Ratio Mathematica Volume 51, 2024

Some Aspects of Soft μ-α-open sets and Soft μ-β-open sets in Soft Generalized Topological Spaces

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Abstract

In this paper, we studied and analysed some of the properties of soft μ-α-open sets and soft μ-β-open sets in soft generalized topological spaces. Also we investigate the inter-relationship among several weaker forms of soft μ-open sets in soft generalized topological spaces.

Keywords: soft $\alpha_{(\mu,\eta)}$-continuous; soft $\beta_{(\mu,\eta)}$-continuous; soft $\rho_{(\mu,\eta)}$-continuous; soft $\delta_{(\mu,\eta)}$-continuous.

2020 AMS subject classifications: 54A05, 54C05, 54G20. [1]

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[1]Received on December 26, 2023. Accepted on January 20, 2024. Published on January 30, 2024. DOI:10.23755/rm.v51i0.1550. ISSN: 1592-7415. eISSN: 2282-8214.

1 Introduction

Topology is an important area of mathematics, with many applications in the domain of computer science and physical sciences. In 1999 D. Molodtsov [13] initiated the concept of soft set theory as a mathematical tool for modelling uncertainties. A soft set is a collection of approximate descriptions of an object. Molodtsov [14] applied the soft set theory in several areas of mathematics such as game theory, probability, Perron and Riemann Integration and theory of measurements. He has analysed that the soft set is used to eradicate problems occurring in the field of economics, social science, engineering, medical science, etc. Maji et al. [11] have further improved the theory of soft sets. N. Cagman et al.[1] modified the definition of soft sets which is similar to that of Molodtsov and also Cagman [5] formulated the maximum decision making method by means of soft matrix theory. Many researchers have worked on the topological structure of soft sets. Bashir Ahmad et al.[4] have defined the soft topological structures by using soft points. B.chen [6] was the first one who studied the weak forms of soft open sets and scrutinized soft semiopen sets in soft topological spaces and studied some of its properties. Arockiarani and Arokialancy[3] determined soft β-open sets and prolonged to study weak forms of soft open sets in soft topological space. Gunduz Aras et al.[2] investigated some soft continuous functions for a fixed number of parameters on the initial universe. Mahanta and Das [10] classified many forms of soft mappings such as irresolute, semicontinuous and semiopen soft mappings. Muhammad Shabir et al. [15] introduced soft topological spaces. In 2002 A. Csaszar [7] introduced the concept of generalized topology and also studied some of its properties.

Soft generalized topology is relatively new and promising domain wich lead to the development of new mathematical models and innovative approaches that will give solution to complex problems in engineering and environment. Sunil Jacob John et al. [8] introduced the concept of soft generalized topological spaces in 2014. sunil Jacob John [9] also introduced some interesting properties of the soft mapping $\pi : S(U)_E \to S(U)_E$ which satisfy the condition $\pi F_B \subset \pi F_D$ whenever $F_B \subset F_D \subset F_{\tilde{E}}$ in soft π-open sets in soft generalized topological spaces in 2015. These concepts promote us in elaborating our further study in soft μ-α-open sets and soft μ-β-open sets in soft generalized topological spaces.

2 Preliminaries

Definition 2.1. [9] A soft set F_A on the universe U is defined by the set of ordered pairs $F_A = \{(e, f_A(e)) \ / \ e \in E, f_A(e) \in \mathcal{P}(U)\}$, where $f_A : E \to \mathcal{P}(U)$ such that $f_A(e) = \emptyset$ if $e \notin A$. Here f_A is called an approximate function of the soft set F_A.

The value of $f_A(e)$ may be arbitrary. Some of them may be empty, some may have nonempty intersection. The set of all soft sets over U with E as the parameter set will be denoted by S(U)$_E$ or simply S(U).

Definition 2.2. [9] Let $F_A \in$ S(U). If $f_A(e) = \emptyset$, for all e $\in$ E, then F_A is called an empty soft set, denoted by $F_\emptyset$. $f_A(e) = \emptyset$ means there is no element in U related to the parameter e in E. Therefore we donot display such elements in the soft sets as it is meaningless to consider such parameters.

Definition 2.3. [9] Let $F_A \in$ S(U). If f_A (e) = U for all e $\in$ A, then F_A is called an A-universal soft set, denoted by $F_{\widetilde{A}}$. If A = E, then the A-universal soft set is called an universal soft set, denoted by $F_{\widetilde{E}}$.

Definition 2.4. [8] Let $F_A \in$ S(U). Then, the soft complement of F_A, denoted by $(F_A)^c$, is defined by the approximate function ${f_A}^c(e) = (f_A(e))^c$, where $(f_A(e))^c$ is the complement of the set $f_A(e)$, that is, $(f_A(e))^c = U \setminus f_A(e)$, for all e $\in$ E.

Definition 2.5. [8] Let $F_A \in$ S(U). A Soft Generalized Topology (SGT) on F_A, denoted by μ or μ_{F_A} is a collection of soft subsets of F_A having the following properties:

i. $F_\emptyset \in \mu$

ii. $\{F_{A_i} \subseteq F_A / i \in J \subseteq N\} \subseteq \mu \Rightarrow \bigcup_{i \in J} F_{A_i} \in \mu$

The pair (F_A, μ) is called a Soft Generalized Topological Space (SGTS).
Observe that $F_A \in \mu$ must not hold.

Definition 2.6. [8] Let (F_A, μ) be a SGTS. Then every element of μ is called a soft μ-open set.

Definition 2.7. [8] Let (F_A, μ) be a SGTS and $F_B \subseteq F_A$. Then the soft μ-interior of F_B, denoted by $i_\mu(F_B)$ is defined as the soft union of all soft μ-open subsets of F_B. Note that $i_\mu(F_B)$ is the largest soft μ-open set that is contained in F_B.

Definition 2.8. [8] Let (F_A, μ) be a SGTS and $F_B \subseteq F_A$. Then the soft μ-closure of F_B, denoted by $c_\mu(F_B)$ is defined as the soft intersection of all soft μ-closed super sets of F_B. Note that $c_\mu(F_B)$ is the smallest soft μ-closed superset of F_B.

Definition 2.9. [9] Let $(F_{\widetilde{E}}, \mu)$ be a SGTS. Then a soft set $F_G \subset F_{\widetilde{E}}$ is said to be a soft μ-pre-open set iff $F_G \subset i_\mu c_\mu F_G$ (i.e, the case when $\pi = i_\mu c_\mu$). The class of all soft μ-pre-open sets is denoted by $\rho_{(\mu)}$ *or* ρ_μ.

Definition 2.10. [9] Let $(F_{\widetilde{E}}, \mu)$ be a SGTS. Then a soft set $F_G \subset F_{\widetilde{E}}$ is said to be a soft μ-semi-open set iff $F_G \subset c_\mu i_\mu F_G$ (i.e, the case when $\pi = c_\mu i_\mu$). The class of all soft μ-semi-open sets is denoted by $\delta_{(\mu)}\ or\ \delta_\mu$.

Definition 2.11. [12] A soft mapping $g : A \rightarrow B$ is called soft α-continuous if the inverse image of each soft open subset of B is a soft α-open set in A.

Definition 2.12. [4] A soft mapping $g : A \rightarrow B$ is called soft β-continuous (resp., soft α-continuous, soft precontinuous, and soft semicontinuous) if the inverse image of each soft open set in B is soft β-open (resp., soft α-open, soft preopen, and soft semiopen) set in A.

Remark 2.1. [16] If $\{(G, A)_\alpha \mid \alpha \in I\}$ is a collection of soft sets, then

(i) $\widetilde{\cup}\, \widetilde{s}int(G, A)_\alpha \widetilde{\subseteq}\, \widetilde{s}int(\, \widetilde{\cup}\, (G, A)_\alpha)$

(ii) $\widetilde{\cup}\, \widetilde{s}cl(G, A)_\alpha \,\widetilde{\subseteq}\, \widetilde{s}cl(\, \widetilde{\cup}\, (G, A)_\alpha)$

Throughout this paper, we symbolize soft generalized topological space, soft generalized topological subspace and the set of all soft μ-semi-open sets, soft μ-pre-open sets, soft μ-α-open sets, soft μ-α-closed sets, soft μ-β-open sets, soft μ-β-closed sets, soft μ-interior and soft μ-closure in a soft generalized topological space $(F_{\widetilde{E}}, \mu)$ by $\mathcal{SGTS}$, $\mathcal{SGTSS}$, $\mathcal{S}\mu$-$\mathcal{SO}$ $(F_{\widetilde{E}})$, $\mathcal{S}\mu$-$\mathcal{PO}$ $(F_{\widetilde{E}})$, $\mathcal{S}\mu$-$\alpha\mathcal{O}$ $(F_{\widetilde{E}})$, $\mathcal{S}\mu$-$\alpha\mathcal{C}$ $(F_{\widetilde{E}})$, $\mathcal{S}\mu$-$\beta\mathcal{O}$ $(F_{\widetilde{E}})$, $\mathcal{S}\mu$-$\beta\mathcal{C}$ $(F_{\widetilde{E}})$, $\mathcal{S}\mu$-$\mathcal{I}nt$ $(F_{\widetilde{E}})$ and $\mathcal{S}\mu$-$\mathcal{C}\ell$ $(F_{\widetilde{E}})$ respectively.

