbb{N}$. Then $\lim_{n\to\infty} \mathfrak{A}x_n = \lim_{n\to\infty} \mathfrak{S}x_n = 1$.

Consequently, $\lim_{n\to\infty} M(\mathfrak{A}\mathfrak{S}x_n, \mathfrak{A}x_n, t) = \frac{6t}{6t+1}$ and $\lim_{n\to\infty} M(\mathfrak{S}\mathfrak{S}x_n, \mathfrak{S}x_n, t) = \frac{2t}{2t+1}$.

Hence $\lim_{n\to\infty} M_{KM}(\mathfrak{A}\mathfrak{S}x_n, \mathfrak{A}x_n, t) \geq \lim_{n\to\infty} M_{KM}(\mathfrak{S}\mathfrak{S}x_n, \mathfrak{S}x_n, t)$, for all $t > 0$.

Thus, the pair $(\mathfrak{A}, \mathfrak{S})$ is $\mathcal{A}$-intimate.

On the other hand, the $(\mathfrak{A}, \mathfrak{S})$ are not compatible of type $(\mathcal{A})$, since $\lim_{n\to\infty} \mathrm{M}_{KM}(\mathfrak{A}\mathfrak{S}x_n, \mathfrak{S}\mathfrak{S}x_n, t) = \frac{3t}{3t+1} \neq 1$ and $\lim_{n\to\infty} \mathrm{M}_{KM}(\mathfrak{S}\mathfrak{A}x_n, \mathfrak{A}\mathfrak{A}x_n, t) = \frac{3t}{3t+1} \neq 1$.

Definition 2.14[20]: Define $\mathfrak{A}$ and $\mathfrak{S}$ as two self maps of FMS $(\mathbb{X}, M_{KM}, *)$ then we say that $\mathfrak{A}$ and $\mathfrak{S}$ satisfy the property E.A if there exists a sequence $\langle x_n \rangle \in \mathbb{X}$ such that $\lim_{n\to\infty} \mathfrak{A}x_n = \lim_{n\to\infty} \mathfrak{S}x_n = t$ for some $t \in \mathbb{X}$.

Definition 2.15[21]: Suppose $\mathfrak{A}$, $\mathfrak{P}$, $\mathfrak{B}$ and $\mathfrak{T}$ are four self maps on FMS $(\mathbb{X}, \mathrm{M_{KM}}, *)$ then we say that $(\mathfrak{A}, \mathfrak{P})$ and $(\mathfrak{B}, \mathfrak{T})$ satisfy common property E.A whenever two sequences $\langle \mathrm{x}_n \rangle$ and $\langle \gamma_n \rangle$ in $\mathbb{X}$ satisfying

$$\lim_{n\to\infty} \mathfrak{A}\, x_n = \lim_{n\to\infty} \mathfrak{S}x_n = \lim_{n\to\infty} \mathfrak{B}\, \gamma_n = \lim_{n\to\infty} \mathfrak{T}\gamma_n = t \text{ for some } t \in \mathbb{X}.$$

3. Main results

3.1 Theorem: Let $(\mathbb{X}, \mathrm{M_{KM}}, *)$ be a complete fuzzy metric space. Suppose $\mathfrak{P}$, $\mathfrak{Q}$, $\mathfrak{S}$ and $\mathfrak{A}$ are self maps on $\mathbb{X}$ satisfying the conditions

$(\mathcal{C}-1)$ $\mathfrak{P}(\mathbb{X}) \subseteq \mathfrak{S}(\mathbb{X})$ and $\mathfrak{Q}(\mathbb{X}) \subseteq \mathfrak{A}(\mathbb{X})$

$(\mathcal{C}-2)$ $\mathrm{M}_{KM}(\mathfrak{P}x, \mathfrak{Q}\gamma, \mathrm{k}t) \geq \mathrm{M}_{KM}(\mathfrak{A}x, \mathfrak{S}\gamma, t) * \mathrm{M}_{KM}(\mathfrak{P}x, \mathfrak{A}x, t) * \mathrm{M}_{KM}(\mathfrak{Q}\gamma, \mathfrak{S}\gamma, t) * \mathrm{M}_{KM}(\mathfrak{P}x, \mathfrak{S}\gamma, t)$

where $k \in (0,1)$ and for all $x,\ \gamma \in \mathbb{X}$

$(\mathcal{C}-3)$ $\mathfrak{A}(\mathbb{X})$ is complete

$(\mathcal{C}-4)$ the pair of mappings $\mathfrak{A}$ and $\mathfrak{P}$ $is\ \mathcal{A}-$intimate and the other pair of mappings also $\mathfrak{S}$ and $\mathfrak{Q}$ is $\mathcal{S}-$intimate.

Then $\mathfrak{P}$, $\mathfrak{Q}$, $\mathfrak{S}$ and $\mathfrak{A}$ have a unique common fixed point in $\mathbb{X}$.

