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Rainfall Forecast in Different Methods of Trend Equations by Fuzzy Time Series

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Abstract

Fuzzy time series models have been put forward for Rainfall Prediction from many researchers around the globe. Fuzzy time series methods do not require any assumptions valid for classic time series approaches. The most important disadvantage of fuzzy time series approaches is that it needs subjective decisions, especially in fuzzification stage. This paper proposes a novel improvement of forecasting approach based on using first order fuzzy time series. In contrast to traditional forecasting methods, fuzzy time series can be also applied to problems, in which historical rainfall data of Trichy district. In this study reveals some feature of FTS predicting Rainfall and the results have been compared with other methods.

Keywords: Predicting Rainfall, Fuzzy Time Series; Time Variant, Fuzzy Membership Grade; Fuzzy Logical Relations, Fuzzy Relation.

AMS Subject Classification: 05C78[3].

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1. Introduction

Forecasting activities play an important role in our daily life. We often forecast the weather, earthquakes, stock market, and everything which people want to foresee. People can plan or prevent beforehand by forecasting activities. It is impossible to make a one hundred percent forecast, but we can do our best to increase the accuracy of forecasts. Traditional forecasting methods can deal with many forecasting cases, but they cannot solve forecasting problems in which the historical data are linguistic values. In [9, 10], Song and Chissom presented the definition of fuzzy time series and outlined its modeling by means of fuzzy relational equations and approximate reasoning [18, 25].

Fuzzy time series is a new concept which can be used to deal with forecasting problems in which historical data are linguistic values. In [10], Song and Chissom have applied the fuzzy time series model to forecast the enrollments of the University of Alabama, where a first-order time invariant model is developed and a step-by- step procedure is provided. In [10], they also pointed out that Bintley [1] has successfully applied fuzzy logic and approximate reasoning to a practical case of forecasting, but the concept of fuzzy time series was not applied on the method presented in [1].

The method presented by Song and Chissom [10] uses the following model for forecasting university enrollments: $Ai = Ai - 1 \circ R$, (1) where $Ai - 1$ is the enrollment of year $i - 1$ represented by a fuzzy set [8, 23, 24], Ai is the forecasted enrollment of year i represented by a fuzzy set, "$\circ$" is the max-min composition operator, and R is a fuzzy relation indicating fuzzy relationships between fuzzy time series. However, the forecasted method presented in [3] requires a large amount of computations to derive the fuzzy relation R of (1), and the max-min composition operations of (1) will take a large amount of computation time when the fuzzy relation R of (l) is very big. Thus, we must develop a new method to forecast the enrollments of the university in a more efficient manner. In [10], Sullivan et al. reviewed Making decisions on forecasting is a complicated process. Due to the complexity of meteorological phenomena, accurate prediction of rainfall is challenging one. To plan our day-to-day activities accurate weather predictions are important. Farmers need information about the weather to plan for planting and harvesting their crops.

Weather forecasting helps to keep us out of danger. The first-order time-invariant fuzzy time-series model and the first-order time-invariant model proposed by Song and Chissom, where these models are compared with each other and with a time-variant Markov model using linguistic labels with probability distributions. In 1993, Song and Chissom presented the theory of fuzzy time series. Fuzzy time series model deal with the both linguistic and numerical values.

In this paper, we review the concept of fuzzy time series and the forecasting methods presented in [12, 18, 23, 24]. A time series represents a collection of values of certain events or tasks which are obtained with respect to time. Future prediction of time series events has been attracted people from its beginning. Song and Chissom developed a model, based on uncertainty and imprecise knowledge contained in time series data. They initially used the fuzzy sets concept to represent or manage all these uncertainties and referred this concept as Fuzzy Time Series (FTS). Researchers have developed

numerous models based on the FTS concept to deal with the forecasting problems of short term as well as long term events. The following are some definitions related with fuzzy time series.
In section 2 some basic definitions of fuzzy time series are discussed. In section 3 rainfall predicting model and computational procedures are discussed.

2. Basic Definitions of Fuzzy Time Series

2.1. Time Series. A time series is a sequence of observations collected at regular time intervals when there is correlation among successive observations.