3 Soft μ-α-open sets

Definition 3.1. [9] Let $(F_{\widetilde{E}}, \mu)$ be a SGTS. Then a soft set $F_G \subset F_{\widetilde{E}}$ is said to be a soft μ-α-open set iff $F_G \subset i_\mu c_\mu i_\mu F_G$ (i.e, the case when $\pi = i_\mu c_\mu i_\mu$). The class of all soft μ-α-open sets is denoted by $\alpha_{(\mu)}\ or\ \alpha_\mu$.

Definition 3.2. Let $(F_{\widetilde{E}}, \mu)$ be a $\mathcal{SGTS}$. Then a soft set $F_G \,\widetilde{\subset}\, F_{\widetilde{E}}$ is said to be a soft μ-α-closed set iff its complement is a soft μ-α-open set.

Example 3.1. Let $\mathcal{K} = \{\kappa_1, \kappa_2, \kappa_3\}$, $\mathcal{E} = \{\hat{x_1}, \hat{x_2}\}$ and $\mu = \{F_\emptyset, F_{\mathcal{A}_1}, F_{\mathcal{A}_2}, F_{\mathcal{A}_3}, F_{\mathcal{A}_4}, F_{\mathcal{A}_5}, F_{\mathcal{A}_6}, F_{\widetilde{\mathcal{E}}}\}$ where

$F_{\mathcal{A}_1} = \{(\hat{x_1}, \{\kappa_1, \kappa_2, \kappa_3\}), (\hat{x_2}, \{\kappa_1, \kappa_2\})\}$

$F_{\mathcal{A}_2} = \{(\hat{x_1}, \{\kappa_1, \kappa_2\}), (\hat{x_2}, \{\kappa_1, \kappa_2\})\}$

$F_{\mathcal{A}_3} = \{(\hat{x_1}, \{\kappa_1\}), (\hat{x_2}, \{\kappa_1\})\}$

$F_{\mathcal{A}_4} = \{(\hat{x_1}, \{\kappa_1, \kappa_3\}), (\hat{x_2}, \{\kappa_1, \kappa_2\})\}$

$F_{\mathcal{A}_5} = \{(\hat{x_1}, \{\kappa_2\}), (\hat{x_2}, \{\kappa_2\})\}$

$F_{\mathcal{A}_6} = \{(\hat{x_1}, \{\kappa_2, \kappa_3\}), (\hat{x_2}, \{\kappa_1, \kappa_2\})\}$

Then $(F_{\widetilde{E}}, \mu)$ is a $\mathcal{SGTS}$.
$\mathcal{S}\mu$-$\alpha\mathcal{O}\ (F_{\widetilde{E}}) = \{F_\emptyset, F_{\widetilde{E}}, \{(\hat{x_1}, \{\kappa_1, \kappa_2, \kappa_3\}), (\hat{x_2}, \{\kappa_1, \kappa_2\})\}, \{(\hat{x_1}, \{\kappa_1, \kappa_2\}), (\hat{x_2}, \{\kappa_1, \kappa_2\})\}, \{(\hat{x_1}, \{\kappa_1\}), (\hat{x_2}, \{\kappa_1\})\}, \{(\hat{x_1}, \{\kappa_1, \kappa_3\}), (\hat{x_2}, \{\kappa_1, \kappa_2\})\}, \{(\hat{x_1}, \{\kappa_2\}), (\hat{x_2}, \{\kappa_2\})\}, \{(\hat{x_1}, \{\kappa_2, \kappa_3\}), (\hat{x_2}, \{\kappa_1, \kappa_2\})\}\}$

Remark 3.1. $F_\emptyset$ and $F_{\widetilde{E}}$ are always soft μ-α-open and soft μ-α-closed.

Theorem 3.1. Every soft μ-open set (soft μ-closed) is soft μ-α-open (soft μ-α-closed). But the converse need not be true.

Example 3.2. In the above example 3.3, $\{(\hat{x_1}, \{\kappa_1, \kappa_3\}), (\hat{x_2}, \{\mathcal{K}\})\}$, $\{(\hat{x_1}, \{\kappa_2, \kappa_3\}), (\hat{x_2}, \{\mathcal{K}\})\}$ are soft μ-α-open sets but not soft μ-open.

Theorem 3.2. Arbitrary soft union of soft μ-α-open sets is a soft μ-α-open set.

Proof. Let $\{F_{\mathcal{A}_i} \ / \ i \,\widetilde{\in}\, I\}$ be a collection of soft μ-α-open sets in a $\mathcal{SGTS}$ $(F_{\widetilde{E}}, \mu)$. Then by the definition of soft μ-α-open set,
$F_{\mathcal{A}_i} \,\widetilde{\subseteq}\, \mathcal{S}\mu\text{-}\mathfrak{Int}\ (\mathcal{S}\mu\text{-}\mathcal{C}\ell\ (\mathcal{S}\mu\text{-}\mathfrak{Int}\ (F_{\mathcal{A}_i}))),\ \forall$ i.
Now, $\widetilde{\cup}\, F_{\mathcal{A}_i} \,\widetilde{\subseteq}\, \widetilde{\cup}\, \mathcal{S}\mu\text{-}\mathfrak{Int}\ (\mathcal{S}\mu\text{-}\mathcal{C}\ell\ (\mathcal{S}\mu\text{-}\mathfrak{Int}\ (F_{\mathcal{A}_i}))),\ \forall$ i.
By remark 2.13 (i), $\widetilde{\cup}\, F_{\mathcal{A}_i} \,\widetilde{\subseteq}\, \mathcal{S}\mu\text{-}\mathfrak{Int}\ (\,\widetilde{\cup}\, \mathcal{S}\mu\text{-}\mathcal{C}\ell\ (\mathcal{S}\mu\text{-}\mathfrak{Int}\ (F_{\mathcal{A}_i}))),\ \forall$ i.
By remark 2.13 (ii),$\widetilde{\cup}\, F_{\mathcal{A}_i} \,\widetilde{\subseteq}\, \mathcal{S}\mu\text{-}\mathfrak{Int}\ (\mathcal{S}\mu\text{-}\mathcal{C}\ell\ (\,\widetilde{\cup}\, \mathcal{S}\mu\text{-}\mathfrak{Int}\ (F_{\mathcal{A}_i}))),\ \forall$ i.
By remark 2.13 (i), $\widetilde{\cup}\, F_{\mathcal{A}_i} \,\widetilde{\subseteq}\, \mathcal{S}\mu\text{-}\mathfrak{Int}\ (\mathcal{S}\mu\text{-}\mathcal{C}\ell\ (\mathcal{S}\mu\text{-}\mathfrak{Int}\ (\,\widetilde{\cup}\, F_{\mathcal{A}_i}))),\ \forall$ i.
Hence $\widetilde{\cup}\, F_{\mathcal{A}_i} \,\widetilde{\subseteq}\, \mathcal{S}\mu\text{-}\mathfrak{Int}\ (\mathcal{S}\mu\text{-}\mathcal{C}\ell\ (\mathcal{S}\mu\text{-}\mathfrak{Int}\ (\,\widetilde{\cup}\, F_{\mathcal{A}_i}))),\ \forall$ i. □

Remark 3.2. Finite soft intersection of soft μ-α-open sets need not be a soft μ-α-open set.

Example 3.3. In the above example 3.3, $F_{\mathcal{C}} = \{(\hat{x_1}, \{\kappa_2, \kappa_3\}), (\hat{x_2}, \{\kappa_1, \kappa_2\})\}$ and $F_{\mathcal{D}} = \{(\hat{x_1}, \{\kappa_1, \kappa_3\}), (\hat{x_2}, \{\kappa_1, \kappa_2\})\}$ are soft μ-α-open sets but $F_{\mathcal{C}} \,\widetilde{\cap}\, F_{\mathcal{D}} = \{(\hat{x_1}, \{\kappa_3\}), (\hat{x_2}, \{\kappa_1, \kappa_2\})\}$ is not soft μ-α-open.

Theorem 3.3. Arbitrary soft intersection of soft μ-α-closed sets is a soft μ-α-closed set.

Proof. The proof is similar to that of theorem 3.7 by taking complements. □

Remark 3.3. Finite soft union of soft μ-α-closed sets need not be a soft μ-α-closed set.

Example 3.4. In the above example 3.3, $F_{\mathcal{S}} = \{(\hat{x_1}, \{\kappa_1\}), (\hat{x_2}, \{\kappa_3\})\}$ and $F_{\mathcal{T}} = \{(\hat{x_1}, \{\kappa_2\}), (\hat{x_2}, \{\kappa_3\})\}$ are soft μ-α-closed sets but $F_{\mathcal{S}} \,\widetilde{\cup}\, F_{\mathcal{T}} = \{(\hat{x_1}, \{\kappa_1, \kappa_2\}), (\hat{x_2}, \{\kappa_3\})\}$ is not soft μ-α-closed.

Remark 3.4. Let $(F_{\widetilde{E}}, \mu)$ be a $\mathcal{SGTS}$. Then every soft μ-α-open set is soft μ-pre-open, soft μ-semi-open and soft μ-β-open. But the converse need not be true.

Example 3.5. In the above example 3.3, $\{(\hat{x_1}, \{\kappa_1, \kappa_2\}), (\hat{x_2}, \{\kappa_3\})\}$ and $\{(\hat{x_1}, \{\kappa_2, \kappa_3\}), (\hat{x_2}, \{\kappa_1, \kappa_3\})\}$ are soft μ-pre-open sets but not soft μ-α-open sets. Similarly, $\{(\hat{x_1}, \{\kappa_2, \kappa_3\}), (\hat{x_2}, \{\kappa_2\})\}$ and $\{(\hat{x_1}, \{\kappa_1\}), (\hat{x_2}, \{\kappa_1, \kappa_3\})\}$ are soft μ-semi-open sets but not soft μ-α-open sets. Also, $\{(\hat{x_1}, \{\mathcal{K}\}), (\hat{x_2}, \{\kappa_2\})\}$ and $\{(\hat{x_1}, \{\emptyset\}), (\hat{x_2}, \{\kappa_1, \kappa_2\})\}$ are soft μ-β-open sets but not soft μ-α-open sets.