Proof:

Let x_0 be any arbitrary point of $\mathbb{X}$.
Since from the condition $\mathfrak{P}(\mathbb{X}) \subseteq \mathfrak{S}(\mathbb{X})$ of $(\mathcal{C}-1)$, there exists a point $x_1 \in \mathbb{X}$ such that $\mathfrak{P}x_0 = \mathfrak{S}x_1 = \gamma_0$.
Now for this x_1 and applying the $(\mathcal{C}-1)$[i.e $\mathfrak{Q}(\mathbb{X}) \subseteq \mathfrak{A}(\mathbb{X})$] $\exists x_2 \in \mathbb{X}$ such that $\mathfrak{Q}x_1 = \mathfrak{A}x_2 = \gamma_1$.
Inductively, we establish two real sequences $< x_n >$ and $< \gamma_n >$ in $\mathbb{X}$ $\ni \gamma_{2n} = \mathfrak{P}x_{2n} = \mathfrak{S}x_{2n+1}$ and $\gamma_{2n+1} = \mathfrak{Q}x_{2n+1} = \mathfrak{A}x_{2n+2}$ for $n \geq 0$.
By taking $x = x_{2n}, \gamma = x_{2n+1}$ in the inequality $(\mathcal{C}-2)$,

$$\mathrm{M}_{KM}(\mathfrak{P}x_{2\mathrm{n}}, \mathfrak{Q}x_{2n+1}, \mathrm{k}t) \geq \mathrm{M}_{KM}(\mathfrak{A}x_{2\mathrm{n}}, \mathfrak{S}x_{2n+1}, t) * \mathrm{M}_{KM}(\mathfrak{P}x_{2\mathrm{n}}, \mathfrak{A}x_{2n}, t) * \mathrm{M}_{KM}(\mathfrak{Q}x_{2n+1}, \mathfrak{S}x_{2n+1}, t) * \mathrm{M}_{KM}(\mathfrak{P}x_{2\mathrm{n}}, \mathfrak{S}x_{2n+1}, t)$$

which implies that an $n \rightarrow \infty$

$$\mathrm{M}_{KM}(\gamma_{2n}, \gamma_{2n+1}, \mathrm{k}t) \geq \mathrm{M}_{KM}(\gamma_{2n-1}, \gamma_{2n}, t) * \mathrm{M}_{KM}(\gamma_{2n}, \gamma_{2n-1}, t) * \mathrm{M}_{KM}(\gamma_{2n+1}, \gamma_{2n}, t) * \mathrm{M}_{KM}(\gamma_{2n}, \gamma_{2n}, t).$$

This yield

$$\mathrm{M}_{KM}(\gamma_{2n}, \gamma_{2n+1}, \mathrm{k}t) \geq \mathrm{M}_{KM}(\gamma_{2n-1}, \gamma_{2n}, t) * \mathrm{M}_{KM}(\gamma_{2n+1}, \gamma_{2n}, t) * \mathrm{M}_{KM}(\gamma_{2n}, \gamma_{2n-1}, t) * 1.$$

Again, by the condition KMFM-3, we get

$$\mathrm{M}_{KM}(\gamma_{2n}, \gamma_{2n+1}, \mathrm{k}t) \geq \mathrm{M}_{KM}(\gamma_{2n-1}, \gamma_{2n}, t) * \mathrm{M}_{KM}(\gamma_{2n}, \gamma_{2n+1}, t)$$

which implies (since $\mathfrak{a} * \mathfrak{b} = \min\{\mathfrak{a}, \mathfrak{b}\}$.)

$$\mathrm{M}_{KM}(\gamma_{2n}, \gamma_{2n+1}, \mathrm{k}t) \geq \mathrm{M}_{KM}(\gamma_{2n-1}, \gamma_{2n}, t).$$

In general

$$\mathrm{M_{KM}}(\gamma_{n+1}, \gamma_{n+2}, kt) \geq M_{KM}(\gamma_n, \gamma_{n+1}, t) \ldots\ldots (\sigma-1)$$

for all $n = 1,2,3..$, and $t > 0$.
From $(\sigma-1)$,

$[\mathrm{M}_{KM}(\gamma_n, \gamma_{n+1}, t)] \geq \mathrm{M}_{KM}\left(\gamma_{n-1}, \gamma_n, \frac{t}{\mathrm{k}}\right) \geq \mathrm{M}_{KM}\left(\gamma_{n-2}, \gamma_{n-1}, \frac{t}{\mathrm{k}^2}\right) \geq \cdots$

$$\ldots \geq \mathrm{M}_{KM}\left(\gamma_0, \gamma_1, \frac{t}{\mathrm{k}^n}\right) \to 1 \text{ as } n \to \infty \ldots\ldots (\sigma - 2)$$

For any $t > 0$ and $\lambda_{MK} \in (0,1)$ we consider $\forall\, n > n_0 \in \mathbb{N}$ such that $\mathrm{M}_{KM}(\gamma_n, \gamma_{n+1}, t) > (1 - \lambda_{\mathrm{MK}}) \ldots (\sigma - 3)$.

For $m, n \in \mathbb{N}$. Suppose $m \geq n$, then we have that

$[\mathrm{M}_{\mathrm{MK}}(\gamma_n, \gamma_m, t)]$

$$\geq \min\{\mathrm{M}_{\mathrm{MK}}\left(\gamma_n, \gamma_{n+1}, \frac{t}{m-n}\right) * \mathrm{M}_{\mathrm{MK}}\left(\gamma_{n+1}, \gamma_{n+2}, \frac{t}{m-n}\right) * \ldots$$

$\mathrm{M}_{\mathrm{MK}}\left(\gamma_{m-1}, \gamma_m, \frac{t}{m-n}\right) \geq (1 - \lambda_{\mathrm{MK}}) * (1 - \lambda_{\mathrm{MK}}) * \ldots (1 - \lambda_{\mathrm{MK}}) .. (m-n)$ times.

This implies $\mathrm{M}_{\mathrm{MK}}(\gamma_{m-1}, \gamma_m, t) \geq (1 - \lambda_{\mathrm{MK}})$

Therefore $< \gamma_n >$ is cauchy sequence in FMS.

Since $(\mathbb{X}, M_{KM}, *)$ is complete FMS, so sequence $\{\gamma_n\}$ converges to p*∈X.

Further fuzzy cauchy sequence $\{\gamma_n\}$ has convergent subsequence $\{\gamma_{2n+1}\}$ and $\{\gamma_{2n}\}$.

