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Minimum Measurement Uncertainty in Quantum Systems Subject to High Energy Fluctuations

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Abstract

The stochastic generalization of the Madelung quantum hydrodynamics incorporates the fluctuations of the mass density into quantum equations induced by the gravitational background noise, a form of dark energy,. This model successfully addresses key aspects of quantum to classical transition through the definition of quantum potential length of interaction in addition to the De Broglie length, beyond which coherent quantum behavior and wavefunction evolution is perturbed or even deeply modified. The stochastic quantum hydrodynamic model emphasizes that an external environment is unnecessary, asserting that the stochastic behavior leading to wave-function collapse and macroscopic classical behavior with the Einstein realism can be an inherent property of physics in a spacetime with fluctuating metrics. The theory establishes a coherent link between the uncertainty principle and the constancy of light speed, aligning seamlessly with finite information transmission speed. Within quantum mechanics submitted to fluctuations, the stochastic quantum hydrodynamic model derives the indeterminacy relation between energy and time, offering insights into measurement processes impossible within a finite time interval in a truly quantum global system. The model offers also how the uncertainty relations modify themselves in an open quantum system submitted to fluctuations offering the possibility of describing how quantum mechanics can modify itself in nuclear and elementary particle physics up to the behavior of high energy black hole states. The self-consistency of the model lies in its ability to describe the dynamics of wavefunction collapse and the measure process within its mathematical structure. Additionally, the theory resolves the Einstein determinism and the pre-existing reality problem by showing that large-scale systems naturally self-decay into decoherent classical states stable in time without observer or measuring apparatus.

Keywords: quantum to classical transition, Einstein determinism, pre-measure reality, uncertainty relations in open quantum system, hidden variable, IsoRedShift Mechanics.

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1. Introduction

Any quantum theory that seeks to explain the development of a physical system across different scales, regardless of magnitude, must inherently deal with the shift from quantum mechanical characteristics to the classical behavior that emerges at larger scales. The essential differences between these two frameworks are highlighted by the uncertainty principle in quantum mechanics, which indicates the intrinsic impossibility of simultaneously measuring conjugate variables, and by the finite speed at which interactions and information propagate in local classical relativistic mechanics.

Several unresolved aspects persist within quantum theories. The Einstein–Podolsky–Rosen (EPR) paradox [1] and the question of the completeness of quantum mechanics are fundamental issues that have yet to be clearly integrated into a comprehensive theoretical framework.

A significant challenge to this view was posed by the Einstein–Podolsky–Rosen objection, which was subsequently criticized by Bohr [2]. Unable to find a compatible solution, Bohr advocated for the acceptance of the probabilistic foundations of quantum mechanics, regardless of the validity of the principle of causality.

In fact, Bohr's perspective significantly influenced subsequent scientific thought. The idea that quantum mechanics functions as a probabilistic process was revitalized by Nelson's research and has persisted over time. However, Nelson's hypotheses were ultimately limited [3] by the requirement of a specific stochastic derivative with time inversion symmetry, which restricted their general applicability. Additionally, Nelson's theory does not completely match the results of quantum mechanics, as shown by Von Neumann's proof [4-5] of the impossibility of replicating quantum mechanics using theories based on underlying classical probabilistic variables.

This approach was widely explored in local hidden variable theories attempting to replicate quantum mechanics, whose failure was demonstrated by Bell's theorem [6]. To address Bell's arguments, Bohm developed a non-local hidden variable theory that seeks to restore determinism in quantum mechanics by introducing the concept of a pilot wave [7]. The core idea is that, in addition to the particles, there is a "guidance" or influence from the pilot wave function that determines the particles' behavior. Although this pilot wave function is not directly observable, it affects the measurement probabilities of the particles.

The ambiguous definition and unclear logical foundations of the pilot wave have been significant obstacles to the broader acceptance of Bohm's theory among physicists. Nonetheless, Santilli has recently renewed interest in and the potential validity of the Bohm hidden variable hypothesis with his IsoRedShift Mechanics theory. This theory introduces a hidden variable to explain deviations in nuclear quantum mechanics [8] in the deuterium state. It is important to note that Santilli's hidden variable is integrated into quantum non-local theory rather than classical theory. Through this approach, Santilli has demonstrated that it is possible to recover Einstein's determinism [9-11].

More recently, the author [12], utilizing the Madelung hydrodynamic formulation of quantum mechanics—a specific case of Bohm's theory [13]—has demonstrated that by considering the fluctuating gravitational background due to relic metric tensor oscillations from the Big Bang and the dynamics of general relativity bodies, one can achieve a quantum-fluctuating dynamics without needing to introduce an external environment. From this standpoint, the derived stochastic quantum hydrodynamic model [12] (SQHM) highlights that conventional quantum theory, despite its mathematical precision, remains incomplete regarding its foundational postulates. Specifically, SQHM emphasizes that the measurement process cannot be fully explained within the deterministic "Hamiltonian" framework of quantum mechanics. Instead, it is fully described as a phenomenon within the framework of a quantum stochastic generalized approach.

The SQHM reveals that quantum mechanics represents the zero thermal noise limit of a broader stochastic theory capable of describing both wavefunction decay and the measurement process, thus achieving a comprehensive, self-consistent quantum theory. From this perspective, SQHM suggests that the term "probabilistic wave" in quantum mechanics is inaccurately introduced, stemming more from the inherent probabilistic nature of the measurement process than from its mathematical formulation. This approach acknowledges a curved spacetime characterized by stochastic fluctuations in both curvature and geodesic distance. Both SQHM and Bohm's theory restore the principle of determinism in quantum theory, leading to the wavefunction being understood as a "state wave" and explaining the probabilistic outcomes as resulting from an associated stochastic phenomenon (namely the GBN or the pilot wave, respectively).

The SQHM suggests that quantum mechanics serves as the deterministic boundary of a broader theory encompassing quantum stochasticity. The SQHM illustrates that in an ideal quantum deterministic universe, complete decoupling of its components—namely the system and the measuring apparatus—is unattainable. Rather, this scenario is only achievable within a self-fluctuating quantum system, where the interaction of the quantum potential, responsible for quantum entanglement, extends over a finite distance, thereby allowing for a macroscopic classical behavior on a large scale.