Definition 3.3. Let $(F_{\widetilde{E}}, \mu)$ be a $\mathcal{SGTS}$ and $F_{\mathcal{Z}} \widetilde{\subseteq} F_{\widetilde{E}}$.

(i) The soft μ-α-interior of $F_{\mathcal{Z}}$ is defined by
$\mathcal{S}\alpha_{i_\mu}(F_{\mathcal{Z}}) = \widetilde{\cup}\left\{F_{\mathcal{W}} : F_{\mathcal{W}} \widetilde{\subseteq} F_{\mathcal{Z}} \text{ and } F_{\mathcal{W}} \widetilde{\in} \mathcal{S}\mu\text{-}\alpha\mathcal{O}(F_{\widetilde{E}})\right\}$

(ii) The soft μ-α-closure of $F_{\mathcal{Z}}$ is defined by
$\mathcal{S}\alpha_{c_\mu}(F_{\mathcal{Z}}) = \widetilde{\cap}\left\{F_{\mathcal{W}} : F_{\mathcal{Z}} \widetilde{\subseteq} F_{\mathcal{W}} \text{ and } F_{\mathcal{W}} \widetilde{\in} \mathcal{S}\mu\text{-}\alpha\mathcal{C}(F_{\widetilde{E}})\right\}$

(i.e.) $\mathcal{S}\alpha_{i_\mu}(F_{\mathcal{Z}})$ is the largest soft μ-α-open set contained in $F_{\mathcal{Z}}$ and $\mathcal{S}\alpha_{c_\mu}(F_{\mathcal{Z}})$ is the smallest soft μ-α-closed set containing $F_{\mathcal{Z}}$.

Theorem 3.4. Let $(F_{\widetilde{E}}, \mu)$ be a $\mathcal{SGTS}$ and $F_{\mathcal{U}},\ F_{\mathcal{V}}$ be two soft sets over $F_{\widetilde{E}}$. Then

(i) $F_{\mathcal{U}} \widetilde{\subseteq} F_{\mathcal{V}} \Rightarrow \mathcal{S}\alpha_{i_\mu}(F_{\mathcal{U}}) \widetilde{\subseteq} \mathcal{S}\alpha_{i_\mu}(F_{\mathcal{V}})$

(ii) $F_{\mathcal{U}} \widetilde{\subseteq} F_{\mathcal{V}} \Rightarrow \mathcal{S}\alpha_{c_\mu}(F_{\mathcal{U}}) \widetilde{\subseteq} \mathcal{S}\alpha_{c_\mu}(F_{\mathcal{V}})$

(iii) $\mathcal{S}\alpha_{c_\mu}(F_{\mathcal{U}} \widetilde{\cup} F_{\mathcal{V}}) = \mathcal{S}\alpha_{c_\mu}(F_{\mathcal{U}}) \widetilde{\cup} \mathcal{S}\alpha_{c_\mu}(F_{\mathcal{V}})$

(iv) $\mathcal{S}\alpha_{i_\mu}(F_{\mathcal{U}} \widetilde{\cap} F_{\mathcal{V}}) = \mathcal{S}\alpha_{i_\mu}(F_{\mathcal{U}}) \widetilde{\cap} \mathcal{S}\alpha_{i_\mu}(F_{\mathcal{V}})$

(v) $\mathcal{S}\alpha_{c_\mu}(F_{\mathcal{U}} \widetilde{\cap} F_{\mathcal{V}}) \widetilde{\subseteq} \mathcal{S}\alpha_{c_\mu}(F_{\mathcal{U}}) \widetilde{\cap} \mathcal{S}\alpha_{c_\mu}(F_{\mathcal{V}})$

(vi) $\mathcal{S}\alpha_{i_\mu}(F_{\mathcal{U}} \widetilde{\cup} F_{\mathcal{V}}) \widetilde{\supseteq} \mathcal{S}\alpha_{i_\mu}(F_{\mathcal{U}}) \widetilde{\cup} \mathcal{S}\alpha_{i_\mu}(F_{\mathcal{V}})$

Proof. (i) Since $F_{\mathcal{U}}$ is a soft μ-open set, $\mathcal{S}\alpha_{i_\mu}(F_{\mathcal{U}}) \widetilde{\subseteq} F_{\mathcal{U}} \widetilde{\subseteq} F_{\mathcal{V}}$ which implies $\mathcal{S}\alpha_{i_\mu}(F_{\mathcal{U}}) \widetilde{\subseteq} F_{\mathcal{V}}$ and $\mathcal{S}\alpha_{i_\mu}(F_{\mathcal{V}})$ is the largest soft μ-α-open set contained in $F_{\mathcal{V}}$. Hence $\mathcal{S}\alpha_{i_\mu}(F_{\mathcal{U}}) \widetilde{\subseteq} \mathcal{S}\alpha_{i_\mu}(F_{\mathcal{V}})$.

(ii) since $F_{\mathcal{U}} \widetilde{\subseteq} \mathcal{S}\alpha_{c_\mu}(F_{\mathcal{V}})$ and $F_{\mathcal{V}} \widetilde{\subseteq} \mathcal{S}\alpha_{c_\mu}(F_{\mathcal{V}})$, $F_{\mathcal{U}} \widetilde{\subseteq} F_{\mathcal{V}} \widetilde{\subseteq} \mathcal{S}\alpha_{c_\mu}(F_{\mathcal{V}})$. Hence $F_{\mathcal{U}} \widetilde{\subseteq} \mathcal{S}\alpha_{c_\mu}(F_{\mathcal{V}})$. Also, $\mathcal{S}\alpha_{c_\mu}(F_{\mathcal{U}})$ is the smallest soft μ-α-closed containing $F_{\mathcal{U}}$. Therefore, $\mathcal{S}\alpha_{c_\mu}(F_{\mathcal{U}}) \widetilde{\subseteq} \mathcal{S}\alpha_{c_\mu}(F_{\mathcal{V}})$.

(iii) We have $F_{\mathcal{U}} \widetilde{\subseteq} F_{\mathcal{U}} \widetilde{\cup} F_{\mathcal{V}}$ and $F_{\mathcal{V}} \widetilde{\subseteq} F_{\mathcal{U}} \widetilde{\cup} F_{\mathcal{V}}$. From (ii), $\mathcal{S}\alpha_{c_\mu}(F_{\mathcal{U}}) \widetilde{\subseteq} \mathcal{S}\alpha_{c_\mu}(F_{\mathcal{U}} \widetilde{\cup} F_{\mathcal{V}})$ and $\mathcal{S}\alpha_{c_\mu}(F_{\mathcal{V}}) \widetilde{\subseteq} \mathcal{S}\alpha_{c_\mu}(F_{\mathcal{U}} \widetilde{\cup} F_{\mathcal{V}})$ which implies $\mathcal{S}\alpha_{c_\mu}(F_{\mathcal{U}}) \widetilde{\cup} \mathcal{S}\alpha_{c_\mu}(F_{\mathcal{V}}) \widetilde{\subseteq} \mathcal{S}\alpha_{c_\mu}(F_{\mathcal{U}} \widetilde{\cup} F_{\mathcal{V}})$. Since $F_{\mathcal{U}} \widetilde{\subseteq} \mathcal{S}\alpha_{c_\mu}(F_{\mathcal{U}})$ and $F_{\mathcal{V}} \widetilde{\subseteq} \mathcal{S}\alpha_{c_\mu}(F_{\mathcal{V}})$, $F_{\mathcal{U}} \widetilde{\cup} F_{\mathcal{V}} \widetilde{\subseteq} \mathcal{S}\alpha_{c_\mu}(F_{\mathcal{V}}) \widetilde{\cup} \mathcal{S}\alpha_{c_\mu}(F_{\mathcal{V}})$. Since $\mathcal{S}\alpha_{c_\mu}(F_{\mathcal{U}}) \widetilde{\cup} \mathcal{S}\alpha_{c_\mu}(F_{\mathcal{V}})$ is a soft μ-α-closed set containing $F_{\mathcal{U}} \widetilde{\cup} F_{\mathcal{V}}$, $\mathcal{S}\alpha_{c_\mu}(F_{\mathcal{U}} \widetilde{\cup} F_{\mathcal{V}})$ is the smallest soft μ-α-closed set containing $F_{\mathcal{U}} \widetilde{\cup} F_{\mathcal{V}}$. Therefore, $\mathcal{S}\alpha_{c_\mu}(F_{\mathcal{U}} \widetilde{\cup} F_{\mathcal{V}}) \widetilde{\subseteq} \mathcal{S}\alpha_{c_\mu}(F_{\mathcal{U}}) \widetilde{\cup} \mathcal{S}\alpha_{c_\mu}(F_{\mathcal{V}})$. Hence $\mathcal{S}\alpha_{c_\mu}(F_{\mathcal{U}} \widetilde{\cup} F_{\mathcal{V}}) = \mathcal{S}\alpha_{c_\mu}(F_{\mathcal{U}}) \widetilde{\cup} \mathcal{S}\alpha_{c_\mu}(F_{\mathcal{V}})$.