From the above argument,

$\gamma_{2n+1} = \mathfrak{Q}x_{2n+1} = \mathfrak{A}x_{2n+2} \to p*$ and

$$\gamma_{2n} = \mathfrak{P}x_{2n} = \mathfrak{S}x_{2n+1} \to p* \text{ as } n \to \infty \ldots (\sigma - 4)$$

Now suppose that the range set $\mathfrak{A}$(X) is complete then ∃ a point u∈$\mathbb{X}$ ∋ $\mathfrak{A}$u=p*..$(\sigma - 5)$.

Now we claim that $\mathfrak{P}$u=p* from the inequality, put $x = u$ and $\gamma = x_{2n+1}$ we have

$\mathrm{M}_{KM}(\mathfrak{P}\mathrm{u}, \mathfrak{Q}n_{2n+1}, \mathrm{k}t) \geq \mathrm{M}_{KM}(\mathfrak{A}\mathrm{u}, \mathfrak{S}x_{2n+1}, t) * \mathrm{M}_{KM}(\mathfrak{P}\mathrm{u}, \mathfrak{A}\mathrm{u}, t)$
$* \mathrm{M}_{KM}(\mathfrak{Q}x_{2n+1}, \mathfrak{S}x_{2n+1}, t) * \mathrm{M}_{KM}(\mathfrak{P}\mathrm{u}, \mathfrak{S}\mathrm{S}x_{2n+1}, t)$.

Taking limit as $n \to \infty$

$\mathrm{M}_{KM}(\mathfrak{P}\mathrm{u}, \mathrm{p}*, \mathrm{k}t) \geq \mathrm{M}_{KM}(\mathrm{p}*, \mathrm{p}*, t) * \mathrm{M}_{KM}(\mathfrak{P}\mathrm{u}, \mathrm{p}*, t)$
$* \mathrm{M}_{KM}(\mathrm{p}*, \mathrm{p}*, t) * \mathrm{M}_{KM}(\mathfrak{P}\mathrm{u}, \mathrm{p}*, t)$.

This gives $\mathfrak{P}$u=p*. That is $\mathfrak{P}$u=$\mathfrak{A}$u=p*…... $(\sigma - 6)$

Let us prove that Qv=p*.

Using the equation (($\sigma - 6$) with contained inequality $\mathfrak{P}(\mathbb{X}) \subseteq \mathfrak{S}(\mathbb{X})$,

p*=$\mathfrak{P}$u ∈ $\mathfrak{P}(\mathbb{X}) \subseteq \mathfrak{S}(\mathbb{X})$ then ∃ a point v∈$\mathbb{X}$ ∋

$\mathfrak{S}$v=$\mathfrak{P}$u=p*…. $(\sigma - 7)$.

Put x=u and $\gamma = v$ in $(\mathcal{C} - 2)$ then we obtain

$\mathrm{M}_{KM}(\mathfrak{P}\mathrm{u}, \mathfrak{Q}\mathrm{v}, \mathrm{k}t)$

$$\geq \mathrm{M}_{KM}(\mathrm{Au}, \mathfrak{S}\mathrm{v}, t) * \mathrm{M}_{KM}(\mathfrak{P}\mathrm{u}, \mathfrak{A}\mathrm{u}, t) * \mathrm{M}_{KM}(\mathfrak{Q}\mathrm{v}, \mathfrak{S}\mathrm{v}, t)$$
$$* \mathrm{M}_{KM}(\mathfrak{P}\mathrm{u}, \mathfrak{S}\mathrm{v}, t).$$

By using$(\sigma - 7)$ we get

$\mathrm{M}_{KM}(\mathrm{p}*, \mathfrak{Q}\mathrm{v}, \mathrm{k}t) \geq \mathrm{M}_{KM}(\mathrm{p}*, \mathfrak{S}\mathrm{v}, t) * \mathrm{M}_{KM}(\mathrm{p}*, \mathrm{p}*, t)$
$* \mathrm{M}_{KM}(\mathfrak{Q}\mathrm{v}, \mathrm{p}*, t) * \mathrm{M}_{KM}(\mathrm{p}*, \mathrm{p}*, t)$

this gives

$\mathrm{M}_{KM}(\mathrm{p}*, \mathfrak{Q}\mathrm{v}, kt) \geq \mathrm{M}_{KM}(\mathfrak{Q}\mathrm{v}, \mathrm{p}*, \mathrm{k}t)$.

Consequently $\mathrm{M}_{KM}(\mathrm{p}*, \mathfrak{Q}\mathrm{v}, \mathrm{k}t) \geq \mathrm{M}_{KM}(\mathrm{p}*, \mathfrak{Q}\mathrm{v}, \mathrm{k}t)$

this implies $\mathfrak{Q}$v=p*.

This shows that $\mathfrak{Q}$v=$\mathfrak{S}$v=p*…... $(\sigma-8)$

Since $\mathfrak{P}$u=$\mathfrak{A}$u=p* and

$(\mathfrak{A},\mathfrak{P})$ is $\mathcal{A}$-intimate we have $M_{KM}(\mathfrak{A}p^*, p^*, t) \geq M_{KM}(\mathfrak{P}p^*, p^*, t)$…. $(\sigma-9)$.

Suppose that $\mathfrak{P}p^* \neq p^*$.