Therefore, in a hypothetical quantum universe devoid of fluctuating spacetime, measuring procedures are impracticable. Conversely, in a macroscopic classical realm formed through quantum decoherence triggered by fluctuating gravitational forces, measurement becomes viable.

While Bohm's theory attributes measurement uncertainty to an indeterminable pilot wave, the SQHM attributes it to fluctuating gravitational dark energy, which, due to its elusive nature, practically assumes the form of a genuine hidden variable. In this context, the author demonstrates that the hidden variable parameter of Santilli's IsoRedShift Mechanics arises from minor disturbances to quantum mechanics induced by fluctuating gravitational dark energy [14].

Moreover, it's important to note that the SQHM solves the enduring query concerning the existence of pre-measurement reality, thus bolstering Einsteinian realism. At this point, the SQHM introduces a nuanced advancement: the concept that the universe can undergo self-decay via macroscopic-scale decoherence, leading to the formation of stable macroscopic states. These states, remaining stable despite fluctuations, establish a continuous reality that predates measurement. Conversely, the Copenhagen interpretation contends that the system only transitions into a stable state through the act of measurement, thereby establishing a continuous reality over time. Consequently, within this paradigm, it remains uncertain whether a continuous reality prevails prior to measurement.

2. The Stochastic Madelung Approach

The Madelung quantum hydrodynamic representation transforms the Schrodinger equation [15-17]

$$-i\hbar\partial_t\psi = \left(\frac{\hbar^2}{2m}\partial_i\partial_i - V_{(q)}\right)\psi \tag{1}$$

for the complex wave function $\psi = |\psi| e^{-\frac{iS}{\hbar}}$, into two equations of the real variables, $|\psi|^2$ and $p_i = \partial_i S_{(q,t)}$: the conservation equation for the mass density $|\psi|^2$

$$\partial_t |\psi|^2 + \partial_i(|\psi|^2 \dot{q}_i) = 0 \tag{2}$$

and the motion equation for the momentum,

$$\frac{\dot{p}}{m} = \ddot{q}_{j(t)} = -\frac{1}{m}\partial_j\left(V_{(q)} + V_{qu(n)}\right) \tag{3}$$

where $S_{(q,t)} = -\frac{\hbar}{2} ln \frac{\psi}{\psi^*}$ and where

$$V_{qu} = -\frac{\hbar^2}{2m}\frac{1}{|\psi|}\partial_i\partial_i |\psi|. \tag{4}$$

These equations were derived by Madelung in a flat space. However, in the general relativity framework, the fluctuating energy content of gravitational background noise (GBN) causes fluctuations in the metric tensor, leading to local stochastic variations in mass density. These mass density fluctuations can be explained by recognizing that gravitons are metric fluctuations that cause space itself to oscillate. This oscillation contracts and elongates distances, similar to the effects observed with gravitational waves. As a result, when a mass element experiences a change in distance, its density decreases or increases accordingly. As demonstrated below, this causes the quantum potential to generate a stochastic force, thereby expanding the Madelung hydrodynamic analogy into a quantum-stochastic framework.. As shown in [12] the stochastic quantum hydrodynamic model (SQHM) is defined on the following assumptions:

1. The additional mass density generated by the GBN are described by the wave function ψ_{gbn} with density $|\psi_{gbn}|^2$;

2. The associated energy density E of GBN is proportional to $|\psi_{gbn}|^2$;

3. The additional mass m_{gbn} is defined by the identity $E = m_{gbn}c^2 |\psi_{gbn}|^2$

4. The additional mass is assumed to not interact with the mass of the physical system (since the gravitational interaction is sufficiently weak to be disregarded).

Under this assumption, the wave function of the overall system ψ_{tot} reads

$$\psi_{tot} \cong \psi\psi_{gbn} \tag{5}$$

Moreover, given that the energy density of the GBN E is quite small, the mass density $m_{gbn}|\psi_{gbn}|^2$ is presumed to be significantly smaller than the body mass density $m|\psi|^2$ typically encountered in physical problems . Hence, considering the mass m_{gbn} much smaller than the mass of the system and assuming in Equation (3-4) $m_{tot} = m_{gbn} + m \cong m$, the overall quantum potential can be expressed as follows

$$\begin{aligned} V_{qu(n_{tot})} &= -\frac{\hbar^2}{2m_{tot}} |\psi|^{-1}|\psi_{gbn}|^{-1} \partial_i\partial_i |\psi||\psi_{gbn}| = \\ &= -\frac{\hbar^2}{2m}\begin{pmatrix} |\psi|^{-1}\partial_i\partial_i|\psi| + |\psi_{gbn}|^{-1}\partial_i\partial_i|\psi_{gbn}| \\ +2|\psi|^{-1}|\psi_{gbn}|^{-1}\partial_i|\psi_{gbn}|\partial_i|\psi| \end{pmatrix} \end{aligned} . \tag{6}$$

By analyzing the variation in quantum potential energy generated by each Fourier component of mass fluctuation and applying the Maxwell-Boltzmann law, we can derive the spectrum of this noise and subsequently determine its correlation function. $G(\lambda)$.

To accomplish this, let's examine the fluctuating mass density with a wavelength λ

$$|\psi_{gbn}|^2_{(\lambda)} \propto cos^2 \frac{2\pi}{\lambda} q \tag{7}$$

related to the wave function of the mass fluctuation

$$\psi_{gbn(\lambda)} \propto \pm cos \frac{2\pi}{\lambda} q \, . \tag{8}$$

By inserting (7-8) in (6), the energy fluctuations of the quantum potential

$$\delta\bar{E}_{qu} = \int_V n_{tot(q,t)} \delta V_{qu(q,t)} dV \, , \tag{9}$$

following the procedure in reference [12], reads:

$$\delta \overline{E}_{qu(\lambda)} \cong \frac{\hbar^2}{2m}\left(\frac{2\pi}{\lambda}\right)^2 \tag{10}$$

that, in 3D space, read

$$\delta \overline{E}_{qu(\lambda)} \cong \frac{\hbar^2}{2m}\sum_i (k_i)^2 = \frac{\hbar^2}{2m}|k|^2 \tag{11}$$

The result shown in Equation (11) indicates that the energy arising from fluctuations in mass density increases inversely with the square of the wavelength λ. Moreover, since fluctuations in the quantum potential with extremely short wavelengths for $\lambda \to 0$ diverge they can lead to a finite contribution even when the noise amplitudes approach zero (i.e., $T \to 0$). This situation raises concerns regarding the achievement of the deterministic zero noise limit (2-4) that represents quantum mechanics.