(iv) We have $F_{\mathcal{U}} \widetilde{\cap} F_{\mathcal{V}} \widetilde{\subseteq} F_{\mathcal{U}}$ and $F_{\mathcal{U}} \widetilde{\cap} F_{\mathcal{V}} \widetilde{\subseteq} F_{\mathcal{V}}$. From (i), $\mathcal{S}\alpha_{i_\mu}(F_{\mathcal{U}} \widetilde{\cap} F_{\mathcal{V}}) \widetilde{\subseteq} \mathcal{S}\alpha_{i_\mu}(F_{\mathcal{U}})$ and $\mathcal{S}\alpha_{i_\mu}(F_{\mathcal{U}} \widetilde{\cap} F_{\mathcal{V}}) \widetilde{\subseteq} \mathcal{S}\alpha_{i_\mu}(F_{\mathcal{V}})$ which implies $\mathcal{S}\alpha_{i_\mu}(F_{\mathcal{U}} \widetilde{\cap} F_{\mathcal{V}}) \widetilde{\subseteq} \mathcal{S}\alpha_{i_\mu}(F_{\mathcal{U}}) \widetilde{\cap} \mathcal{S}\alpha_{i_\mu}(F_{\mathcal{V}})$. Since $\mathcal{S}\alpha_{i_\mu}(F_{\mathcal{U}}) \widetilde{\subseteq} F_{\mathcal{U}}$ and $\mathcal{S}\alpha_{i_\mu}(F_{\mathcal{V}}) \widetilde{\subseteq} F_{\mathcal{V}}, \mathcal{S}\alpha_{i_\mu}(F_{\mathcal{U}}) \widetilde{\cap} \mathcal{S}\alpha_{i_\mu}(F_{\mathcal{V}}) \widetilde{\subseteq} F_{\mathcal{U}} \widetilde{\cap} F_{\mathcal{V}}$. As $\mathcal{S}\alpha_{i_\mu}(F_{\mathcal{U}}) \widetilde{\cap} \mathcal{S}\alpha_{i_\mu}(F_{\mathcal{V}})$ is a soft μ-α-open set contained in $F_{\mathcal{U}} \widetilde{\cap} F_{\mathcal{V}}$, $\mathcal{S}\alpha_{i_\mu}(F_{\mathcal{U}} \widetilde{\cap} F_{\mathcal{V}})$ is the largest soft μ-α-open set contained in $F_{\mathcal{U}} \widetilde{\cap} F_{\mathcal{V}}$. Therefore, $\mathcal{S}\alpha_{i_\mu}(F_{\mathcal{U}}) \widetilde{\cap} \mathcal{S}\alpha_{i_\mu}(F_{\mathcal{V}}) \widetilde{\subseteq} \mathcal{S}\alpha_{i_\mu}(F_{\mathcal{U}} \widetilde{\cap} F_{\mathcal{V}})$. Hence $\mathcal{S}\alpha_{i_\mu}(F_{\mathcal{U}} \widetilde{\cap} F_{\mathcal{V}}) = \mathcal{S}\alpha_{i_\mu}(F_{\mathcal{U}}) \widetilde{\cap} \mathcal{S}\alpha_{i_\mu}(F_{\mathcal{V}})$.

(v) We have $F_{\mathcal{U}} \widetilde{\cap} F_{\mathcal{V}} \widetilde{\subseteq} F_{\mathcal{U}}$ and $F_{\mathcal{U}} \widetilde{\cap} F_{\mathcal{V}} \widetilde{\subseteq} F_{\mathcal{V}}$ which implies $\mathcal{S}\alpha_{c_\mu}(F_{\mathcal{U}} \widetilde{\cap} F_{\mathcal{V}}) \widetilde{\subseteq} \mathcal{S}\alpha_{c_\mu}(F_{\mathcal{U}})$ and $\mathcal{S}\alpha_{c_\mu}(F_{\mathcal{U}} \widetilde{\cap} F_{\mathcal{V}}) \widetilde{\subseteq} \mathcal{S}\alpha_{c_\mu}(F_{\mathcal{V}})$. Hence $\mathcal{S}\alpha_{c_\mu}(F_{\mathcal{U}} \widetilde{\cap} F_{\mathcal{V}}) \widetilde{\subseteq} \mathcal{S}\alpha_{c_\mu}(F_{\mathcal{U}}) \widetilde{\cap} \mathcal{S}\alpha_{c_\mu}(F_{\mathcal{V}})$.

(vi) We have $F_{\mathcal{U}} \widetilde{\subseteq} F_{\mathcal{U}} \widetilde{\cup} F_{\mathcal{V}}$ and $F_{\mathcal{V}} \widetilde{\subseteq} F_{\mathcal{U}} \widetilde{\cup} F_{\mathcal{V}}$ which implies $\mathcal{S}\alpha_{i_\mu}(F_{\mathcal{U}}) \widetilde{\subseteq} \mathcal{S}\alpha_{i_\mu}(F_{\mathcal{U}} \widetilde{\cup} F_{\mathcal{V}})$ and $\mathcal{S}\alpha_{i_\mu}(F_{\mathcal{V}}) \widetilde{\subseteq} \mathcal{S}\alpha_{i_\mu}(F_{\mathcal{U}} \widetilde{\cup} F_{\mathcal{V}})$. Hence $\mathcal{S}\alpha_{i_\mu}(F_{\mathcal{U}}) \widetilde{\cup} \mathcal{S}\alpha_{i_\mu}(F_{\mathcal{V}}) \widetilde{\subseteq} \mathcal{S}\alpha_{i_\mu}(F_{\mathcal{U}} \widetilde{\cup} F_{\mathcal{V}})$.
(i.e.) $\mathcal{S}\alpha_{i_\mu}(F_{\mathcal{U}} \widetilde{\cup} F_{\mathcal{V}}) \widetilde{\supseteq} \mathcal{S}\alpha_{i_\mu}(F_{\mathcal{U}}) \widetilde{\cup} \mathcal{S}\alpha_{i_\mu}(F_{\mathcal{V}})$. □

Theorem 3.5. Let $(F_{\widetilde{E}}, \mu)$ be a $\mathcal{SGTS}$ and $F_{\mathcal{N}}$ be a soft set over $F_{\widetilde{E}}$. Then

(i) $F_{\mathcal{N}}$ is a soft μ-α-closed set.

(ii) $\mathcal{S}\mu$-$\mathfrak{Int}$ $(\mathcal{S}\mu$-$\mathfrak{Cl}$ $(\mathcal{S}\mu$-$\mathfrak{Int}$ $(F_{\mathcal{N}})^c)) \widetilde{\supseteq} (F_{\mathcal{N}})^c$.

(iii) $\mathcal{S}\mu$-$\mathfrak{Cl}$ $(\mathcal{S}\mu$-$\mathfrak{Int}$ $(\mathcal{S}\mu$-$\mathfrak{Cl}$ $(F_{\mathcal{N}}))) \widetilde{\subseteq} F_{\mathcal{N}}$.

(iv) $(F_{\mathcal{N}})^c$ is a soft μ-α-open set.

Proof. (i) $\Rightarrow$ (ii):
Since $F_{\mathcal{N}}$ is a soft μ-α-closed set, $\mathcal{S}\mu$-$\mathfrak{Cl}$ $(\mathcal{S}\mu$-$\mathfrak{Int}$ $(\mathcal{S}\mu$-$\mathfrak{Cl}$ $(F_{\mathcal{N}}))) \widetilde{\subseteq} F_{\mathcal{N}}$ which implies $(F_{\mathcal{N}})^c \widetilde{\subseteq} \mathcal{S}\mu$-$\mathfrak{Int}$ $(\mathcal{S}\mu$-$\mathfrak{Cl}$ $(\mathcal{S}\mu$-$\mathfrak{Int}$ $(F_{\mathcal{N}})^c))$.
(i.e.) $\mathcal{S}\mu$-$\mathfrak{Int}$ $(\mathcal{S}\mu$-$\mathfrak{Cl}$ $(\mathcal{S}\mu$-$\mathfrak{Int}$ $(F_{\mathcal{N}})^c)) \widetilde{\supseteq} (F_{\mathcal{N}})^c$.

(ii) $\Rightarrow$ (iii):
Suppose $\mathcal{S}\mu$-$\mathfrak{Int}$ $(\mathcal{S}\mu$-$\mathfrak{Cl}$ $(\mathcal{S}\mu$-$\mathfrak{Int}$ $(F_{\mathcal{N}})^c)) \widetilde{\supseteq} (F_{\mathcal{N}})^c$. Then by taking the complement on both sides, we get $\mathcal{S}\mu$-$\mathfrak{Cl}$ $(\mathcal{S}\mu$-$\mathfrak{Int}$ $(\mathcal{S}\mu$-$\mathfrak{Cl}$ $(F_{\mathcal{N}}))) \widetilde{\subseteq} F_{\mathcal{N}}$.

(iii) $\Rightarrow$ (iv):
The result follows from the definition 3.2.

(iv) $\Rightarrow$ (i):
The result follows from the definition 3.1. □

Definition 3.4. Let $(F_{\widetilde{E}}, \mu)$ and $(F_{\widetilde{K}}, \eta)$ be any two $\mathcal{SGTS}$'s. A soft function $\Psi_\chi : (F_{\widetilde{E}}, \mu) \to (F_{\widetilde{K}}, \eta)$ is said to be soft $\alpha_{(\mu,\eta)}$-continuous (briefly soft (μ, η)-alpha-continuous), if the inverse image of each soft η-open subset in $F_{\widetilde{K}}$ is soft μ-α-open in $F_{\widetilde{E}}$.