Put $x = p*, \gamma = v$ in $(\mathcal{C}-2)$ then we get,

$$M_{KM}(\mathfrak{P}p*, \mathfrak{Q}v, kt) \geq M_{KM}(\mathfrak{A}p*, \mathfrak{S}v, t) * M_{KM}(\mathfrak{P}p*, \mathfrak{A}p*, t) * M_{KM}(\mathfrak{Q}v, \mathfrak{S}v, t) * M_{KM}(\mathfrak{P}p*, \mathfrak{S}v, t).$$

Using $(\sigma-8)$ we get,

$$M_{KM}(\mathfrak{P}p*, p*, kt) \geq M_{KM}(\mathfrak{A}p*, p*, t) * M_{KM}(\mathfrak{P}p*, \mathfrak{A}p*, t) * M_{KM}(p*, p*, t) * M_{KM}(\mathfrak{P}p*, p*, t).$$

By applying (KMFM-iv) we get

$$M_{KM}(\mathfrak{P}p*, p*, kt) \geq M_{KM}(\mathfrak{P}p*, p*, t) * M_{KM}(\mathfrak{P}p*, p*, t/2) * M_{KM}(p*, \mathfrak{A}p*, t/2) * M_{KM}(p*, p*, t) * M_{KM}(\mathfrak{P}p*, p*, t).$$

By using $(\sigma-9)$ we get

$M_{KM}(\mathfrak{P}p*, p*, kt) \geq M_{KM}(\mathfrak{P}p*, p*, t/2)$.

This gives $\mathfrak{P}$p*=p* …...$(\sigma-10)$.

From $(\sigma-9)$ and $(\sigma-10)$ we write $M_{KM}(\mathfrak{A}p^*, p^*, t) \geq 1$

this gives $\mathfrak{A}$p*=p*……$(\sigma-11)$

using $(\sigma-10)$ and $(\sigma-11)$ we get

$\mathfrak{A}$p*=$\mathfrak{P}$p*=p*…... $(\sigma-12)$

Also, $\mathfrak{Q}$v=$\mathfrak{S}$v=p* and using the pair $(\mathfrak{S}, \mathfrak{Q})$ as $\mathcal{S}$-intimate then we have

$M_{KM}(\mathfrak{S}p*, p*, t) \geq M_{KM}(\mathfrak{Q}p*, p*, kt)$…. $(\sigma-13)$

Suppose that $\mathfrak{Q}p^* \neq p^*$.

Put $x = u$ and $\gamma = p*$ in the inequality

$$M_{KM}(\mathfrak{P}u, \mathfrak{Q}p*, kt) \geq M_{KM}(\mathfrak{A}u, \mathfrak{S}p*, t) * M_{KM}(\mathfrak{P}u, \mathfrak{A}u, t) * M_{KM}(\mathfrak{Q}p*, \mathfrak{S}p*, t) * M_{KM}(\mathfrak{P}u, \mathfrak{S}p*, t)$$

using $(\sigma-6)$ and (KMFM-iv) we get,

$$M_{KM}(p*, \mathfrak{Q}p*, kt) \geq M_{KM}(p*, \mathfrak{S}p*, t) * M_{KM}(p*, p*, t) * M_{KM}\left(\mathfrak{P}p*, p*, \frac{t}{2}\right) * M_{KM}\left(p*, \mathfrak{S}p*, \frac{t}{2}\right) * M_{KM}(p*, \mathfrak{S}p*, t)$$

on using $(\sigma-13)$ we get

$$M_{KM}(p*, \mathfrak{Q}p*, kt) \geq M_{KM}(p*, \mathfrak{Q}p*, t) * M_{KM}\left(\mathfrak{Q}p*, p*, \frac{t}{2}\right) * M_{KM}(\mathfrak{Q}p*, p*, t/2) * M_{KM}(p*, \mathfrak{Q}p*, t).$$

This implies $M_{KM}(p*, \mathfrak{Q}p*, kt) \geq M_{KM}(p*, \mathfrak{Q}p*, t/2)$.

This gives $\mathfrak{Q}$p*=p*…$(\sigma - 14)$.
From $(\sigma - 13)$ and $(\sigma - 14)$ we get
$M_{KM}(\mathfrak{S}p^*, p^*, t) \geq 1$
$\mathfrak{S}$p*=p*…...$(\sigma - 15)$.
Using $(\sigma - 14)$ and $(\sigma - 15)$ we get
$\mathfrak{Q}$p*=$\mathfrak{S}$p*=p*. …. $(\sigma - 16)$.
Using $(\sigma - 12)$ and $(\sigma - 16)$ we conclude that $\mathfrak{A}$p*=$\mathfrak{P}$p*=$\mathfrak{Q}$p*=$\mathfrak{S}$p*=p*. Hence the result.
We can prove the uniqueness of the fixed point easily.

Example 3.1.1: Suppose ($\mathbb{X}$, M_{KM}, *) is a standard FMS with $a * a \geq a$ $\forall a \in [0,1]$, where $\mathfrak{A}$, $\mathfrak{S}$, $\mathfrak{P}$ and $\mathfrak{Q}$:$\mathbb{X} \to \mathbb{X}$ as

$$\mathfrak{P}(x) = \mathfrak{Q}(x) = \begin{cases} x + 0.125 & \text{if } 0 \leq x < 0.125 \\ 0.25 & \text{if } 0.125 \leq x \leq 1 \end{cases}$$

$$\mathfrak{A}(x) = \mathfrak{S}(x) = \begin{cases} 2x & \text{if } 0 \leq x < 0.125 \\ 0.25 & \text{if } 0.125 \leq x \leq 1 \end{cases}$$