2.2. Fuzzy Time Series. Let $X(t)$ $(t = 0,1,2,\cdots)$ be the universe of discourse and the fuzzy set defined on $X(t)$be$f_i(t)$ $(t = 0,1,2,\cdots)$. Then $F(t) = f_i(t)$ $t = 0,1,2,\cdots$, $i = 1,2,\cdots$ the collection of all fuzzy sets defined on $X(t)$ is called a fuzzy time series of$X(t)$ $(t = 0,1,2,\cdots)$.

2.3. Fuzzy Relation. If $F(t)$ is caused by$F(t-1)$, denoted by$F(t) \rightarrow F(t-1)$, then this relationship can be represented by$F(t) = F(t-1) \circ R(t,t-1)$, where $\circ$ denotes the composition operator and $R(t,t-1)$ is a fuzzy relation between $F(t)$ and$F(t-1)$.

2.4. First Order Model. The model in which the relation $R\ (t,t-1)$ is a fuzzy relation between $F(t)$ and $F(t-1)$ is called the first order model of$F(t)$.

2.5. Time Invariant Fuzzy Time Series. If in first order model of $F\ (t)$ relation $R\ (t,t-1) = R\ (t-1,t-2)$ for any time$t$, then $F(t)$ is called time invariant fuzzy time series.

2.6. Time Variant Fuzzy Time Series. If in first order model of $F(t)$ relation $R\ (t,t-1) \neq R\ (t-1,t-2)$ for any time t, then $F(t)$ is called time invariant fuzzy time series.

2.7. n-order Fuzzy time series. Let $F(t)$ be a fuzzy time series. If $F(t)$ is caused by$F(t-1), F(t-2), \ldots F(t-n)$, then this fuzzy relationship is represented by $F(tn), \ldots, F(t-2), F(t-1) \rightarrow F(t)$ and is called an n- order fuzzy time series.

2.8. Fuzzy variables. Imprecise data at equally spaced discrete time points are modeled as fuzzy variables. The set of this discrete fuzzy data forms a fuzzy time series.

Theorem 2.9.1. If $F(t)$ is a fuzzy time series, for any $t, F(t)$ has only finite elements $f_i(t)$ $(t = 1,2,\ldots n)$ and $F(t) = F(t-1)$ then $F(t)$ is a time –invariant fuzzy time series.

Given that, $F(t)$ is a fuzzy time series, for any $t, F(t)$ has only finite elements $f_i(t)$ $(t = 1,2,\ldots n)$and $F(t) = F(t-1)$.

To prove that $F(t)$ is a time –invariant fuzzy time series.

Proof. Generality, rearrange the subscript of each element of $F(t)\,(t = 1,2,\ldots n)$ such that these with the same values at different time t's have the same subscript. For simplicity, double subscripts for each element. According to definition, "suppose $F(t)$is a fuzzy time series($t = \ldots,0,1,2,3,4,\ldots$)and $t_1 \neq t_2$.If for any $f_i(t_1) \in F(t_1)$ there exists an $f_j(t_2) \in F(t_2)$ such that $f_i(t_1) = f_j(t_2)$ and vice versa, then define$F(t_1) = F(t_2)$", the following is true.

$$\begin{aligned}
&\ldots = f_{10}(t) = f_{11}(t-1) = f_{12}(t-2) = f_{13}(t-3) = f_{14}(t-4) = \cdots = f_{1n}(t-n) = \ldots \\
&\ldots = f_{20}(t) = f_{21}(t-1) = f_{22}(t-2) = f_{23}(t-3) = f_{24}(t-4) = \cdots = f_{2n}(t-n) = \ldots \\
&\ldots = f_{30}(t) = f_{31}(t-1) = f_{32}(t-2) = f_{33}(t-3) = f_{34}(t-4) = \cdots = f_{3n}(t-n = \ldots \quad (1) \\
&\ldots = f_{40}(t) = f_{41}(t-1) = f_{42}(t-2) = f_{43}(t-3) = f_{44}(t-4) = \cdots = f_{4n}(t-n) = \ldots \\
&\ldots = f_{n0}(t) = f_{n1}(t-1) = f_{n2}(t-2) = f_{n3}(t-3) = f_{n4}(t-4) = \cdots = f_{nn}(t-n) = \ldots.
\end{aligned}$$