Theorem 3.6. Each soft (μ, η)-continuous is soft $\alpha_{(\mu,\eta)}$-continuous. But the converse neednot be true.

Proof. Let $\Psi_\chi : (F_{\widetilde{E}}, \mu) \to (F_{\widetilde{K}}, \eta)$ be a soft (μ, η)-continuous mapping. Let $F_{\mathcal{D}}$ be any soft η-open set in $F_{\widetilde{K}}$. since Ψ_χ is soft (μ, η)-continuous, its inverse image $\Psi_\chi^{-1}(F_{\mathcal{D}})$ is a soft μ-open set in $F_{\widetilde{E}}$ and every soft μ-open set is soft μ-α-open, $\Psi_\chi^{-1}(F_{\mathcal{D}})$ is soft μ-α-open in $F_{\widetilde{E}}$. Hence Ψ_χ is soft $\alpha_{(\mu,\eta)}$-continuous. □

Example 3.6. Let $\mathcal{L} = \{\ell_1, \ell_2, \ell_3\}$, $\mathcal{P} = \{\hat{p}_1, \hat{p}_2\}$ then $\mu = \{F_\emptyset, F_{\widetilde{\mathcal{P}}}, F_{\mathcal{M}_1}, F_{\mathcal{M}_2}, F_{\mathcal{M}_3}, F_{\mathcal{M}_4}, F_{\mathcal{M}_5}, F_{\mathcal{M}_6}\}$ is a $\mathcal{SGTS}$ where

$F_{\mathcal{M}_1} = \{(\hat{p}_1, \{\mathcal{L}\}), (\hat{p}_2, \{\ell_1, \ell_2\})\}$
$F_{\mathcal{M}_2} = \{(\hat{p}_1, \{\ell_1, \ell_2\}), (\hat{p}_2, \{\ell_1, \ell_2\})\}$
$F_{\mathcal{M}_3} = \{(\hat{p}_1, \{\ell_1, \ell_3\}), (\hat{p}_2, \{\ell_1, \ell_2\})\}$
$F_{\mathcal{M}_4} = \{(\hat{p}_1, \{\ell_1\}), (\hat{p}_2, \{\ell_1\})\}$
$F_{\mathcal{M}_5} = \{(\hat{p}_1, \{\ell_2\}), (\hat{p}_2, \{\ell_2\})\}$
$F_{\mathcal{M}_6} = \{(\hat{p}_1, \{\ell_2, \ell_3\}), (\hat{p}_2, \{\ell_1, \ell_2\})\}$
and

Let $\mathcal{D} = \{d_1, d_2, d_3\}$, $\mathcal{Q} = \{\hat{q}_1, \hat{q}_2\}$ then $\eta = \{F_\emptyset, F_{\widetilde{\mathcal{Q}}}, F_{\mathcal{N}_1}, F_{\mathcal{N}_2}, F_{\mathcal{N}_3}, F_{\mathcal{N}_4}, F_{\mathcal{N}_5}\}$ is a $\mathcal{SGTS}$ where

$F_{\mathcal{N}_1} = \{(\hat{q}_1, \{d_2, d_3\}), (\hat{q}_2, \{d_1, d_3\})\}$
$F_{\mathcal{N}_2} = \{(\hat{q}_1, \{d_2, d_3\}), (\hat{q}_2, \{d_1, d_2\})\}$
$F_{\mathcal{N}_3} = \{(\hat{q}_1, \{d_2, d_3\}), (\hat{q}_2, \{\mathcal{D}\})\}$
$F_{\mathcal{N}_4} = \{(\hat{q}_1, \{\mathcal{D}\}), (\hat{q}_2, \{d_1, d_3\})\}$
$F_{\mathcal{N}_5} = \{(\hat{q}_1, \{d_3\}), (\hat{q}_2, \{d_3\})\}$

A mapping $\Psi : \mathcal{L} \to \mathcal{D}$ is defined by $\Psi(\ell_1) = d_2$, $\Psi(\ell_2) = d_3$, $\Psi(\ell_3) = d_1$ and $\chi : \mathcal{P} \to \mathcal{Q}$ by $\chi(p_1) = q_2$, $\chi(p_2) = q_1$. Then $\Psi_\chi : (F_{\widetilde{\mathcal{P}}}, \mu) \to (F_{\widetilde{\mathcal{Q}}}, \eta)$ is soft $\alpha_{(\mu,\eta)}$-continuous. But it is not soft (μ, η)-continuous.

Theorem 3.7. A soft mapping $\Psi_\chi : (F_{\widetilde{\mathcal{E}}}, \mu) \to (F_{\widetilde{\mathcal{K}}}, \eta)$ is soft $\alpha_{(\mu,\eta)}$-continuous if the inverse image of each soft η-closed set in $F_{\widetilde{\mathcal{K}}}$ is soft μ-α-closed in $F_{\widetilde{\mathcal{E}}}$.

Proof. Let F_I be any soft η-closed set in $F_{\widetilde{\mathcal{K}}}$. Then $(F_I)^c$ is a soft η-open set in $F_{\widetilde{\mathcal{K}}}$. Since Ψ_χ is soft $\alpha_{(\mu,\eta)}$-continuous, $\Psi_\chi^{-1}((F_I)^c)$ is a soft μ-α-open set in $F_{\widetilde{\mathcal{E}}}$. Hence $(\Psi_\chi^{-1}((F_I)^c))^c$ is a soft μ-α-closed set in $F_{\widetilde{\mathcal{E}}}$. □

Theorem 3.8. Let $(F_{\widetilde{E}}, \mu)$ and $(F_{\widetilde{K}}, \eta)$ be two $\mathcal{SGTS}$'s. If $\Psi_\chi : (F_{\widetilde{E}}, \mu) \to (F_{\widetilde{K}}, \eta)$ is a soft $\alpha_{(\mu,\eta)}$-continuous mapping and $(F_{\mathcal{G}}, \xi)$ is a $\mathcal{SGTSS}$ of $F_{\widetilde{E}}$, then the restricted map $\Psi_{\chi|F_{\mathcal{G}}} : (F_{\mathcal{G}}, \xi) \to (F_{\widetilde{K}}, \eta)$ is soft $\alpha_{(\xi,\eta)}$-continuous.

Proof. Let $(F_{\mathcal{B}_i})$ be any η-open set in F_B. Since Ψ_χ is soft $\alpha_{(\mu,\eta)}$-continuous, $\Psi_\chi^{-1}(F_{\mathcal{B}_i})$ is a soft μ-α-open set in $F_{\widetilde{E}}$. Then $\Psi_{\chi|F_{\mathcal{G}}}^{-1}(F_{\mathcal{B}_i}) = F_{\mathcal{G}} \hat{\bigcap} \Psi_\chi^{-1}(F_{\mathcal{B}_i})$, where $F_{\mathcal{G}} \hat{\bigcap} \Psi_{\chi}^{-1}(F_{\mathcal{B}_i})$ is a soft ξ-open set in $F_{\mathcal{G}}$ which implies $\Psi_{\chi|F_{\mathcal{G}}}^{-1}(F_{\mathcal{B}_i})$ is a soft ξ-open set in $F_{\mathcal{G}}$. Hence $\Psi_{\chi|F_{\mathcal{G}}} : (F_{\mathcal{G}}, \xi) \to (F_{\widetilde{K}}, \eta)$ is soft $\alpha_{(\xi,\eta)}$-continuous. $\square$

Theorem 3.9. A soft mapping $\Psi_{\chi} : (F_{\widetilde{E}}, \mu) \to (F_{\widetilde{K}}, \eta)$ is soft $\alpha_{(\mu,\eta)}$-continuous iff for every soft μ-singleton set $F_{\mathcal{U}}$ in $F_{\widetilde{E}}$ and every soft η-open set $F_{\mathcal{V}}$ in $F_{\widetilde{K}}$ and $\Psi_{\chi}(F_{\mathcal{U}}) \widetilde{\subseteq} F_{\mathcal{V}}, \exists$ a soft μ-α-open set $F_{\mathcal{H}}$ in $F_{\widetilde{E}} \ni\ : F_{\mathcal{U}} \widetilde{\subseteq} F_{\mathcal{H}}$ and $\Psi_{\chi}(F_{\mathcal{H}}) \widetilde{\subseteq} F_{\mathcal{V}}$.