$\mathfrak{P}(\mathbb{X}) = \mathfrak{Q}(\mathbb{X}) = [0.125, 0.25]$ and $\mathfrak{A}(\mathbb{X}) = \mathfrak{S}(\mathbb{X}) = [0, 0.25]$ these sets satisfy the condition $(\mathcal{C} - 1)$.
Now assume $\langle x_n \rangle = \left\{0.125 + \frac{1}{n}\right\}$ then $\lim_{n\to\infty} \mathfrak{A}x_n = \lim_{n\to\infty} \mathfrak{P}x_n = 0.25$.
Also we have, $\lim_{n\to\infty} \mathfrak{A}\mathfrak{P}x_n = \lim_{n\to\infty} \mathfrak{A}\mathfrak{P}(0.125 + \frac{1}{n}) = \lim_{n\to\infty} \mathfrak{A}(0.25) = 0.125$.
$\lim_{n\to\infty} M_{KM}(\mathfrak{A}\mathfrak{P}x_n, \mathfrak{A}x_n, t) \geq \lim_{n\to\infty} M_{KM}(\mathfrak{P}\mathfrak{P}x_n, \mathfrak{P}x_n, t)$, for $t > 0$.
Thus, the pair *($\mathfrak{A}$,$\mathfrak{P}$)* is $\mathcal{A}$-intimate.
Further $\lim_{n\to\infty} M_{KM}(\mathfrak{S}\mathfrak{Q}x_n, \mathfrak{S}x_n, t) \geq \lim_{n\to\infty} M_{KM}(\mathfrak{Q}\mathfrak{Q}x_n, \mathfrak{Q}x_n, t)$.
Thus, the pair ($\mathfrak{S}$,$\mathfrak{Q}$) is $\mathcal{S}$-intimate.
Moreover, it satisfies the contraction condition of the theorem. Clearly 0.25 is the unique common fixed point for these four mappings.

Theorem.3.2: Let $(\mathbb{X}, M_{KM}, *)$be a fuzzy metric space. Suppose $\mathfrak{P}$,$\mathfrak{Q}$, $\mathfrak{S}$ and $\mathfrak{A}$ are self maps on $\mathbb{X}$ satisfies the conditions $(\mathcal{C} - 1)$, $(\mathcal{C} - 2)$, $(\mathcal{C} - 3)$ and $(\mathcal{C} - 4)$ with
$(\mathcal{C} - 5)$:$(\mathfrak{P}, \mathfrak{A})$ or $(\mathfrak{Q}, \mathfrak{S})$ satisfy E.A property
then $\mathfrak{P}$, $\mathfrak{Q}$,$\mathfrak{S}$ and $\mathfrak{A}$ have a unique common fixed point in $\mathbb{X}$.

Proof: Suppose the pair $(\mathfrak{Q}, \mathfrak{S})$ satisfies E.A property then $\exists$ sequence $\langle x_n \rangle$ in $\mathbb{X}$ such that $\lim_{n\to\infty} \mathfrak{Q}x_n = \lim_{n\to\infty} \mathfrak{S}x_n = p*$ for some $p* \in \mathbb{X}$.
Since $\mathfrak{Q}(\mathbb{X}) \subseteq \mathfrak{A}(\mathbb{X})$ then $\exists$ $\langle x_n \rangle$ in $\mathbb{X}$ such that $\mathfrak{Q}x_n = \mathfrak{A}y_n$.
Hence $\lim_{n\to\infty} \mathfrak{A}\gamma_n = p*$. ....$(\varphi - 1)$.
Now we show that $\lim_{n\to\infty} \mathfrak{P}\gamma_n = p*$.
Put $x = \gamma_n$ and $\gamma = x_n$ we obtain,

$M_{KM}(\mathfrak{P}\gamma_n, \mathfrak{Q}x_n, k\mathcal{t})$
$\geq M_{KM}(\mathfrak{A}\gamma_n, \mathfrak{S}x_n, \mathcal{t}) * M_{KM}(\mathfrak{P}\gamma_n, \mathfrak{A}\gamma_n, \mathcal{t}) * M_{KM}(\mathfrak{Q}x_n, \mathfrak{S}x_n, \mathcal{t})$
$* M_{KM}(\mathfrak{P}\gamma_n, \mathfrak{S}x_n, \mathcal{t})$.

Letting $n \longrightarrow \infty$ and using $p\gamma_n \longrightarrow p *$ we get
$\lim_{n\to\infty} \mathfrak{Q}x_n = \lim_{n\to\infty} \mathfrak{S}x_n = \lim_{n\to\infty} \mathfrak{A}\gamma_n = \lim_{n\to\infty} \mathfrak{P}\gamma_n = p *$.

Suppose that $\mathfrak{A}(\mathbb{X})$ is closed subspace of $\mathbb{X}$, $\exists$ $u \in \mathbb{X}$ such that
p*=$\mathfrak{A}u\ldots(\varphi - 2)$.

We show that $\mathfrak{A}u = \mathfrak{P}u$.

Put $x = u$ and $\gamma = x_n$ in $(\mathcal{C} - 2)$ then we get

$M_{KM}(\mathfrak{P}u, \mathfrak{Q}x_n, k\mathcal{t})$
$\geq M_{KM}(\mathfrak{A}u, \mathfrak{S}x_n, \mathcal{t}) * M_{KM}(\mathfrak{P}u, \mathfrak{A}u, \mathcal{t}) * M_{KM}(\mathfrak{Q}x_n, \mathfrak{S}x_n, \mathcal{t})$
$* M_{KM}(\mathfrak{P}u, \mathfrak{S}x_n, \mathcal{t})$.

This implies $\mathfrak{P}u = p * \ldots(\varphi - 3)$.

From $(\varphi - 2)$ and $(\varphi - 2)$ we get
$\mathfrak{A}u = \mathfrak{P}u = p * \ldots.(\varphi - 4)$.

And since $(\mathfrak{A}, \mathfrak{P})$ is $\mathcal{A} - i$ntimate then we get $\mathfrak{A}p *= \mathfrak{P}p *= p * \ldots\ldots(\varphi - 5)$.