Let $R(t,t-1) = \cup_{i,j} R_{ij}\,(t,t-1)$ and, $R\,(t-1,t-2) = \cup_{i,j} R_{ij}(t-1,t-2)$.For any $j \in J$ and$i \in I$, suppose $R_{ij}(t,t-1) = f_{i1}(t-1)\,x\,f_{j0}(t)$ which is equivalent to $f_{j0}(t) = f_{i1}(t-1) \circ R_{ij}(t,t-1)$.According to (1), there exist an $f_{i2}(t-2)$ and $f_{j1}(t-1)$ such that$f_{j0}(t) = f_{j1}(t-1)$and$f_{i1}(t-1) = f_{i2}(t-2)$.Therefore**,**

$$\begin{aligned}
R_{ij}(t,t-1) &= f_{i1}(t-1)x\,f_{j0}(t) = f_{i2}(t-2)x\,f_{j1}(t-1) \\
&= R_{ij}(t-1,t-2)
\end{aligned}$$

This means that $R_{ij}(t,t-1) \subseteq R_{ij}(t-1,t-2)$.suppose$R_{ij}(t-1,t-2) = f_{i2}(t-2)x\,f_{j1}(t-1)$. According to (1), there exist an $f_{j0}(t)$and$f_{j1}(t-1)$ such that$f_{j0}(t) = f_{j1}(t-1)$ and $f_{i1}(t-1) = f_{i2}(t-2)$.Therefore,

$$Rij(t-1,t-2) = fi2(t-2)x\,fj1(t-1) = fi1(t-2)x\,fj0(t) = Rij(t,t-1).$$

This implies that$R_{ij}(t-1,t-2) \subseteq R_{ij}(t,t-1)$. Hence, $Rij(t,t-1) = R_{ij}((t-1),(t-1)-1)$,$R_{ij}(t,t-2) = R_{ij}((t-1),(t-1)-2)$,$R_{ij}(t,t-3) = R_{ij}((t-1),(t-1)-3)$,$R_{ij}(t,t-4) = R_{ij}((t-1),(t-1)-4)\ldots.$ Similarly, we get for any $t, R_a(t,t-m) = R_{ij}((t-1),(t-1-m))andR_0\,(t,t-m) = R \circ ((t-1),(t-1-m))$. According to definition," If $F(t) = F(t-1) \circ R(t,t-1)$ or $F(t) = (F(t-l) \cup F(t-2) \cup \cdots \cup F(t-m)) \circ R_0\,(t,t-m)$ or $F(t) = (F(t-l)\,x\,F(t-2)\,x\,\cdots x\,F(t-m)) \circ Ra(t,t-m)$, the fuzzy relation $R(t,t-1)\,R_a\,(t,t-m)$ or $R_0\,(t,t-m)$ of $F(t)$ is independent of time t, that is to say for different times t_1 and t_2, $R(t_1,t_1-1) = R(t_2,t_2-1)$, or $R_a(t_1,t_1-m) = R_a(t_2,t_2-m)$, or $R_0(t_1,t_1-m) = R_0(t_1,t_1-m)$, then F(t) is called a time –invariant fuzzy time series. Otherwise, it is called a time –variant fuzzy time series.
In the case of time –invariant fuzzy time series,

$$R(t,t-1) = R$$
$$R_a(t,t-m) = R_a(m),$$

$$R_0\,(t, t-m) \;=\; R_0\,(m).$$

It should be noted that generally at different times t_1 and $t_2, R(t_1, t_1 - 1) \neq R(t_2, t_2 - 1)$ or $R_a(t_1, t_1 - m) \neq R_a(t_2, t_2 - m)andR_0\,(t_1, t_1 - m) \neq R_0\,(t_2, t_2 - m)$.There are two reasons for this: first the universe of discourse on which the fuzzy sets are defined may be different at different times; second, the value of $F(t)$at different times may be different. Therefore, the classifications of fuzzy time series are meaningful." Hence, proved that $F(t)$ is called a time – invariant fuzzy time series.

Theorem 2.9.2 If $F(t)$ is a fuzzy time series, for any $t, F(t) = F(t-1)$ and $F(t)$ has only finite elements $f_i(t)$ $(i = 1, 2, \ldots n)$ then $R(t, t-1)$ can be expressed as $R = R(t, t-1) = \cdots \cup f_{i1}(t-1)x\, f_{j0}(t) \cup f_{i2}(t-2) \times f_{j1}(t-1) \cup \ldots \cup f_{im}(t - m) \times f_{jm} - 1(t - m + 1) \cup$ ...where $m > o$and each pair of fuzzy sets are different.