Proof. Let $F_{\mathcal{V}}$ be any soft η-open set in $F_{\widetilde{K}}$. Since Ψ_{χ} is soft $\alpha_{(\mu,\eta)}$-continuous, $\Psi_{\chi}^{-1}(F_{\mathcal{V}})$ is soft μ-α-open in $F_{\widetilde{E}}$. By hypothesis, $\Psi_{\chi}(F_{\mathcal{U}}) \widetilde{\subseteq} F_{\mathcal{V}}$ which implies $F_{\mathcal{U}} \widetilde{\subseteq} \Psi_{\chi}^{-1}(F_{\mathcal{V}})$. Take $F_{\mathcal{H}} = \Psi_{\chi}^{-1}(F_{\mathcal{V}})$ which implies $F_{\mathcal{U}} \widetilde{\subseteq} F_{\mathcal{H}}$ and $\Psi_{\chi}(F_{\mathcal{H}}) \widetilde{\subseteq} F_{\mathcal{V}}$. Conversely, Let $F_{\mathcal{V}}$ be any soft η-open set in $F_{\widetilde{K}}$ and $\Psi_{\chi}(F_{\mathcal{U}}) \widetilde{\subseteq} F_{\mathcal{V}}, \exists$ a soft μ-α-open set $F_{\mathcal{H}}$ in $F_{\widetilde{E}}$ such that $F_{\mathcal{U}} \widetilde{\subseteq} F_{\mathcal{H}}$ and $\Psi_{\chi}(F_{\mathcal{H}}) \widetilde{\subseteq} F_{\mathcal{V}}$. Hence $F_{\mathcal{U}} \widetilde{\subseteq} F_{\mathcal{H}} \widetilde{\subseteq} \Psi_{\chi}^{-1}(F_{\mathcal{V}}) = \widetilde{\bigcup} (F_{\mathcal{H}}) \widetilde{\in} \mathcal{S}\mu\text{-}\alpha\mathcal{O}\ (F_{\widetilde{E}})$. Hence Ψ_{χ} is soft $\alpha_{(\mu,\eta)}$-continuous. $\square$

Definition 3.5. Let $(F_{\widetilde{E}}, \mu)$ and $(F_{\widetilde{K}}, \eta)$ be any two $\mathcal{SGTS}$'s. A soft function $\Psi_{\chi} : (F_{\widetilde{E}}, \mu) \to (F_{\widetilde{K}}, \eta)$ is said to be soft $\rho_{(\mu,\eta)}$-continuous (briefly soft (μ,η)-pre-continuous), if the inverse image of each soft η-open subset in $F_{\widetilde{K}}$ is soft μ-pre-open in $F_{\widetilde{E}}$.

Definition 3.6. Let $(F_{\widetilde{E}}, \mu)$ and $(F_{\widetilde{K}}, \eta)$ be any two $\mathcal{SGTS}$'s. A soft function $\Psi_{\chi} : (F_{\widetilde{E}}, \mu) \to (F_{\widetilde{K}}, \eta)$ is said to be soft $\delta_{(\mu,\eta)}$-continuous (briefly soft (μ,η)-semi-continuous), if the inverse image of each soft η-open subset in $F_{\widetilde{K}}$ is soft μ-semi-open in $F_{\widetilde{E}}$.

Remark 3.5. Every soft $\alpha_{(\mu,\eta)}$-continuous is soft $\rho_{(\mu,\eta)}$-continuous and soft $\delta_{(\mu,\eta)}$-continuous. But the converse neednot be true.

Example 3.7. Let $\mathcal{L} = \{\ell_1, \ell_2, \ell_3\}, \mathcal{P} = \{\hat{p}_1, \hat{p}_2\}$ then $\mu = \{F_{\emptyset}, F_{\widetilde{\mathcal{P}}}, F_{\mathcal{M}_1}, F_{\mathcal{M}_2}, F_{\mathcal{M}_3}, F_{\mathcal{M}_4}, F_{\mathcal{M}_5}, F_{\mathcal{M}_6}\}$ is a $\mathcal{SGTS}$ where

$$\begin{aligned}
F_{\mathcal{M}_1} &= \{(\hat{p}_1, \{\mathcal{L}\}), (\hat{p}_2, \{\ell_1, \ell_2\})\} \\
F_{\mathcal{M}_2} &= \{(\hat{p}_1, \{\ell_1, \ell_2\}), (\hat{p}_2, \{\ell_1, \ell_2\})\} \\
F_{\mathcal{M}_3} &= \{(\hat{p}_1, \{\ell_1, \ell_3\}), (\hat{p}_2, \{\ell_1, \ell_2\})\} \\
F_{\mathcal{M}_4} &= \{(\hat{p}_1, \{\ell_1\}), (\hat{p}_2, \{\ell_1\})\} \\
F_{\mathcal{M}_5} &= \{(\hat{p}_1, \{\ell_2\}), (\hat{p}_2, \{\ell_2\})\} \\
F_{\mathcal{M}_6} &= \{(\hat{p}_1, \{\ell_2, \ell_3\}), (\hat{p}_2, \{\ell_1, \ell_2\})\}
\end{aligned}$$

and

Let $\mathcal{D} = \{d_1, d_2, d_3\}, \mathcal{Q} = \{\hat{q}_1, \hat{q}_2\}$ then $\eta = \{F_{\emptyset}, F_{\widetilde{\mathcal{Q}}}, F_{\mathcal{N}_1}, F_{\mathcal{N}_2}, F_{\mathcal{N}_3}, F_{\mathcal{N}_4}\}$ is a $\mathcal{SGTS}$ where

$$\begin{aligned}
F_{\mathcal{N}_1} &= \{(\hat{q}_1, \{d_2\}), (\hat{q}_2, \{d_1, d_3\})\} \\
F_{\mathcal{N}_2} &= \{(\hat{q}_1, \{d_1, d_2\}), (\hat{q}_2, \{d_2, d_3\})\} \\
F_{\mathcal{N}_3} &= \{(\hat{q}_1, \{d_1, d_2\}), (\hat{q}_2, \{\mathcal{D}\})\}
\end{aligned}$$

$F_{\mathcal{N}_4} = \left\{ \left(\hat{q}_1, \{d_2, d_3\}\right), \left(\hat{q}_2, \{d_1, d_3\}\right) \right\}$

A mapping $\Psi : \mathcal{L} \to \mathcal{D}$ is defined by $\Psi(\ell_1) = d_2,\ \Psi(\ell_2) = d_3,\ \Psi(\ell_3) = d_1$ and $\chi : \mathcal{P} \to \mathcal{Q}$ by $\chi(p_1) = q_2,\ \chi(p_2) = q_1$. Then $\Psi_\chi : (F_{\widetilde{\mathcal{P}}}, \mu) \to (F_{\widetilde{\mathcal{Q}}}, \eta)$ is soft $\rho_{(\mu,\eta)}$-continuous. But it is not soft $\alpha_{(\mu,\eta)}$-continuous.

Example 3.8. Let $\mathcal{V} = \{\nu_1, \nu_2, \nu_3\}, \mathcal{E} = \{\hat{e}_1, \hat{e}_2\}$ then $\mu = \left\{F_\emptyset, F_{\widetilde{\mathcal{E}}}, F_{\mathcal{A}_1}, F_{\mathcal{A}_2}, F_{\mathcal{A}_3}, F_{\mathcal{A}_4}, F_{\mathcal{A}_5}, F_{\mathcal{A}_6}\right\}$ is a $\mathcal{SGTS}$ where
$F_{\mathcal{A}_1} = \{(\hat{e}_1, \{\mathcal{V}\}), (\hat{e}_2, \{\nu_1, \nu_2\})\}$
$F_{\mathcal{A}_2} = \{(\hat{e}_1, \{\nu_1, \nu_2\}), (\hat{e}_2, \{\nu_1, \nu_2\})\}$
$F_{\mathcal{A}_3} = \{(\hat{e}_1, \{\nu_1, \nu_3\}), (\hat{e}_2, \{\nu_1, \nu_2\})\}$
$F_{\mathcal{A}_4} = \{(\hat{e}_1, \{\nu_1\}), (\hat{e}_2, \{\nu_1\})\}$
$F_{\mathcal{A}_5} = \{(\hat{e}_1, \{\nu_2\}), (\hat{e}_2, \{\nu_2\})\}$
$F_{\mathcal{A}_6} = \{(\hat{e}_1, \{\nu_2, \nu_3\}), (\hat{e}_2, \{\nu_1, \nu_2\})\}$
and

Let $\mathcal{W} = \{w_1, w_2, w_3\}, \mathcal{Z} = \{\hat{z}_1, \hat{z}_2\}$ then $\eta = \left\{F_\emptyset, F_{\widetilde{\mathcal{Z}}}, F_{\mathcal{B}_1}, F_{\mathcal{B}_2}, F_{\mathcal{B}_3}, F_{\mathcal{B}_4}\right\}$ is a $\mathcal{SGTS}$ where
$F_{\mathcal{B}_1} = \{(\hat{z}_1, \{w_2, w_3\}), (\hat{z}_2, \{\mathcal{W}\})\}$
$F_{\mathcal{B}_2} = \{(\hat{z}_1, \{w_2, w_3\}), (\hat{z}_2, \{w_2, w_3\})\}$
$F_{\mathcal{B}_3} = \{(\hat{z}_1, \{w_3\}), (\hat{z}_2, \{w_1, w_3\})\}$
$F_{\mathcal{B}_4} = \{(\hat{z}_1, \{w_1, w_3\}), (\hat{z}_2, \{w_1, w_3\})\}$

A mapping $\Psi : \mathcal{V} \to \mathcal{W}$ is defined by $\Psi(\nu_1) = w_2,\ \Psi(\nu_2) = w_3,\ \Psi(\nu_3) = w_1$ and $\chi : \mathcal{E} \to \mathcal{Z}$ by $\chi(e_1) = z_2,\ \chi(e_2) = z_1$. Then $\Psi_\chi : (F_{\widetilde{\mathcal{E}}}, \mu) \to (F_{\widetilde{\mathcal{Z}}}, \eta)$ is soft $\delta_{(\mu,\eta)}$-continuous. But it is not soft $\alpha_{(\mu,\eta)}$-continuous.

Remark 3.6. The composition of two soft $\alpha_{(\mu,\eta)}$-continuous function neednot be soft $\alpha_{(\mu,\eta)}$-continuous.

Theorem 3.10. Let $\Phi_\zeta : (F_{\widetilde{E}}, \mu) \to (F_{\widetilde{K}}, \eta)$ be a soft $\alpha_{(\mu,\eta)}$-continuous function and $\Psi_\chi : (F_{\widetilde{K}}, \eta) \to (F_{\widetilde{P}}, \xi)$ be a soft (η, ξ)-continuous map. Then $\Psi_\chi \; o \; \Phi_\zeta : (F_{\widetilde{E}}, \mu) \to (F_{\widetilde{P}}, \xi)$ is a soft $\alpha_{(\mu,\xi)}$-continuous map.