Since $\mathfrak{P}(\mathbb{X}) \subseteq \mathfrak{S}(\mathbb{X})$ then there exists a point $v \in \mathbb{X}$ such that
$\mathfrak{P}u = \mathfrak{S}v = p * \ldots\ldots(\varphi - 6)$.

Now put $x = u$ and $\gamma = v$ in $(\mathcal{C} - 2)$ then this gives
$M_{KM}(\mathfrak{P}u, \mathfrak{Q}v, k\mathcal{t}) \geq M_{KM}(\mathfrak{A}u, \mathfrak{S}v, \mathcal{t}) * M_{KM}(\mathfrak{P}u, \mathfrak{A}u, \mathcal{t}) * M_{KM}(\mathfrak{Q}v, \mathfrak{S}v, \mathcal{t}) *$
$M_{KM}(\mathfrak{P}u, \mathfrak{S}v, \mathcal{t})$ implies
$M_{KM}(p *, \mathfrak{Q}v, k\mathcal{t})$
$\geq M_{KM}(p *, p *, \mathcal{t}) * M_{KM}(p *, p *, \mathcal{t}) * M_{KM}(\mathfrak{Q}v, p *, \mathcal{t})$
$* M_{KM}(p *, p *, \mathcal{t})$.

This implies $\mathfrak{Q}v = p *$ therefore $\mathfrak{S}v = \mathfrak{Q}v = p * \ldots\ldots(\varphi - 7)$,
and since $(\mathfrak{S}, \mathfrak{Q})$ is $\mathcal{S} -$ intimate then we get
$\mathfrak{S}p *= \mathfrak{Q}p *= p * \ldots.(\varphi - 8)$.

Using $(\varphi - 7)$ $(\varphi - 8)$ and we conclude that $\mathfrak{A}p *= \mathfrak{P}p *= \mathfrak{Q}p *= \mathfrak{S}p *= p *$.
We can prove the uniqueness of the common fixed point easily.

Example 3.2.1: Suppose ($\mathbb{X}$, M_{KM}, *) is a standard FMS with $a * a \geq a$ $\forall a \in [1,11)$, where $\mathfrak{A}$, $\mathfrak{S}$, $\mathfrak{P}$ and $\mathfrak{Q}:\mathbb{X}\to\mathbb{X}$ as

$$\mathfrak{P}(x) = \mathfrak{Q}(x) = \begin{cases} 1 & \text{if } x \in \{1\} \cup (3,11) \\ 1 + x & \text{if } 1 < x \leq 3 \end{cases}$$

$$\mathfrak{S}(x) = \begin{cases} 1 & \text{if } x = 1 \\ 6 & \text{if } 1 < x \leq 3 \\ x - 2 & \text{if } 3 < x < 11 \end{cases} \qquad \mathfrak{A}(x) = \begin{cases} 1 & \text{if } x = 1 \\ 4 & \text{if } 1 < x \leq 3 \\ \frac{3x-1}{8} & \text{if } 3 < x < 11 \end{cases}$$

$\mathfrak{P}(\mathbb{X}) = \mathfrak{Q}(\mathbb{X}) = \{1\} \cup (2,4]$ and $\mathfrak{S}(\mathbb{X}) = \{1\} \cup \{6\} \cup (1,9)$ $\mathfrak{A}(\mathbb{X}) = \{1\} \cup \{4\} \cup (1,4) = [1,4]$ these sets satisfy the conditions $(\mathcal{C}-2)$and $(\mathcal{C}-3)$.

Now assume $\langle x_n \rangle = \left\{3 + \frac{1}{n}\right\}$ then $\lim_{n\to\infty} \mathfrak{A}x_n = \lim_{n\to\infty} \mathfrak{P}x_n = 1$ and this implies $(\mathfrak{P}, \mathfrak{A})$ satisfies E.A property and also we have, $\lim_{n\to\infty} \mathfrak{A}\mathfrak{P}x_n = \lim_{n\to\infty} \mathfrak{P}\mathfrak{P}x_n = 1$. This gives $\lim_{n\to\infty} M_{KM}(\mathfrak{A}\mathfrak{P}x_n, \mathfrak{A}x_n, t) \geq \lim_{n\to\infty} M_{KM}(\mathfrak{P}\mathfrak{P}x_n, \mathfrak{P}x_n, t)$ for $t > 0$.

Thus, the pair $(\mathfrak{A},\mathfrak{P})$ is $\mathcal{A}$-intimate.

Since $\lim_{n\to\infty} \mathfrak{S}x_n = \lim_{n\to\infty} \mathfrak{Q}x_n = 1$ and $\lim_{n\to\infty} \mathfrak{S}\mathfrak{Q}x_n = \lim_{n\to\infty} \mathfrak{Q}\mathfrak{Q}x_n = 1$ this gives $\lim_{n\to\infty} M_{KM}(\mathfrak{S}\mathfrak{Q}x_n, \mathfrak{S}x_n, t) \geq \lim_{n\to\infty} M_{KM}(\mathfrak{Q}\mathfrak{Q}x_n \mathfrak{Q}x_n, t)$.

Thus, the pair $(\mathfrak{S},\mathfrak{Q})$ is $\mathcal{S}$-intimate. Moreover, it satisfies the contraction condition of the theorem. Clearly 1 is the unique common fixed point for these four mappings.

Finally, we discuss another theorem.

3.3 Theorem: Let$(\mathbb{X}, M_{KM},*)$ be a FMS. Suppose $\mathfrak{P}$,$\mathfrak{Q}$, $\mathfrak{S}$ and $\mathfrak{A}$ are self maps on $\mathbb{X}$ satisfying the conditions $(\mathcal{C}-2)$ and $(\mathcal{C}-4)$ in addition to

$(\mathcal{C}-6)$ $\mathfrak{A}(\mathbb{X})$ and $\mathfrak{S}(\mathbb{X})$ are closed subsets of $\mathbb{X}$

$(\mathcal{C}-7)$the pairs $(\mathfrak{P}, \mathfrak{A})$ and $(\mathfrak{Q}, \mathfrak{S})$ share the common property E. A.