Given that:$F(t)$ is a fuzzy time series, for any $t, F(t) = F(t-1)$ and $F(t)$ has only finite elements $f_i(t)$ $(t = 1, 2, \ldots n)$.

To prove that: $R = R(t, t-1) = \ldots \cup f_{i1}(t-1) \times f_{j0}(t) \cup fi2(t-2) \times f_{j1}(t-1) \cup \ldots \cup f_{im}(t-m) \times f_{jm} - 1(t - m + 1) \cup$ ...where $m > o$and each pair of fuzzy sets are different.

Proof. We know that, "If $F(t)$ is a fuzzy time series, for any $t, F(t)$ has only finite elements $f_i(t)$ $(i = 1, 2, \ldots n)$ and $F(t) = F(t-1)$ then $F(t)$ is a time –invariant fuzzy time series." Hence, $F(t)$ is a time –invariant fuzzy time series.According to definition, "If $F(t) = F(t-1) \circ R(t, t-1)$ or $F(t) = (F(t-l) \cup F(t-2) \cup \cdots \cup F(t-m)) \circ R_0\,(t, t-m)$ or $F(t) = (F(t-l) \times F(t-2) \times \cdots \times F(t-m)) \circ R_a(t, t-m)$, the fuzzy relation $R(t, t-1)\, R_a\,(t, t-m)$ or $R_0\,(t, t-m)$ of $F(t)$ is independent of time t, that is to say for different times t_1 and t_2, $R(t_1, t_1 - 1) = R(t_2, t_2 - 1)$, or $R_a(t_1, t_1 - m) = R_a(t_2, t_2 - m)$, or $R_0\,(t_1, t_1 - m) = R_0\,(t_1, t_1 - m)$, then $F(t)$ is called a time –invariant fuzzy time series. Otherwise, it is called a time –variant fuzzy time series. In the case of time –invariant fuzzy time series, $R(t, t-1) = R, R_a(t, t-m) = R_a(m), R_0\,(t, t-m) = R_0\,(m)$".

It should be noted that generally at different times $t_1$and$t_2, R(t_1, t_1 - 1) \neq R(t_2, t_2 - 1)$ or $R_a(t_1, t_1 - m) \neq R_a(t_2, t_2 - m)$ and $R_0\,(t_1, t_1 - m) \neq R_0\,(t_1, t_1 - m)$.There are two reasons for this: first the universe of discourse on which the fuzzy sets are defined may be different at different times; second, the value of $F(t)$at different times may be different. Therefore, the classifications of fuzzy time series are meaningful.

$R(t, t-1)$ is independent of t and therefore, $R(t, t-1) = R$.

Because of this, the following holds:

$$\ldots = R(t+1, t) = R(t, t-1) = R(t-1, t-2) = R(t-2, t-3) = \ldots$$

Since $R(t,t-1) = \cup_{ij} (R_{ij}(t,t-1)) = f_i(t-1) \times f_j(t)$ $(t = \ldots 0,1,2,\ldots n)$ and $F(t)$ has only n elements, the maximum number of different components,$(R_{ij}(t,t-1)$'s, of$R(t,t-1)$, isn^2. Clearly, all$(R_{ij}(t,t-1))$'s have the same components.

Let us consider a selection process. First at an arbitrary and fixed time, select a pair of elements of $F(t)$ and$F(t-1)$, say,$f_{j0}(t)$ and$f_{i1}(t-1)$, and then calculate$f_{i1}(t-1) \times f_{j0}(t)$. At$t-1$, select a different pair of elements of $F(t-1)$ and$F(t-2)$, say,$f_{i1}(t-1)$ and$f_{i2}(t-2)$, and then calculate$f_{i2}(t-2) \times f_{j1}(t-1)$. Suppose at time $t-k(k>1)$,we have selected a pair of elements, say,$f_{jk}-1(t-k+1)$ and $f_{ik}(t-k)$, of $F(t-k+1)$ and $F(t-k)$ which are different from any pairs selected before, and then calculate $f_{ik}(t-k) \times f_{jk}-1(t-k+1)$.since R has at most n^2 components, the above procedure, at most n^2steps are needed to select all the different components needed to calculate R. Hence, proved that $R = R(t,t-1) = \ldots \cup f_{i1}(t-1) \times f_{j0}(t) \cup f_{i2}(t-2) \times f_{j1}(t-1) \cup \ldots \cup f_{im}(t-m) \times f_{jm}-1(t-m+1) \cup$ ...where $m > o$ and each pair of fuzzy sets are different.