Proof. Let $F_{\mathfrak{C}_i}$ be any soft ξ-open subset of $F_{\widetilde{P}}$. Since Ψ_χ is soft (η, ξ)-continuous, $\Psi_\chi^{-1}(F_{\mathfrak{C}_i})$ is a soft η-open subset of $F_{\widetilde{K}}$. Also, we have Φ_ζ is $\alpha_{(\mu,\eta)}$-continuous, $\Phi_\zeta^{-1}(\Psi_\chi^{-1}(F_{\mathfrak{C}_i}))$ is a soft μ-α-open set in $F_{\widetilde{E}}$. Hence $\Psi_\chi \; o \; \Phi_\zeta$ is a soft $\alpha_{(\mu,\xi)}$-continuous map. $\square$

4 Soft μ-β-open sets

Definition 4.1. [9] Let $(F_{\widetilde{E}}, \mu)$ be a SGTS. Then a soft set $F_G \subset F_{\widetilde{E}}$ is said to be a soft μ-β-open set iff $F_G \subset c_\mu i_\mu c_\mu F_G$ (i.e, the case when $\pi = c_\mu i_\mu c_\mu$). The class of all soft μ-β-open sets is denoted by $\beta_{(\mu)}$ *or* β_μ.

Definition 4.2. Let $(F_{\widetilde{E}}, \mu)$ be a $\mathcal{SGTS}$. Then a soft set $F_G \mathrel{\widetilde{\subset}} F_{\widetilde{E}}$ is said to be a soft μ-β-closed set iff its complement is a soft μ-β-open set.

Example 4.1. In the above example 3.3, $\mathcal{S}\mu$-$\beta\mathcal{O}\ (F_{\widetilde{E}}) = \big\{F_\emptyset, F_{\widetilde{E}}, \{(\hat{x_1}, \{\kappa_2\}), (\hat{x_2}, \{\kappa_3\})\}, \{(\hat{x_1}, \{\kappa_3\}), (\hat{x_2}, \{\kappa_1\})\}, \{(\hat{x_1}, \{\kappa_2, \kappa_3\}), (\hat{x_2}, \{\kappa_1, \kappa_3\})\}, \{(\hat{x_1}, \{\emptyset\}), (\hat{x_2}, \{\kappa_1, \kappa_2\})\}, \{(\hat{x_1}, \{\emptyset\}), (\hat{x_2}, \{\kappa_2, \kappa_3\})\}, \{(\hat{x_1}, \{\kappa_1, \kappa_3\}), (\hat{x_2}, \{\emptyset\})\}, \{(\hat{x_1}, \{\kappa_3\}), (\hat{x_2}, \{\kappa_1, \kappa_3\})\}, \{(\hat{x_1}, \{\kappa_1, \kappa_3\}), (\hat{x_2}, \{\kappa_2, \kappa_3\})\}, \{(\hat{x_1}, \{\kappa_2\}), (\hat{x_2}, \{\kappa_2, \kappa_3\})\}\big\}$

Remark 4.1. Both $F_\emptyset$ and $F_{\widetilde{E}}$ are soft μ-β-open and soft μ-β-closed.

Theorem 4.1. Every soft μ-open set (soft μ-closed) is soft μ-β-open (soft μ-β-closed). But the converse need not be true.

Example 4.2. In the above example 4.3, $\{(\hat{x_1}, \{\kappa_1, \kappa_3\}), (\hat{x_2}, \{\emptyset\})\}$, $\{(\hat{x_1}, \{\kappa_1, \kappa_3\}), (\hat{x_2}, \{\kappa_2, \kappa_3\})\}$ are soft μ-β-open sets but not soft μ-open.

Theorem 4.2. Arbitrary soft union of soft μ-β-open sets is a soft μ-β-open set.

Proof. Let $\big\{F_{\mathcal{A}_i}\ /\ i \mathrel{\widetilde{\in}} I\big\}$ be a collection of soft μ-β-open sets in a $\mathcal{SGTS}$ $(F_{\widetilde{E}}, \mu)$. Then by the definition of soft μ-β-open set,
$F_{\mathcal{A}_i} \mathrel{\widetilde{\subseteq}} \mathcal{S}\mu\text{-}\mathcal{C}\ell\ (\mathcal{S}\mu\text{-}\mathcal{I}nt\ (\mathcal{S}\mu\text{-}\mathcal{C}\ell\ (F_{\mathcal{A}_i}))),\ \forall$ i.
Now, $\widetilde{\cup}\, F_{\mathcal{A}_i} \mathrel{\widetilde{\subseteq}} \widetilde{\cup}\, \mathcal{S}\mu\text{-}\mathcal{C}\ell\ (\mathcal{S}\mu\text{-}\mathcal{I}nt\ (\mathcal{S}\mu\text{-}\mathcal{C}\ell\ (F_{\mathcal{A}_i}))),\ \forall$ i.
By remark 2.13 (ii), $\widetilde{\cup}\, F_{\mathcal{A}_i} \mathrel{\widetilde{\subseteq}} \mathcal{S}\mu\text{-}\mathcal{C}\ell\ (\ \widetilde{\cup}\, \mathcal{S}\mu\text{-}\mathcal{I}nt\ (\mathcal{S}\mu\text{-}\mathcal{C}\ell\ (F_{\mathcal{A}_i}))),\ \forall$ i.
By remark 2.13 (i),$\widetilde{\cup}\, F_{\mathcal{A}_i} \mathrel{\widetilde{\subseteq}} \mathcal{S}\mu\text{-}\mathcal{C}\ell\ (\mathcal{S}\mu\text{-}\mathcal{I}nt\ (\ \widetilde{\cup}\, \mathcal{S}\mu\text{-}\mathcal{C}\ell\ (F_{\mathcal{A}_i}))),\ \forall$ i.
By remark 2.13 (ii), $\widetilde{\cup}\, F_{\mathcal{A}_i} \mathrel{\widetilde{\subseteq}} \mathcal{S}\mu\text{-}\mathcal{C}\ell\ (\mathcal{S}\mu\text{-}\mathcal{I}nt\ (\mathcal{S}\mu\text{-}\mathcal{C}\ell\ (\ \widetilde{\cup}\, F_{\mathcal{A}_i}))),\ \forall$ i.
Hence $\widetilde{\cup}\, F_{\mathcal{A}_i} \mathrel{\widetilde{\subseteq}} \mathcal{S}\mu\text{-}\mathcal{C}\ell\ (\mathcal{S}\mu\text{-}\mathcal{I}nt\ (\mathcal{S}\mu\text{-}\mathcal{C}\ell\ (\ \widetilde{\cup}\, F_{\mathcal{A}_i}))),\ \forall$ i. □

Remark 4.2. Finite soft intersection of soft μ-β-open sets need not be a soft μ-β-open set.

Example 4.3. In the above example 4.3, $F_{\mathcal{C}} = \{(\hat{x_1}, \{\kappa_2, \kappa_3\}), (\hat{x_2}, \{\kappa_1, \kappa_3\})\}$ and $F_{\mathcal{D}} = \{(\hat{x_1}, \{\kappa_1, \kappa_3\}), (\hat{x_2}, \{\kappa_2, \kappa_3\})\}$ are soft μ-β-open sets but $F_{\mathcal{C}} \mathrel{\widetilde{\cap}} F_{\mathcal{D}} = \{(\hat{x_1}, \{\kappa_3\}), (\hat{x_2}, \{\kappa_3\})\}$ is not soft μ-β-open.

Theorem 4.3. Arbitrary soft intersection of soft μ-β-closed sets is a soft μ-β-closed set.

Proof. The proof is similar to that of theorem 4.7 by taking its complements. □

Remark 4.3. Finite soft union of soft μ-β-closed sets need not be a soft μ-β-closed set.

Example 4.4. In example 4.3, $F_{\mathcal{S}} = \{(\hat{x_1}, \{\kappa_2, \kappa_3\}), (\hat{x_2}, \{\kappa_2\})\}$ and $F_{\mathcal{T}} = \{(\hat{x_1}, \{\kappa_3\}), (\hat{x_2}, \{\kappa_1\})\}$ are soft μ-β-closed sets but $F_{\mathcal{S}} \widetilde{\cup} F_{\mathcal{T}} = \{(\hat{x_1}, \{\kappa_2, \kappa_3\}), (\hat{x_2}, \{\kappa_1, \kappa_2\})\}$ is not soft μ-β-closed.

Definition 4.3. Let $(F_{\widetilde{E}}, \mu)$ be a $\mathcal{SGTS}$ and $F_{\mathcal{Z}} \widetilde{\subseteq} F_{\widetilde{E}}$.

(i) The soft μ-β-interior of $F_{\mathcal{Z}}$ is defined by
$\mathcal{S}\beta_{i_\mu}(F_{\mathcal{Z}}) = \widetilde{\cup}\left\{F_{\mathcal{W}} : F_{\mathcal{W}} \widetilde{\subseteq} F_{\mathcal{Z}} \text{ and } F_{\mathcal{W}} \widetilde{\in} \mathcal{S}\mu\text{-}\beta\mathcal{O}(F_{\widetilde{E}})\right\}$

(ii) The soft μ-β-closure of $F_{\mathcal{Z}}$ is defined by
$\mathcal{S}\beta_{c_\mu}(F_{\mathcal{Z}}) = \widetilde{\cap}\left\{F_{\mathcal{W}} : F_{\mathcal{Z}} \widetilde{\subseteq} F_{\mathcal{W}} \text{ and } F_{\mathcal{W}} \widetilde{\in} \mathcal{S}\mu\text{-}\beta\mathcal{C}(F_{\widetilde{E}})\right\}$

(i.e.) $\mathcal{S}\beta_{i_\mu}(F_{\mathcal{Z}})$ is the largest soft μ-β-open set contained in $F_{\mathcal{Z}}$ and $\mathcal{S}\beta_{c_\mu}(F_{\mathcal{Z}})$ is the smallest soft μ-β-closed set containing $F_{\mathcal{Z}}$.