Moreover, since fluctuations in the quantum potential with extremely short wavelengths diverge, they can result in a finite contribution even when the noise amplitudes approach zero (i.e., $T \to 0$). This situation raises concerns about reaching the deterministic zero noise limit that characterizes quantum mechanics.

This happens because the quantum potential's output, due to its second derivative, depends on the correlation distance of the noise. Therefore, to nullify the fluctuations in the quantum potential and achieve convergence to the deterministic limit of quantum mechanics for $T \to 0$, a condition on the correlation function $G(\lambda)$ of the quantum potential noise must be established for $\lambda \to 0$ [18].

Deriving the shape of the correlation function $G(\lambda)$ involves complex stochastic calculations [18], which can be obtained by considering the probability of uncorrelated fluctuations occurring at increasingly shorter distances.

A simpler and more direct method for calculating $G(\lambda)$ is through the noise spectrum, represented as $S(k)$, as outlined in [12].

$$S_{(k)} \propto probability_{(k=\frac{2\pi}{\lambda})} = exp\left[-\frac{\delta \overline{E}_{qu(\lambda)}}{kT}\right] = exp\left[-\left(\frac{k\lambda_c}{2}\right)^2\right] \tag{12}$$

In Equation (12), k represents the Boltzmann constant, and T stands for the temperature (mean amplitude) of mass density fluctuations. It's important to highlight that Equation (12) displays a non-white characteristic, where the probability of wavelengths λ smaller than λ_c quickly diminishes toward zero. From (12), $G(\lambda)$ reads [12]:

$$G_{(\lambda)} \propto \int_{-\infty}^{+\infty} exp[ik\lambda] S_{(k)} dk \propto \int_{-\infty}^{+\infty} exp[ik\lambda]\, exp\left[-\left(k\frac{\lambda_c}{2}\right)^2\right] dk \\ \propto \frac{\pi^{1/2}}{\lambda_c} exp\left[-\left(\frac{\lambda}{\lambda_c}\right)^2\right] . \quad (13)$$

where

$$\lambda_c = \sqrt{2}\frac{\hbar}{(mkT)^{1/2}} . \quad (14)$$

At least for $\sqrt{2}$, λ_c represents the De Broglie length. The expression (12) reveals that uncorrelated mass density fluctuations are suppressed on increasingly shorter distances than λ_c.As a result, we reveal a novel function of the quantum potential within an open system: progressively damping fluctuations (due to their rapidly escalating energy) on a microscopic level. This clarifies the empirical observation that quantum mechanics governs the micro-scale.

This probability-mediated suppression enables the proposition of conventional quantum mechanics as the zero-noise 'deterministic' limit of the SQHM). Moreover, since this phenomenon pertains to systems with a physical length substantially smaller than the De Broglie length λ_c, directly extending quantum mechanical behavior to macroscopic large-scale scenarios is not feasible at $T > 0$. This is because, within the framework of SQHM, the De Broglie length λ_c disrupts scale invariance.

In the presence mass density fluctuations, $|\psi|^2$ becomes a stochastic function we name as $\tilde{n}$, where $lim_{T\to 0}\,\tilde{n} \to |\psi|^2$. On this, we can conceptually separate $\tilde{n}$ into two parts: $\tilde{n} = \overline{n} + \delta n$, where δn is the fluctuating part and $\overline{n}$ is the regular part obeying to the condition $lim_{T\to 0}\,\tilde{n} = lim_{T\to 0}\,\overline{n} = |\psi|^2$.

Moreover, the features of the Madelung quantum potential in the presence of stochastic noise, can be determined by positing it as comprising a regular component $\overline{V_{qu(\tilde{n})}}$ (to be defined) along with a fluctuating component V_{st}, such as

$$V_{qu(\tilde{n})} = -\frac{\hbar^2}{2m}\tilde{n}^{-1/2}\partial_i\partial_i\tilde{n}^{1/2} = \overline{V_{qu(\tilde{n})}} + V_{st} \quad (15)$$

where the stochastic part of the quantum potential V_{st} results in force noise

$$-\partial_i V_{st} = m\varpi_{(q,t,T)} \quad (16)$$

leading to the stochastic motion equation

$$\ddot{q}_{j(t)} = -\frac{1}{m}\partial_j\left(V_{(q)} + \overline{V_{qu(\tilde{n})}}\right) + \varpi_{(q,t,T)}. \tag{17}$$

Moreover, the regular part $\overline{V_{qu(\tilde{n})}}$ for microscopic systems ($\frac{\mathcal{L}}{\lambda_c} \ll 1$), without loss of generality, can be reorganized as

$$\overline{V_{qu(\tilde{n})}} = -\frac{\hbar^2}{2m}\overline{\left(\tilde{n}^{-1/2}\partial_i\partial_i\tilde{n}^{1/2}\right)} = -\frac{\hbar^2}{2m}\frac{1}{\rho^{1/2}}\partial_i\partial_i\rho^{1/2} + \overline{\Delta V} = V_{qu(\rho)} + \overline{\Delta V} \tag{18}$$

leading to the motion equation

$$\ddot{q}_{j(t)} = -\frac{1}{m}\partial_j\left(V_{(q)} + V_{qu(\rho)} + \overline{\Delta V}\right) + \varpi_{(q,t,T)} \tag{19}$$

where $\rho_{(q,t)}$ represents the probability mass density function (PMD) associated with the stochastic process (17) [19], obeying to the condition $lim_{T\to 0}\,\rho_{(q,t)} = lim_{T\to 0}\,\tilde{n} = lim_{T\to 0}\,\overline{n} = |\psi|^2$.