3. Fuzzy Time Series Rainfall Predicting Model

3.1. General description of the model

First, we tested the model [2] with rainfall data to forecast the distribution. Then we applied the methods of [6], [3] and [4]. Steps for finding forecasting fuzzy time series (Chen's Method) Chen [2] proposed a simple method to forecast the enrollment in fuzzy time series. The seven steps involved in the method is given below:

Step 1: Define the universe of discourse to accommodate the time series.
Step 2: Define the universe of discourse and partition it into equally lengthy intervals.
Step 3: Fuzzify historical data.
Step 4: Identify fuzzy logical relationship (FLR's).
Step 5: Establish fuzzy logical relationship groups (FLRG's).
Step 6: Computing the predicted rainfall (fuzzy output)
Step 7: Defuzzification of the fuzzy output for crisp predicting

3.2. Computational Procedures

The descriptive statistics and block wise rainfall data are given in Table I & II.

Table 3.1. Actual Enrollments and Linguistic of Historical data

Blocks	**Actual Enrollments**	**Blocks**	**Actual Enrollments**
Andanallur	1085.23	Pullambadi	1163.59
Lalgudi	1136.79	T.pet	710.16
Manachanallur	569.44	Thiruverumbur	960.9
Manapparai	837.43	Thottiam	866.73
Manikandam	379.79	Thuraiur	942.11
Marungapuri	942.16	Uppiliyapuram	473.35
Musiri	794.31	Vaiyampatty	922.27

Table 3.2. Descriptive Statistics of the Rainfall Data from 2004 - 2010

Minimum(min)	= 379.79
Maximum(max)	= 1163.59
Range(R)	= 783.8
Count(n)	= 14
Sum	= 11784.26
Mean($\bar{x}$)	$= 841.733$
Median($\tilde{x}$)	$= 894.5$
Mode	= No mode
Standard Deviation(s)	= 237.65
Variance(s^2)	= 56479.51

3.2.1. Specification of the universe of discourse U (Step 1)

Generally, to define the universe of discourse U, find the minimum enrollmentD_{min}, and the maximum enrollment D_{max} from the known historical data. Based on D_{min}, and D .... Define U as $[D_{min} - D_1, D_{max} + D_2]$ where D_1 and D_2 are two proper positive numbers. So far, we have collected historical enrollment data from 2004 to 2010. The minimum enrollment D_{min} and the maximum enrollment D_{max} are 379.79 and 1163.59

respectively. Therefore, we choose $D_1 = 79.79$ and $D_2 = 36.41$ to make$U = [300, 1200]$.

3.2.2. Partitioning U into Several Even Length Intervals (Step 2)

The process is the same as in [3]. We have six intervals which are given below.

Table 3.3**.** Length of Intervals

u_1	[300 - 450]
u_2	[450 - 600]
u_3	[600 - 750]
u_4	[750 - 900]
u_5	[900 -1050]
u_6	[1050-1200]

3.2.3. Fuzzify historical data (Step 3).

Consider six fuzzy sets which are A_1 = (very few), A_2 = (very very few), A_3 = (Moderate), A_4 = (High), A_5 = (very High), and A_6 = (very very High). To define these fuzzy sets, we can use the same process as in [3]. Thus, all the fuzzy sets A_i (i =1 to 6) can be defined as follows
The fuzzy sets A_i are expressed as follows:

$$A_1 = {}^{1}/_{u_1} + {}^{0.5}/_{u_2} + {}^{0}/_{u_3} + {}^{0}/_{u_4} + {}^{0}/_{u_5} + {}^{0}/_{u_6}$$
$$A_2 = {}^{0.5}/_{u_1} + {}^{1}/_{u_2} + {}^{0.5}/_{u_3} + {}^{0}/_{u_4} + {}^{0}/_{u_5} + {}^{0}/_{u_6}$$
$$A_3 = {}^{0}/_{u_1} + {}^{0.5}/_{u_2} + {}^{1}/_{u_3} + {}^{0.5}/_{u_4} + {}^{0}/_{u_5} + {}^{0}/_{u_6}$$
$$A_4 = {}^{0}/_{u_1} + {}^{0}/_{u_2} + {}^{0.5}/_{u_3} + {}^{1}/_{u_4} + {}^{0.5}/_{u_5} + {}^{0}/_{u_6}$$
$$A_5 = {}^{0}/_{u_1} + {}^{0}/_{u_2} + {}^{0}/_{u_3} + {}^{0.5}/_{u_4} + {}^{1}/_{u_5} + {}^{0.5}/_{u_6}$$
$$A_6 = {}^{0}/_{u_1} + {}^{0}/_{u_2} + {}^{0}/_{u_3} + {}^{0}/_{u_4} + {}^{0.5}/_{u_5} + {}^{1}/_{u_6}$$

Where u_i (i = l to 6) is the element and the number below / is the membership of u_i to Aj (j = 1 to 6). For simplicity, we will also use A1, A2, A3, A4, A5 and A6 as row vectors whose elements are the memberships of the corresponding fuzzy sets.

3.2.4. Identify fuzzy logical relationship (step 4)

To fuzzify the historical data, we can apply the same method as employed in [3]. For the block Andanallur and Lalgudi, the enrollments are 1085.23 and 1136.79 respectively, which are almost equal to those of Pullambadi and Thiruverumbur respectively and therefore the fuzzified enrollments for Andanallur and Lalgudi are A_6

and A_6 ([3] for details of the fuzzification process). All the fuzzified historical data are listed in Table III where A_i (i = 1 to 6) are defined in Section 4.4

Table 3.4. Fuzzified Historical Enrollments Based on Variations

Blocks	Rainfall (mm)	Fuzzified	Blocks	Rainfall (mm)	Fuzzified
Andanallur	1085.23	A_6	Pullambadi	1163.59	A_6
Lalgudi	1136.79	A_6	T.pet	710.16	A_4
Manachanallur	569.44	A_2	Thiruverumbur	960.9	A_5
Manapparai	837.43	A_4	Thottiam	866.73	A_4
Manikandam	379.79	A_1	Thuraiur	942.11	A_5
Marungapuri	942.16	A_5	Uppiliyapuram	473.35	A_2
Musiri	794.31	A_4	Vaiyampatty	922.27	A_5

3.2.5. Establish fuzzy logical relationship groups (FLRG's) (step 5).

The fuzzy logical relationships are as follows (the repeated relationships are omitted)

Table 3.5. Predictions of Fuzzy Logical Relationships

No.	Relationship	No.	Relationship
1	$A_6 \rightarrow A_6$	8	$A_6 \rightarrow A_4$
2	$A_6 \rightarrow A_2$	9	$A_4 \rightarrow A_5$
3	$A_2 \rightarrow A_4$	10	$A_5 \rightarrow A_4$
4	$A_4 \rightarrow A_1$	11	$A_4 \rightarrow A_5$
5	$A_1 \rightarrow A_5$	12	$A_5 \rightarrow A_2$
6	$A_5 \rightarrow A_4$	13	$A_2 \rightarrow A_5$
7	$A_4 \rightarrow A_6$		

3.2.6. Computing the predicted rainfall (fuzzy output) (step6)

Generally, to determine the fuzzy relations, we need to find two consecutive blocks' data, say, t and $t-1$. If the data for t is$A$, and $A_{t-l}$for $t-1$, then the fuzzy logical relationship will be$A_{t-1} \rightarrow A_t$, and the fuzzy relation between $A_{t-1}$and A_t will be $A_{t-1}^T \times At$,.The fuzzy logical relationships are as follows (the repeated relationships are omitted).