Then $\mathfrak{P}$, $\mathfrak{Q}$,$\mathfrak{S}$ and $\mathfrak{A}$ have a unique common fixed point in $\mathbb{X}$.

Proof: In view of the condition $(\mathcal{C}-7)$there exists two sequences $\langle x_n \rangle$ and $\langle \gamma_n \rangle$ in $\mathbb{X}$ such that $\lim_{n\to\infty} \mathfrak{P}x_n = \lim_{n\to\infty} \mathfrak{A}x_n = \lim_{n\to\infty} \mathfrak{Q}\gamma_n = \lim_{n\to\infty} \mathfrak{S}\gamma_n = p*$ for some $p* \in \mathbb{X}$.

From the $(\mathcal{C}-6)$ we have$\mathfrak{A}(\mathbb{X})$ is closed subset of $\mathbb{X}$,consequently $\lim_{n\to\infty} \mathfrak{P}x_n = p* \in \mathfrak{A}(\mathbb{X})$. This means there exists appoint $u \in \mathbb{X}$ such that $\mathfrak{A}u = p*$.

Now we assert that $\mathfrak{P}u = \mathfrak{A}u$.

Put $x = u$ and $\gamma = \gamma_n$, we get

$$M_{KM}(\mathfrak{P}u, \mathfrak{Q}y_n, kt)$$
$$\geq M_{KM}(\mathfrak{A}u, \mathfrak{S}y_n, t) * M_{KM}(\mathfrak{P}u, \mathfrak{A}u, t) * M_{KM}(\mathfrak{Q}y_n, \mathfrak{S}y_n, t) * M_{KM}(\mathfrak{P}u, \mathfrak{S}y_n, t)$$

Which on making $n \to \infty$, with $\mathfrak{A}u = p*$ reduces to $\mathfrak{P}u = p*$. This implies $\mathfrak{P}u = \mathfrak{A}u = p*$ which signifies that u is coincident point of the pair $(\mathfrak{P}, \mathfrak{A})$.

On the other hand, $\mathfrak{S}(\mathbb{X})$ is closed subset of $\mathbb{X}$ therefore $\lim_{n\to\infty} \mathfrak{S}\gamma_n = p* \in \mathfrak{S}(\mathbb{X})$ and hence we can find a point $w \in \mathbb{X} \ni \mathfrak{S}w = p*$.

Now we show that $\mathfrak{S}w = \mathfrak{Q}w$. On using condition $(\mathcal{C}-2)$ with $x = u$ and $\gamma = w$ then we get

$$M_{KM}(\mathfrak{P}u, \mathfrak{Q}w, kt) \geq M_{KM}(\mathfrak{A}u, \mathfrak{S}w, t) * M_{KM}(\mathfrak{P}u, \mathfrak{A}u, t) * M_{KM}(\mathfrak{Q}w, \mathfrak{S}w, t) * M_{KM}(\mathfrak{P}u, \mathfrak{S}w, t).$$

This implies $\mathfrak{Q}w = p*$.This gives $\mathfrak{S}w = \mathfrak{Q}w = p*$.

Since the pair $(\mathfrak{Q}, \mathfrak{S})$ is $\mathcal{S}-$ intimate this gives

$M_{KM}(\mathfrak{S}p*, p*, t) \geq M_{KM}(\mathfrak{Q}p*, p*, t)$.

Suppose that $\mathfrak{S}p* \neq p*$.
Put $x = \mathrm{u}$ and $\gamma = \mathrm{p}*$ in contraction condition $(\mathcal{C}-2)$

$$\mathrm{M}_{KM}(\mathfrak{P}\mathrm{u}, \mathfrak{Q}p*, \mathrm{k}t) \geq \mathrm{M}_{KM}(\mathfrak{A}\mathrm{u}, \mathfrak{S}p*, t) * \mathrm{M}_{KM}(\mathfrak{P}\mathrm{u}, \mathfrak{A}\mathrm{u}, t) * \mathrm{M}_{KM}(\mathfrak{Q}p*, \mathfrak{S}p*, t) * \mathrm{M}_{KM}(\mathfrak{P}\mathrm{u}, \mathfrak{S}p*, t)$$

implies $\mathfrak{Q}\mathrm{p}* = \mathrm{p}*$.
Using $\mathrm{M}_{\mathrm{KM}}(\mathfrak{S}\mathrm{p}*, \mathrm{p}*, \mathrm{t}) \geq \mathrm{M}_{\mathrm{KM}}(\mathrm{p}*, \mathrm{p}*, t)$
we get $\mathfrak{S}\mathrm{p}* = \mathrm{p}*$.
Therefore $\mathfrak{Q}\mathrm{p}* = \mathfrak{S}\mathrm{p}* = \mathrm{p}* \ldots\ldots\ldots (\psi - 1)$.
Since $\mathfrak{P}\mathrm{u} = \mathfrak{A}\mathrm{u} = \mathrm{p}*$ and using $(\mathfrak{P}, \mathfrak{A})$ is $\mathcal{A}$ −intimate then we get $\mathfrak{A}\mathrm{p}* = \mathrm{p}*$.
By putting $x = \gamma = \mathrm{p}*$ we get

$$\mathrm{M}_{\mathrm{KM}}(\mathfrak{P}\mathrm{p}*, \mathfrak{Q}\mathrm{p}*, \mathrm{k}t) \geq \mathrm{M}_{\mathrm{KM}}(\mathfrak{A}\mathrm{p}*, \mathfrak{S}\mathrm{p}*, t) * \mathrm{M}_{\mathrm{KM}}(\mathfrak{P}\mathrm{p}*, \mathfrak{A}\mathrm{p}*, t) * \mathrm{M}_{\mathrm{KM}}(\mathfrak{Q}\mathrm{p}*, \mathfrak{S}\mathrm{p}*, t) * \mathrm{M}_{\mathrm{KM}}(\mathfrak{P}\mathrm{p}*, \mathfrak{S}\mathrm{p}*, t).$$