For the sufficiently general case to be practically relevant, it can be assumed that the correlation function of $\varpi_{(q,t,T)}$ possesses zero correlation time, is isotropic in space, and is independent among different coordinates, taking the form

$$\lim_{T\to 0} < \varpi_{(q_\alpha,t)}, \varpi_{(q_\beta+\lambda,t+\tau)} > \cong < \varpi_{(q_\alpha)}, \varpi_{(q_\beta)} >_{(T)} \; G(\lambda)\delta(\tau)\delta_{\alpha\beta} \tag{20}$$

with

$$\lim_{T\to 0} < \varpi_{(q_\alpha)}, \varpi_{(q_\beta)} >_{(T)} = 0 \quad . \tag{21}$$

Furthermore, given that for microscopic systems ($\frac{\mathcal{L}}{\lambda_c} \ll 1$) it follows that

$$lim_{T\to 0}\, G_{(\lambda)} \propto \frac{1}{\lambda_c} = \frac{1}{\hbar}\sqrt{\frac{mkT}{2}} \tag{22}$$

and that

$$\lim_{T\to 0} < \varpi_{(q_\alpha,t)}, \varpi_{(q_\beta+\lambda,t+\tau)} > \approx \; < \varpi_{(q_\alpha)}, \varpi_{(q_\beta)} >_{(T)} \frac{1}{\hbar}\sqrt{\frac{mkT}{2}}\delta(\tau)\delta_{\alpha\beta}. \tag{23}$$

Therefore, equation (17) takes the stochastic form of the Markov process [19]

$$\ddot{q}_{j(t)} = -\kappa\dot{q}_{j(t)} - \frac{1}{m}\frac{\partial\left(V_{(q)} + V_{qu(\rho)}\right)}{\partial q_j} + \kappa D^{1/2}\xi_{(t)}. \tag{24}$$

where

$$D^{1/2} = \left(\frac{\pounds}{\lambda_c}\right)\left(\gamma_D \frac{\hbar}{2m}\right)^{1/2} = \sqrt{\gamma_D}\,\frac{\pounds}{2}\sqrt{\frac{kT}{2\hbar}} \qquad (25)$$

where γ_D is a non-zero pure number.

In this case, $\rho_{(q,t)}$ is the probability mass density determined by the probability transition function (PTF) $P(q,z\,|\,t,0)$ of the Markov process (24) [19] through the relation $\rho(q,t) = \int P(q,z\,|\,t,0)\rho_{(z,0)} d^{6N}z$ where $P(q,z\,|\,t,0)$ obeys to the Smoluchowski conservation equation [19]

$$P_{(q,q_0|t+\tau,t_0)} = \int_{-\infty}^{\infty} P_{(q,z|\tau,t)}\, P_{(z,q_0|t-t_0,t_0)}\, d^{6N}z\,. \qquad (26)$$

Finally, in summary, for the complex field

$$\boldsymbol{\psi} = \rho^{1/2} e^{-\frac{iS}{\hbar}} \qquad (27)$$

the SQHM system of equation reads:

$$\rho(q,t) = \int P(q,z\,|\,t,0)\rho_{(z,0)} d^r z \qquad (28)$$

$$m\dot{q}_i = p_i = \partial_i \boldsymbol{S}_{(q,t)}\,, \qquad (29)$$

$$\ddot{q}_{j(t)} = -\kappa \dot{q}_{j(t)} - \frac{1}{m}\frac{\partial\left(V_{(q)} + V_{qu(\rho)}\right)}{\partial q_j} + \kappa D^{1/2}\xi_{(t)} \qquad (30)$$

$$V_{qu} = -\frac{\hbar^2}{2m}\frac{1}{\rho^{1/2}}\partial_i\partial_i\rho^{1/2}\,. \qquad (31)$$

where

$$\boldsymbol{S}_{(q,t)} = -\frac{\hbar}{2} ln \frac{\boldsymbol{\psi}}{\boldsymbol{\psi}^*}\,. \qquad (32)$$

3. Emerging classical behavior on large size systems

When attempting to nullify the quantum potential manually in the equations of motion for quantum hydrodynamics (2-4), the classical equation of motion emerges. However, despite the apparent validity of this assertion, such a manipulation lacks mathematical rigor as it alters the fundamental characteristics of the quantum hydrodynamic equations. Specifically, this process results in the removal of stationary configurations, such as eigenstates, as the counteracting force of the quantum potential against the Hamiltonian force —which establishes their stationary mass density distribution condition—is eliminated. Consequently, even a minor quantum potential is not disregarded in conventional quantum mechanics, as depicted by the zero-noise "deterministic" limit of SQHM (2-4).

Conversely, in the stochastic generalization, it is feasible to appropriately disregard the quantum potential in (3,24) when its force is significantly smaller than the force noise, such as $|\frac{1}{m}\partial_i V_{qu(\rho)}| \ll |\varpi_{(q,t,T)}|$ that by (25) leads to condition

$$|\frac{1}{m}\partial_i V_{qu(\rho)}| \ll \kappa\left(\frac{\mathcal{L}}{\lambda_c}\right)\left(\gamma_D\frac{\hbar}{2m}\right)^{1/2} = \kappa\left(\frac{\mathcal{L}\sqrt{mkT}}{2\hbar}\right)\left(\gamma_D\frac{\hbar}{2m}\right)^{1/2}, \tag{38}$$

and hence, in a coarse-grained description with elemental cell side Δq, to

$$lim_{q\to\Delta q}\left|\partial_i V_{qu(\rho)}\right| \ll m\kappa\sqrt{\gamma_D}\frac{\mathcal{L}}{2}\sqrt{\frac{kT}{2\hbar}}, \tag{39}$$

where $\mathcal{L}$ is the physical length of the system.

It's important to note that, although the noise $\varpi_{(q,t,T)}$ has a zero mean, the mean of the fluctuations in the quantum potential, denoted as $\bar{V}_{st(n,S)} \cong \kappa S$, is not null. This non-null mean gives rise to the dissipative force $-\kappa\dot{q}_{(t)}$ in equation (24). As a result, the stochastic sequence of noise inputs disrupts the coherent evolution of the quantum superposition of states, leading them to decay to a stationary mass density distribution with $\dot{q}_{(t)} = 0$.