Table 3.6. Predictions of Fuzzy Logical Relationships Groups

No.	Group of Relationships
1	$A_1 \rightarrow A_5$
2	$A_2 \rightarrow A_4, A_5$
3	$A_3 \rightarrow 0$
4	$A_2 \rightarrow A_1, A_5, A_5, A_6$
5	$A_5 \rightarrow A_2, A_4$
6	$A_6 \rightarrow A_2, A_4, A_6$

Combine the fuzzy relationship into fuzzy logical relation group starting from identical left-hand sides (Table VI).After that calculate R_i, i=1, 2, 3…6, as a union of logical relationship in each group.

$$R_1 = A_1^T \times A_5$$

$$R_2 = A_2^T \times A_4 \cup A_2^T \times A_5$$

$$\dots \dots \dots \dots \dots \dots \dots \dots \dots$$

$$\dots \dots \dots \dots \dots \dots \dots \dots \dots$$

$$\dots \dots \dots \dots \dots \dots \dots \dots \dots$$

$$R_6 = A_6^T \times A_2 \cup A_6^T \times A_2 \cup A_6^T \times A_6$$

Table 3.7. Fuzzification of Time Series Data of Rainfall

Blocks	Actual	A_1	A_2	A_3	A_4	A_5	A_6	Fuzzified
Andanallur	1085.23	0	0	0.2	0.5	0.8	1	A_6
Lalgudi	1136.79	0	0	0	0.4	0.7	1	A_6
Manachanallur	569.44	0	1	0.5	0.2	0	0	A_2
Manapparai	837.43	0	0.3	0.7	1	0.5	0.2	A_4
Manikandam	379.79	1	0.5	0.5	0	0	0	A_1
Marungapuri	942.16	0	0	0.3	0.5	1	0.2	A_5
Musiri	794.31	0	0.2	0.7	1	0.5	0	A_4
Pullambadi	1163.59	0	0	0.3	0.5	0.8	1	A_6
Thathaingarpet	710.16	0	0.3	0.7	1	0.5	0	A_4
Thiruverumbur	960.9	0	0	0.2	0.5	1	0.5	A_5
Thottium	866.73	0	0.2	0.7	1	0.5	0.3	A_4
Thurayur	942.11	0	0	0.4	0.5	1	0.5	A_5
Uppiliyapurm	473.35	0	1	0.5	0.2	0	0	A_2

Vaiyampaty	922.27	0	0.2	0.4	05	1	0.3	A_5

Table 3.8. Forecasted Outputs and Enrollments from 2004 to 2010

No.	Blocks	Actual	Fuzzy Outputs					
1.	Andanallur	1085.23	0.5	1	0.5	1	0.5	1
2.	Lalgudi	1136.79	0.5	1	0.5	1	0.5	1
3.	Manachanallur	569.44	0.5	0.5	0.5	1	1	0.5
4.	Manapparai	837.43	1	1	0.5	0.5	1	1
5.	Manikandam	379.79	0.5	0.5	0.5	0.5	0.5	0.5
6.	Marungapuri	942.16	0.5	0.5	0.5	1	0.5	0.5
7.	Musiri	794.31	1	1	0.5	0.5	1	1
8.	Pullambadi	1163.59	0.5	1	0.5	1	0.5	1
9.	Thathaingarpet	710.16	1	1	0.5	0.5	1	1
10.	Thiruverumbur	960.9	0.5	1	0.5	1	0.5	0.5
11.	Thottium	866.73	1	1	0.5	0.5	1	1
12.	Thurayur	942.11	0.5	1	0.5	1	0.5	0.5
13.	Uppiliyapurm	473.35	0.5	0.5	0.5	1	1	0.5
14.	Vaiyampaty	922.27	0.5	1	0.5	1	0.5	0.5

The prediction rainfall obtained by the four models have been place in the following table and presented in figure1.

3.2.7. Defuzzification of the fuzzy output for crisp predicting (step7)

Defuzzification is the process by which fuzzy output of model is transformed to crisp values for getting the predicted values. The output of each of the four methods are defuzzified in the following ways.