This implies $\mathfrak{P}\mathrm{p}* = \mathrm{p}*$ and this gives $\mathfrak{A}\mathrm{p}* = \mathfrak{P}\mathrm{p}* = \mathrm{p}* \ldots\ldots (\psi - 2)$.
From $(\psi - 1)$ and $(\psi - 2)$ we conclude that $\mathfrak{A}\mathrm{p}* = \mathfrak{P}\mathrm{p}* = \mathfrak{Q}\mathrm{p}* = \mathfrak{S}\mathrm{p}* = \mathrm{p}*$.
We can prove the uniqueness of the fixed point easily.

Example 3.3.1: Suppose $(\mathbb{X}, \mathrm{M}_{\mathrm{KM}}, *)$ is a standard FMS with $a * a \geq a \ \forall a \in [1,20]$, where $\mathfrak{A}$, $\mathfrak{S}$, $\mathfrak{P}$ and $\mathfrak{Q}:\mathbb{X}\to\mathbb{X}$ as

$$\mathfrak{P}(x) = \mathfrak{Q}(x) = \begin{cases} 1 & \text{if } x = 1,\ \ 2 \leq x < 20 \\ x & \text{if } 1 \leq x < 2 \end{cases}$$

$$\mathfrak{S}(x) = \begin{cases} 1 & \text{if } x = 1 \\ 12 & \text{if } 1 < x < 2 \\ \dfrac{x+1}{3} & \text{if } 2 \leq x \leq 20 \end{cases} \qquad \mathfrak{A}(\mathrm{x}) = \begin{cases} 1 & \text{if } x = 1 \\ 7 & \text{if } 1 < x < 2 \\ \dfrac{2x+5}{9} & \text{if } 2 \leq x \leq 20 \end{cases}$$

$\mathfrak{P}(\mathbb{X}) = \mathfrak{Q}(\mathbb{X}) = \{1\} \cup (1,2)$, $\mathfrak{S}(\mathbb{X}) = \{1\} \cup \{12\} \cup [1,5]$ and
$\mathfrak{A}(\mathbb{X}) = \{1\} \cup \{7\} \cup [1,9]$ these sets satisfy the conditions $(\mathcal{C}-1)$ and $(\mathcal{C}-3)$.
Now assume $\langle x_n \rangle = \left\{2 + \frac{1}{n}\right\}$ and $\langle \gamma_n \rangle = \{1\}$ then

$$\lim_{n\to\infty} \mathfrak{A}x_n = \lim_{n\to\infty} \mathfrak{P}x_n = \lim_{n\to\infty} \mathfrak{S}\gamma_n = \lim_{n\to\infty} \mathfrak{Q}\gamma_n = 1.$$

This implies the pairs $(\mathfrak{P}, \mathfrak{A})$ and $(\mathfrak{S},\mathfrak{Q})$ share the common E. A property and also we have, $\lim_{n\to\infty} \mathfrak{A}\mathfrak{P}x_n = \lim_{n\to\infty} \mathfrak{P}\mathfrak{P}x_n = 1$ this gives

$$\lim_{n\to\infty} \mathrm{M}_{\mathrm{KM}}(\mathfrak{A}\mathfrak{P}x_n, \mathfrak{A}x_n, t) \geq \lim_{n\to\infty} \mathrm{M}_{\mathrm{KM}}(\mathfrak{P}\mathfrak{P}x_n, \mathfrak{P}x_n, t), \text{for } \ t > 0.$$

Thus, the pair $(\mathfrak{A},\mathfrak{P})$ is $\mathcal{A}$-intimate.
Since $\lim_{n\to\infty} \mathfrak{S}x_n = \lim_{n\to\infty} \mathfrak{Q}x_n = 1$ and $\lim_{n\to\infty} \mathfrak{S}\mathfrak{Q}x_n = \lim_{n\to\infty} \mathfrak{Q}\mathfrak{Q}x_n = 1$
this gives $\lim_{n\to\infty} \mathrm{M}_{\mathrm{KM}}(\mathfrak{S}\mathfrak{Q}x_n, \mathfrak{S}x_n, t) \geq \lim_{n\to\infty} \mathrm{M}_{\mathrm{KM}}(\mathfrak{Q}\mathfrak{Q}x_n, \mathfrak{Q}x_n, t)$.
Thus, the pair $(\mathfrak{S},\mathfrak{Q})$ is $\mathcal{S}$-intimate. Moreover, it satisfies the contraction condition of the theorem. Clearly 1 is the unique common fixed point for these four mappings.

4 Conclusion

This paper aimed to prove three common fixed point theorems to generalize the class of compatible mappings by using the calss of non compatible mappings like different forms of E.A properties along with intimate mappings in fuzzy metric space. In Theorem 3.1, one of the range of mappings is assumed to be complete.Further, in Theorem 3.2, one of the pairs is assumed to satisfy E.A property along with one of the range of mappings is complete without being complete fuzzy metric space. Finally in Theorem 3.3, improved version of EA property namely common EA property is assumed along with completeness of fuzzy metric space. Moreover, all these results are justified with suitable examples.

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