Furthermore, considering that the stochastic noise

$$\kappa\left(\frac{\mathcal{L}}{\lambda_c}\right)\left(\gamma_D\frac{\hbar}{2m}\right)^{1/2} \cong \varpi \tag{40}$$

grows with the size of the system, for macroscopic systems (i.e., $\frac{\mathcal{L}}{\lambda_c}\to\infty$), condition (38) is satisfied if

$$lim_{\frac{q}{\lambda_c}\to\infty\left(\frac{\mathcal{L}}{\lambda_c}=\infty\right)}\left|\frac{1}{m}\partial_i V_{qu(n_{(q)})}\right| < \infty \ . \tag{41}$$

Furthermore, for achieving a comprehensive depiction without quantum correlations in any large-scale system of physical length $\mathcal{L}$, a more stringent criterion can be imposed, such as

$$lim_{\frac{q}{\lambda_c}\to\infty}\left|\frac{1}{m}\partial_i V_{qu(\rho_{(q)})}\right| = lim_{\frac{q}{\lambda_c}\to\infty}\frac{1}{m}\sqrt{\partial_i V_{qu(\rho_{(q)})}\partial_i V_{qu(\rho_{(q)})}} = 0 \, . \tag{42}$$

Therefore, since for linear systems

$$lim_{q\to\infty} V_{qu(q)} \propto q^2 , \tag{43}$$

from the standpoint of SQHM promptly follows that they do not lead to the macroscopic classical phase.

Generally speaking, as the Hamiltonian potential strengthens, the wave function localization increases, and the quantum potential behavior becomes stronger.

This comes out clearly by considering the MDD

$$|\psi|^2 \propto exp\left[-P^k_{(q)}\right] , \tag{44}$$

where $P^k{}_{(q)}$ is a polynomial of order *k*, since a finite range of quantum potential interaction is achieved for $k < \frac{3}{2}$.

Linear systems, as well as the ballistic Gaussian coherent states, characterized by $k = 2$, have the quantum potential that is relevant even at infinite distance.

Conversely, for gas phases where particles interact via the Lennard-Jones potential, whose long-distance wave function reads [20]

$$lim_{r\to\infty}|\psi| \propto a^{-1/2}\frac{1}{r} , \tag{45}$$

the quantum potential reads

$$lim_{r\to\infty} V_{qu(\rho)} \cong lim_{q\to\infty}\frac{\hbar^2}{2m}\frac{1}{|\psi|}\partial_r\partial_r\,|\psi| = \frac{1}{r^2} = \frac{\hbar^2}{m}a\,|\psi|^2 \tag{46}$$

leading to the quantum force

$$lim_{r\to\infty} - \partial_r V_{qu(\rho)} = lim_{q\to\infty}\frac{\hbar^2}{2m}\partial_r\frac{1}{|\psi|}\partial_r\partial_r\,|\psi| = \frac{\hbar^2}{2m}\partial_r r \partial_r\partial_r\frac{1}{r} = -2\frac{\hbar^2}{m}\frac{1}{r^3} = 0 \, , \tag{47}$$

that satisfying conditions (38, 42), can lead to large-scale classical behavior [12] in a sufficiently rarefied phase.

It's worth mentioning that in equation (46), the output of the quantum potential corresponds to the hard sphere potential within the "pseudo-potential Hamiltonian model" of the Gross-Pitaevskij equation [21-22], where $\frac{a}{4\pi}$ represents the boson-boson s-wave scattering length.

By observing that, to satisfy (42), it is sufficient to have that

$$\int_0^\infty r^{-1} \left| \frac{1}{m} \partial_i V_{qu(\rho(q))} \right|_{(r,\theta,\varphi)} dr < \infty \qquad \forall \theta,\varphi \quad , \tag{48}$$

the finite value of the range of interaction of the quantum potential λ_{qu} such as [12]

$$\lambda_{qu} = \lambda_c \frac{\int_0^\infty r^{-1} \left| \partial_i V_{qu(\rho(q))} \right|_{(r,\theta,\varphi)} dr}{\left| \partial_i V_{qu(\rho(q))} \right|_{(r=\lambda_c,\theta,\varphi)}} = \lambda_c I_{qu} \quad . \qquad I_{qu} > 1$$

(49)

warrants the existence of classical macroscopic phase.

Relation (49) provides a measure of the physical length associated with quantum non-local interactions.

It's worth noting that the quantum non-local interactions extend themselves up to distances on the order of the largest length between λ_{qu} and λ_c. Below λ_c, an even weaker quantum potential arises due to noise damping, while above it but below λ_{qu}, the quantum potential becomes strong enough to surpass the fluctuations, establishing a quantum-stochastic behavior

The reach of quantum non-local effects can be expanded by increasing λ_c, achievable by reducing the temperature or mass of the bodies (refer to equation (14)), or by enhancing λ_{qu} through a more potent Hamiltonian potential. In the latter scenario, for example, augmenting the linear range of Hamiltonian interaction between particles.

4. Maximum Precision in Measure of Conjugated Mechanical Variable in a Quantum System Subject to fluctuations

During the measurement process, the interaction concludes when the measuring apparatus is relocated to a significant distance from the system. Within the SQHM framework, this relocation is crucial and must exceed specified distances λ_c and λ_{qu}. After this relocation, the measuring apparatus is responsible for interpreting and managing the "interaction output." This typically involves a classical, irreversible process characterized by a distinct temporal progression, culminating in the determination of the macroscopic measurement result. As a result, the phenomena of decoherence and quantum decoupling play a crucial role in the measurement process. They enable the establishment of a large-scale classical framework, ensuring genuine quantum isolation between the measuring apparatus and the system, both before and after the measurement event. This quantum-isolated state, at both the initial and final stages, is of paramount importance in determining the temporal duration of the measurement and in collecting statistical data through a series of independent repeated measurements.

It's important to emphasize that, within the framework of SQHM, simply moving the measured system to an infinite distance before and after the measurement, as commonly practiced, does not fully ensure the independence of the system and the measuring apparatus if either condition $\lambda_c = \infty$ or $\lambda_q = \infty$ is met. Therefore, the existence of a macroscopic classical reality remains essential for carrying out the measurement process.

Since the quantum potential suppresses fluctuations with wavelengths smaller than the De Broglie length λ_c, any system fully adheres to conventional quantum mechanics within a physical length $L_q < L_c$, where subparts lack individual identities. Based on this, the independent observer (experimental apparatus), seeking information about the system, must maintain a separation distance larger than L_q both before and after the process.