Table 3.9. Rainfall Forecast in Different Methods

Blocks	Actual	Mid. value of 16th Interval (Mehod1)	Max. value of 9th Interval (Method2)	Tsaur et al. (Method 3)	Chen (Method 4)
Andanallur	1085.23	1125	1100	1050	1150
Lalgudi	1136.79	1125	1200	1150	550
Manachanallur	569.44	525	600	550	850
Manapparai	837.43	825	900	850	950
Manikandam	379.79	375	400	400	683.30
Marungapuri	942.16	975	1000	950	1150
Musiri	794.31	825	800	800	1050
Pullambadi	1163.59	1125	1200	1150	950
Thathaingarpet	710.16	825	800	750	683.30
Thiruverumbur	960.9	975	1000	950	662.10
Thottium	866.73	825	900	850	950
Thurayur	942.11	975	950	950	450

Uppiliyapurm	473.35	525	500	500	950
Vaiyampaty	922.27	975	950	950	950

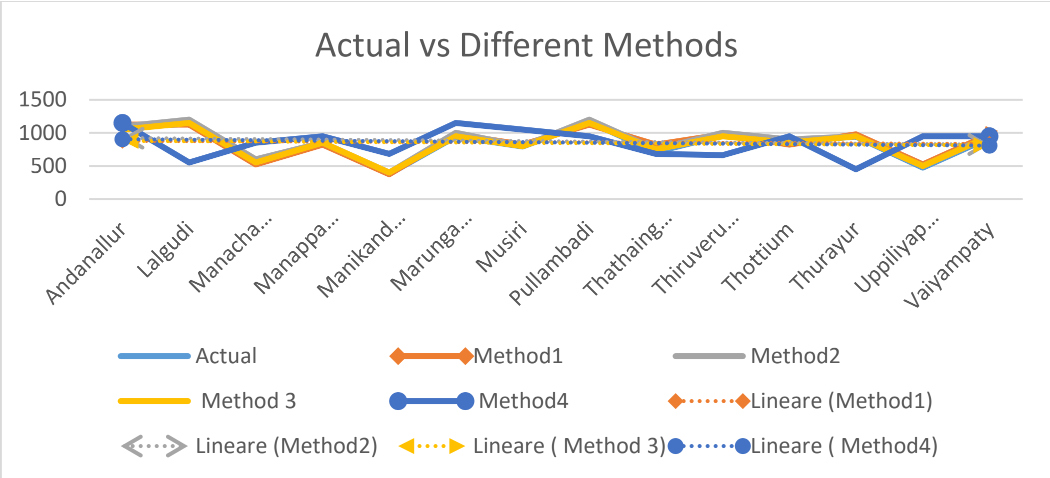

Figure 3.1. Graph of Actual vs. Different methods

Table 3.10. Rainfall Forecast in Different Methods of Trend equations with r^2 value

Methods	**Trend Equations**	**R^2 value**
Actual	$y = -7.2052x + 895.77$	0.0161
Method1	$y = -3.6264x + 884.34$	0.0041
Method2	$y = -7.6923x + 929.12$	0.0175
Method 3	$y = -5.1648x + 885.16$	0.0088
Method4	$y = -7.4859x + 911.77$	0.0209

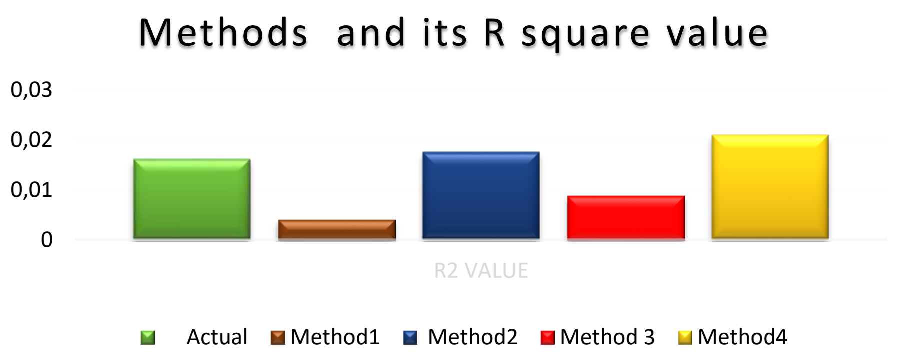

Figure 3.2. Graph of Methods and its R square value

4. Conclusions

In this paper, we have proposed a new method (Method1) for handling forecasting problems based on fuzzy time series, where the data of historical enrollments of the Trichy district rainfall data are adopted to illustrate the forecasting process. We have also shown that the method1 is more efficient than another Method. Furthermore, the linear trend line and R^2 value (Method 1) is smaller than another ones. The forecasting ability is better than traditional forecasting methods of Chen and tsaur et al., in the area of forecasting average rainfall. It has been successfully implemented to the forecasting of the annual average rainfall of a district of Trichy.

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