Therefore, due to the finite speed of propagation of interactions and information, the process cannot be executed in a time shorter than

$$\Delta\tau_{min} > \frac{L_q}{c} \propto \frac{\lambda_c}{c} \propto \frac{2\hbar}{(2mc^2kT)^{1/2}}. \tag{57}$$

Furthermore, considering the Gaussian noise (20) with the diffusion coefficient proportional to kT, we find that the mean value of energy fluctuation is $\delta E_{(T)} = \frac{kT}{2}$ for the degree of freedom. As a result, a scalar structureless particle, with mass $m >> \frac{kT}{c^2}$, exhibits an energy variance ΔE of

$$\Delta E \approx (<(mc^2 + \delta E_{(T)})^2 - (mc^2)^2 >)^{1/2} \cong (<(mc^2)^2 + 2mc^2 \delta E - (mc^2)^2 >)^{1/2}$$
$$\cong (2mc^2 < \delta E >)^{1/2} \cong (mc^2 kT)^{1/2} \quad (58)$$

from which it follows that

$$\Delta E \Delta t > \Delta E \Delta \tau_{min} \propto \frac{(mc^2 kT)^{1/2} \lambda_c}{c}) \propto \sqrt{2}\hbar , \quad (59)$$

It's worth mentioning that the product $\Delta E \Delta \tau$ remains constant, as the increase in energy variance with the square root of T precisely counterbalances the corresponding decrease in the minimum acquisition time τ . This result holds true when establishing the uncertainty relations between the position and momentum of a particle with mass m .

When we obtain information about the spatial position of a particle with precision ΔL , we essentially exclude the space beyond this distance from the quantum non-local interaction of the particle, thereby

$$L_q < \Delta L . \quad (60)$$

the variance Δp of its relativistic momentum $(p^\mu p_\mu)^{1/2} = mc$ reads

$$\Delta p \approx (<(mc + \frac{\delta E_{(T)}}{c})^2 - (mc)^2 >)^{1/2} \cong (<(mc)^2 + 2m\delta E - (mc)^2 >)^{1/2} \quad (61)$$
$$\cong (2m < \delta E >)^{1/2} \cong (mkT)^{1/2}$$

leading to the indeterminacy relation

$$\Delta L \Delta p > L_q (mkT)^{1/2} \propto \lambda_c (mkT)^{1/2}) \propto \sqrt{2}\hbar \quad (62)$$

Equating (62) to the uncertainty value, such as

$$\Delta L \Delta p > L_q (2mkT)^{1/2} = \frac{\hbar}{2} \quad (63)$$

or

$$\Delta E \Delta t > \Delta E \Delta \tau_{min} = \frac{(2mc^2 kT)^{1/2} L_q}{c} = \frac{\hbar}{2} , \quad (64)$$

It follows that $L_q = \frac{\lambda_c}{2\sqrt{2}}$ represents the physical length below which quantum entanglement is fully effective establishing the zero-thermal noise limit of the SQHM, specifically the realization of quantum mechanics.

As far as it concerns the *theoretical minimum uncertainty* of quantum mechanics, obtainable from the *minimum indeterminacy* (59, 62) in the limit of quantum mechanics ($T = 0$ and $\lambda_c \to \infty$) in the non-relativistic limit (), we have that

$$\Delta\tau_{min} = \frac{\lambda_c}{2c\sqrt{2}} \to \infty \qquad (65)$$

$$\Delta E \cong (mc^2kT)^{1/2} = \sqrt{2}\frac{\hbar c}{\lambda_c} \to 0\,, \qquad (66)$$

$$L_q = \frac{\lambda_c}{2\sqrt{2}} \to \infty \qquad (67)$$

$$\Delta p \cong (mkT)^{1/2} \cong \frac{\sqrt{2}\hbar}{\lambda_c} \to 0 \qquad (68)$$

and therefore that

$$\Delta E\Delta t > \Delta E\Delta\tau_{min} = \frac{\hbar}{2} \qquad (69)$$

$$\Delta L\Delta p > L_q(mkT)^{1/2} = \frac{\hbar}{2} \qquad (70)$$

That constitutes the *minimum uncertainty* in quantum mechanics, obtained as the zero noise-limit of the SQHM.

An important point to consider is that the SQHM broadens the uncertainty principles to encompass all paired characteristics of four-dimensional space-time. Unlike traditional quantum mechanics, where determining the uncertainty between energy and time is unattainable due to the lack of a defined time operator..

Furthermore, it is interesting to note that in the relativistic limit of quantum mechanics ($T = 0$ and $\lambda_c \to \infty$), by the finite speed of light, the minimum acquisition time of information in the quantum limit reads

$$\Delta\tau_{min} = \frac{L_q}{c} \to \infty\,. \qquad (71)$$

The outcome of equation (71) demonstrates that conducting a measurement within a completely deterministic quantum mechanical global system is impractical since it would require an infinite duration.

Considering that non-locality is confined to domains with physical lengths on the order of $\frac{\lambda_c}{2\sqrt{2}}$, and given that information about a quantum system cannot travel faster than the speed of light (otherwise violating the uncertainty principle), local realism holds true within the coarse-grained macroscopic physics, where domains of order of $\lambda_c{}^3$ reduce to a point.

The paradox of "spooky action at a distance" is limited to microscopic distances (smaller than $\frac{\lambda_c}{2\sqrt{2}}$), where quantum mechanics is described in the low-velocity limit, assuming $c \to \infty$ and $\lambda_c \to \infty$. This results in the seemingly instantaneous transmission of interaction over a distance.

It is also noteworthy that in the presence of noise, the measured indeterminacy undergoes a relativistic correction, as $\Delta E \cong (mc^2 kT \left(1 + \frac{kT}{4mc^2} \right))^{1/2}$, leading at $T > 0$:

$$\Delta E \Delta t > \frac{\hbar}{2} \left(1 + \frac{kT}{4mc^2} \right)^{1/2} \tag{72}$$

and

$$\Delta L \Delta p > \frac{\hbar}{2} \left(1 + \frac{kT}{4mc^2} \right)^{1/2} \tag{73}$$

This can become significant for light nuclear and elementary particles (with $m \to 0$) and at high temperature T .

5. Cosmological and high energy physics consequences

Within the framework of the SQHM, utilizing the measure indeterminacy relations and the maximum attainable velocity of the speed of light

$$\dot{x} \le c , \tag{74}$$

it follows that

$$\dot{x} \ge \Delta \dot{x} = \frac{\Delta p}{m} = \frac{\hbar}{2m\Delta x} , \tag{75}$$

leading to $\frac{\hbar}{2m\Delta x} \leq c$ and to

$$\Delta x > \frac{\hbar}{2mc} = \frac{R_c}{2} \ . \tag{76}$$

where R_c is the Compton's length.

Equation (76) unveils that the highest density of mass within a body is found within an elemental volume with a side length equivalent to half of its Compton wavelength.

This discovery carries profound implications for the formation of black holes (BHs). In order for a BH to form, all of the mass must be compressed within its gravitational radius R_g, leading to the following relationship:

$$R_g = \frac{2Gm}{c^2} > \frac{\Delta x}{2} = r_{\min} = \frac{R_c}{4}, \tag{77}$$

which further leads to the condition:

$$\frac{R_c}{4R_g} = \frac{\hbar}{8mcR_g} = \frac{\hbar c}{8m^2G} = \frac{{m_p}^2}{m^2} < 1 \tag{78}$$

showing that the BH mass

$$m > \sqrt{\frac{\hbar c}{8G}} = \sqrt{\pi} m_p \ . \tag{79}$$

where $m_p = \sqrt{\frac{\hbar c}{8\pi G}}$ is the reduced Planck's mass.

This holds true in a vacuum at zero background temperature, If we consider positive temperature as in (72-73) it follows that

$$\dot{x} \geq \Delta \dot{x} = \frac{\Delta p}{m} = \frac{\hbar}{2m\Delta x}\left(1 + \frac{kT}{4mc^2}\right)^{1/2} , \tag{80}$$

leads to $\frac{\hbar}{2m\Delta x}\left(1 + \frac{kT}{4mc^2}\right)^{1/2} \leq c$ and, consequently, to

$$\Delta x > \frac{\hbar}{2mc}\left(1 + \frac{kT}{4mc^2}\right)^{1/2} = \frac{R_c}{2}\left(1 + \frac{kT}{4mc^2}\right)^{1/2} \ . \tag{81}$$

which leads to the condition:

$$\frac{R_c}{4R_g}\left(1+\frac{kT}{4mc^2}\right)^{1/2} = \frac{\hbar}{8mcR_g}\left(1+\frac{kT}{4mc^2}\right)^{1/2}$$
$$= \frac{\hbar c}{8m^2G}\left(1+\frac{kT}{4mc^2}\right)^{1/2} = \frac{m_p{}^2}{m^2}\left(1+\frac{kT}{4mc^2}\right)^{1/2} < 1 \qquad (82)$$

with

$$m > \sqrt{\frac{\hbar c}{G}}\left(1+\frac{kT}{16mc^2}\right) = \sqrt{\pi}m_p\left(1+\frac{kT}{16mc^2}\right) \qquad (83)$$

Form (83) we can see that in presence of thermal fluctuations, the black hole of Planck mass becomes unstable and more mass is necessary to stabilize the BH.

If we consider black holes of mass hugely larger than m_p, considered that they are constituted by N sub-units (particles of mass m_i), we have that the energy fluctuations results $\delta E_{(T)} = 3N\frac{kT}{2}$ and relation (83) reads

$$m > \sqrt{\pi}m_p\left(1+\frac{NkT}{16mc^2}\right) \qquad (84)$$

Applying this result to the pre-big-bang BH (PBBH), we can derive an order of magnitude of the temperature necessary at the big bang to make thermally unstable the PBBH letting to expelling matter outside.

Assuming the universe energy $m_{PBBH}c^2 \approx 10^{53}$, the PBBH made of N particles becomes unstable if

$$\left(1+\frac{NkT}{16mc^2}\right) > \sqrt{\pi}\frac{m_p}{m} \qquad (85)$$

and therefore, for a temperature such as

$$\frac{NkT}{16m_{PBBH}c^2} = \frac{NkT}{16\times 10^{53}} > \frac{m_{PBBH}-m_p}{m_p} \approx \frac{m_{PBBH}}{m_p} \approx 4,6\times 10^{44} \qquad (86)$$

leading to the value

$$T > \frac{10^{120}}{N}\ {}^{\circ}\mathrm{K}. \qquad (87)$$

If we assume that the elementary particles in the Primordial Black Hole were black holes of Planck mass, it follows that

$$N = \frac{m_{PBBH}}{m_p} \approx 10^{45} \tag{88}$$

we obtain,

$$\frac{kT}{16 m_{PBBH} c^2} = \frac{kT}{16 \times 10^{53}} >\approx 1 \tag{89}$$

$$T_{big-bang} > 10^{77} \ °\mathrm{K} \tag{90}$$

while, if we assume that the particles weigh into the PBBH on the order of the elemental baryonic values, we have that:

$$T_{big-bang} > 10^{63} \ °\mathrm{K} \tag{91}$$

this is larger than the Planck temperature necessary to vaporize a black hole of small Planck mass.

Moreover, in the case of high energy fluctuations, the Marcovian term $\delta E_{(T)} = 3N\frac{kT}{2}$ has to be substituted by the more general expression $\delta E_{(T)} = \frac{3NkT}{2} + \sum_{i=1}^{\infty} \alpha_i \left(\frac{3NkT}{2}\right)^{i+1}$ that for particles with internal structure reads

$$\begin{aligned} \delta E_{(T)} &= f_{s_0} \frac{3NkT}{2} + \sum_{i=1}^{\infty} f_{s_i} \alpha_i \left(\frac{3NkT}{2}\right)^{i+1} \\ &= \sum_{i=0}^{\infty} \beta_i \left(\frac{3NkT}{2}\right)^{i+1} \cong= \sum_{i=0}^{h-1} \beta_i \left(\frac{3NkT}{2}\right)^{i+1} \end{aligned} \tag{92}$$

where f_{s_i} are structural factors. Therefore (72) becomes:

$$\Delta E \Delta t > \frac{\hbar}{2} \left(1 + \frac{3NkT}{4mc^2} \sum_{i=0}^{\infty} \beta_i \left(\frac{3NkT}{2}\right)^i \right)^{1/2} \tag{93}$$

and

$$\Delta L \Delta p > \frac{\hbar}{2} \left(1 + \frac{3NkT}{4mc^2} \sum_{i=0}^{\infty} \beta_i \left(\frac{3NkT}{2}\right)^i \right)^{1/2} \tag{94}$$

In this case it follows that

$$\dot{x} \geq \Delta \dot{x} = \frac{\Delta p}{m} = \frac{\hbar}{2m\Delta x}\left(1+\frac{3NkT}{4mc^2}\sum_{i=0}^{\infty}\beta_i\left(\frac{3NkT}{2}\right)^i\right)^{1/2} , \tag{95}$$

leads to $\frac{\hbar}{2m\Delta x}\left(1+\frac{3NkT}{4mc^2}\sum_{i=0}^{\infty}\beta_i\left(\frac{3NkT}{2}\right)^i\right)^{1/2} \leq c$ and, consequently, to

$$\Delta x > \frac{\hbar}{2mc}\left(1+\frac{3NkT}{4mc^2}\sum_{i=0}^{\infty}\beta_i\left(\frac{3NkT}{2}\right)^i\right)^{1/2} = \frac{R_c}{2}\left(1+\frac{3NkT}{4mc^2}\sum_{i=0}^{\infty}\beta_i\left(\frac{3NkT}{2}\right)^i\right)^{1/2} . \tag{96}$$

which leads to the condition:

$$\begin{aligned}&\frac{R_c}{4R_g}\left(1+\frac{3NkT}{4mc^2}\sum_{i=0}^{\infty}\beta_i\left(\frac{3NkT}{2}\right)^i\right)^{1/2} = \frac{\hbar}{8mcR_g}\left(1+\frac{3NkT}{4mc^2}\sum_{i=0}^{\infty}\beta_i\left(\frac{3NkT}{2}\right)^i\right)^{1/2} \\ &= \frac{\hbar c}{8m^2G}\left(1+\frac{3NkT}{4mc^2}\sum_{i=0}^{\infty}\beta_i\left(\frac{3NkT}{2}\right)^i\right)^{1/2} = \frac{{m_p}^2}{m^2}\left(1+\frac{3NkT}{4mc^2}\sum_{i=0}^{\infty}\beta_i\left(\frac{3NkT}{2}\right)^i\right)^{1/2} < 1\end{aligned} \tag{97}$$

with

$$\begin{aligned} m_{BH} &> \sqrt{\frac{\hbar c}{{m_{BH}}^2 G}}\left(1+\frac{3NkT}{16m_{BH}c^2}\sum_{i=0}^{\infty}\beta_i\left(\frac{3NkT}{2}\right)^i\right) \\ &= m_p\left(1+\frac{3NkT}{16m_{BH}c^2}\sum_{i=0}^{\infty}\beta_i\left(\frac{3NkT}{2}\right)^i\right)\end{aligned} \tag{98}$$

leading to

$$\frac{3NkT}{16m_{PBBH}c^2}\sum_{i=0}^{h-1}\beta_i\left(\frac{3NkT}{2}\right)^i > 1 \tag{99}$$

that, for BH of elemental baryonic particles, leads to

$$T_{big-bang} > \approx 10^{\frac{63}{h}} \ {}^{\circ}\mathrm{K} \tag{100}$$

where h is the polynomial degree of temperature series expansion (92). For quadratic dependence by temperature it follows that

$$T_{big-bang} > \cong 3{,}3 \times 10^{31}\ °\mathrm{K}, \tag{101}$$

while for cubic dependence we have

$$T_{big-bang} > \cong 10^{21}\ °\mathrm{K} \tag{102}$$

The validity of the results (76) and subsequent ones are substantiated by the quantum-gravitational effects produced by the quantum mass distribution within spacetime [23-24]. This demonstration elucidates that when mass density is condensed into a sphere with a diameter equal to half the Compton length, it engenders a quantum potential force that is able to counters the compressive gravitational force within a black hole. Moreover, in presence of thermal fluctuations, for $T > 0$, the repulsive thermal force adds itself to the quantum potential one rendering the Planck's BH unstable and leading to its evaporation.

Regarding high-energy physics, the pure quantum realm is limited to roughly one-third of the De Broglie length. When the system experiences significant thermal energy fluctuations and the De Broglie length approaches its physical length, quantum mechanics begins to be influenced by environmental fluctuations.

From the standpoint of the SQHM, classical behavior begins to emerge at increasingly smaller dimensions, leading to deviations from quantum mechanical behavior. At higher energies and fluctuation amplitudes, it becomes necessary to account for perturbations to the quantum theory. By employing a semiempirical approach at the first order of approximation, this effect can be addressed by introducing a measurable parameter. In this regard, Santilli's IsoRedShift theory, which incorporates hidden variables, seems to provide a solid and dependable model for describing high-energy dynamics in the presence of significant fluctuations.

6. Discussion and Conclusions

The stochastic extension of Madelung quantum hydrodynamics effectively integrates mass density fluctuations into quantum equations, driven by gravitational background noise, which is a form of dark energy. This model adeptly tackles significant elements of the quantum-to-classical transition by introducing the concept of quantum potential interaction length alongside the De Broglie wavelength. Beyond this length, coherent quantum behavior and the evolution of the wavefunction are disrupted or profoundly changed.

The stochastic quantum hydrodynamic framework asserts that an external environment is not required, indicating that the stochastic behavior causing wavefunction collapse and macroscopic classical phenomena, consistent with Einsteinian realism, is an intrinsic aspect of physics in a fluctuating spacetime. The theory provides a coherent connection between the uncertainty principle and the constancy of light speed, corresponding with the finite speed of information transmission. In quantum mechanics subjected to fluctuations, the model derives the uncertainty relationship between energy and time, shedding light on measurement processes that cannot be concluded within a finite time interval in a truly quantum global system.

Moreover, the model elucidates how uncertainty relations evolve in an open quantum system under fluctuations, presenting a framework to describe how quantum mechanics adapts in the contexts of nuclear and elementary particle physics and the behavior of high-energy black hole states. The model's self-consistency is demonstrated through its ability to depict the dynamics of wavefunction collapse and the measurement process within its mathematical structure. Additionally, the theory addresses the issue of Einsteinian determinism and pre-existing reality by showing that large-scale systems naturally transition into stable, decoherent classical states over time, without the necessity for an observer or measurement apparatus. The SQHM presented here is limited to the non-relativistic Madelung theory. The generalization to the relativistic representation of Klein-Gordon equation as well Dirac's one represent the possible future developments.

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