Theorem 4.4. Let $(F_{\widetilde{E}}, \mu)$ be a $\mathcal{SGTS}$ and $F_{\mathcal{Q}},\ F_{\mathcal{R}}$ be two soft sets over $F_{\widetilde{E}}$. Then

(i) $F_{\mathcal{Q}} \widetilde{\subseteq} F_{\mathcal{R}} \Rightarrow \mathcal{S}\beta_{i_\mu}(F_{\mathcal{Q}}) \widetilde{\subseteq} \mathcal{S}\beta_{i_\mu}(F_{\mathcal{R}})$

(ii) $F_{\mathcal{Q}} \widetilde{\subseteq} F_{\mathcal{R}} \Rightarrow \mathcal{S}\beta_{c_\mu}(F_{\mathcal{Q}}) \widetilde{\subseteq} \mathcal{S}\beta_{c_\mu}(F_{\mathcal{R}})$

(iii) $\mathcal{S}\beta_{c_\mu}(F_{\mathcal{Q}} \widetilde{\cup} F_{\mathcal{R}}) = \mathcal{S}\beta_{c_\mu}(F_{\mathcal{Q}}) \widetilde{\cup} \mathcal{S}\beta_{c_\mu}(F_{\mathcal{R}})$

(iv) $\mathcal{S}\beta_{i_\mu}(F_{\mathcal{Q}} \widetilde{\cap} F_{\mathcal{R}}) = \mathcal{S}\beta_{i_\mu}(F_{\mathcal{Q}}) \widetilde{\cap} \mathcal{S}\beta_{i_\mu}(F_{\mathcal{R}})$

(v) $\mathcal{S}\beta_{c_\mu}(F_{\mathcal{Q}} \widetilde{\cap} F_{\mathcal{R}}) \widetilde{\subseteq} \mathcal{S}\beta_{c_\mu}(F_{\mathcal{Q}}) \widetilde{\cap} \mathcal{S}\beta_{c_\mu}(F_{\mathcal{R}})$

(vi) $\mathcal{S}\beta_{i_\mu}(F_{\mathcal{Q}} \widetilde{\cup} F_{\mathcal{R}}) \widetilde{\supseteq} \mathcal{S}\beta_{i_\mu}(F_{\mathcal{Q}}) \widetilde{\cup} \mathcal{S}\beta_{i_\mu}(F_{\mathcal{R}})$

Proof. (i) Since $F_{\mathcal{Q}}$ is a soft μ-open set, $\mathcal{S}\beta_{i_\mu}(F_{\mathcal{Q}}) \widetilde{\subseteq} F_{\mathcal{Q}} \widetilde{\subseteq} F_{\mathcal{R}}$ which implies $\mathcal{S}\beta_{i_\mu}(F_{\mathcal{Q}}) \widetilde{\subseteq} F_{\mathcal{R}}$ and $\mathcal{S}\beta_{i_\mu}(F_{\mathcal{R}})$ is the largest soft μ-β-open set contained in $F_{\mathcal{R}}$. Hence $\mathcal{S}\beta_{i_\mu}(F_{\mathcal{Q}}) \widetilde{\subseteq} \mathcal{S}\beta_{i_\mu}(F_{\mathcal{R}})$.

(ii) since $F_{\mathcal{Q}} \widetilde{\subseteq} \mathcal{S}\beta_{c_\mu}(F_{\mathcal{R}})$ and $F_{\mathcal{R}} \widetilde{\subseteq} \mathcal{S}\beta_{c_\mu}(F_{\mathcal{R}})$, $F_{\mathcal{Q}} \widetilde{\subseteq} F_{\mathcal{R}} \widetilde{\subseteq} \mathcal{S}\beta_{c_\mu}(F_{\mathcal{R}})$. Hence $F_{\mathcal{Q}} \widetilde{\subseteq} \mathcal{S}\beta_{c_\mu}(F_{\mathcal{R}})$ and $\mathcal{S}\beta_{c_\mu}(F_{\mathcal{Q}})$ is the smallest soft μ-β-closed containing $F_{\mathcal{Q}}$. Therefore, $\mathcal{S}\beta_{c_\mu}(F_{\mathcal{Q}}) \widetilde{\subseteq} \mathcal{S}\beta_{c_\mu}(F_{\mathcal{R}})$.

(iii) We have $F_{\mathcal{Q}} \widetilde{\subseteq} F_{\mathcal{Q}} \widetilde{\cup} F_{\mathcal{R}}$ and $F_{\mathcal{R}} \widetilde{\subseteq} F_{\mathcal{Q}} \widetilde{\cup} F_{\mathcal{R}}$. From (ii), $\mathcal{S}\beta_{c_\mu}(F_{\mathcal{Q}}) \widetilde{\subseteq} \mathcal{S}\beta_{c_\mu}(F_{\mathcal{Q}} \widetilde{\cup} F_{\mathcal{R}})$ and $\mathcal{S}\beta_{c_\mu}(F_{\mathcal{R}}) \widetilde{\subseteq} \mathcal{S}\beta_{c_\mu}(F_{\mathcal{Q}} \widetilde{\cup} F_{\mathcal{R}})$ which implies $\mathcal{S}\beta_{c_\mu}\ (F_{\mathcal{Q}})\ \widetilde{\cup}\ \mathcal{S}\beta_{c_\mu}\ (F_{\mathcal{R}})\ \widetilde{\subseteq}\ \mathcal{S}\beta_{c_\mu}\ (F_{\mathcal{Q}}\ \widetilde{\cup}\ F_{\mathcal{R}})$. Since $F_{\mathcal{Q}}\ \widetilde{\subseteq}\ \mathcal{S}\beta_{c_\mu}\ (F_{\mathcal{Q}})$ and $F_{\mathcal{R}}\ \widetilde{\subseteq}\ \mathcal{S}\beta_{c_\mu}\ (F_{\mathcal{R}})$, $F_{\mathcal{Q}}\ \widetilde{\cup}\ F_{\mathcal{R}}\ \widetilde{\subseteq}\ \mathcal{S}\beta_{c_\mu}\ (F_{\mathcal{R}})\ \widetilde{\cup}\ \mathcal{S}\beta_{c_\mu}\ (F_{\mathcal{R}})$. As $\mathcal{S}\beta_{c_\mu}\ (F_{\mathcal{Q}})\ \widetilde{\cup}\ \mathcal{S}\beta_{c_\mu}\ (F_{\mathcal{R}})$ is a soft μ-β-closed set containing $F_{\mathcal{Q}}\ \widetilde{\cup}\ F_{\mathcal{R}}$, $\mathcal{S}\beta_{c_\mu}\ (F_{\mathcal{Q}}\ \widetilde{\cup}\ F_{\mathcal{R}})$ is the smallest soft μ-β-closed set containing $F_{\mathfrak{Q}}\ \widetilde{\cup}\ F_{\mathfrak{R}}$. Therefore, $\mathcal{S}\beta_{c_\mu}\ (F_{\mathcal{Q}}\ \widetilde{\cup}\ F_{\mathcal{R}})\ \widetilde{\subseteq}\ \mathcal{S}\beta_{c_\mu}\ (F_{\mathcal{Q}})\ \widetilde{\cup}\ \mathcal{S}\beta_{c_\mu}\ (F_{\mathcal{R}})$ which implies $\mathcal{S}\beta_{c_\mu}\ (F_{\mathcal{Q}}\ \widetilde{\cup}\ F_{\mathcal{R}}) = \mathcal{S}\beta_{c_\mu}\ (F_{\mathcal{Q}})\ \widetilde{\cup}\ \mathcal{S}\beta_{c_\mu}\ (F_{\mathcal{R}})$.

(iv) We have $F_{\mathcal{Q}}\ \widetilde{\cap}\ F_{\mathcal{R}}\ \widetilde{\subseteq}\ F_{\mathcal{Q}}$ and $F_{\mathcal{Q}}\ \widetilde{\cap}\ F_{\mathcal{R}}\ \widetilde{\subseteq}\ F_{\mathcal{R}}$. From (i), $\mathcal{S}\beta_{i_\mu}\ (F_{\mathcal{Q}}\ \widetilde{\cap}\ F_{\mathcal{R}})\ \widetilde{\subseteq}\ \mathcal{S}\beta_{i_\mu}\ (F_{\mathcal{Q}})$ and $\mathcal{S}\beta_{i_\mu}\ (F_{\mathcal{Q}}\ \widetilde{\cap}\ F_{\mathcal{R}})\ \widetilde{\subseteq}\ \mathcal{S}\beta_{i_\mu}\ (F_{\mathcal{R}})$ which implies $\mathcal{S}\beta_{i_\mu}\ (F_{\mathcal{Q}}\ \widetilde{\cap}\ F_{\mathcal{R}})\ \widetilde{\subseteq}\ \mathcal{S}\beta_{i_\mu}\ (F_{\mathcal{Q}})\ \widetilde{\cap}\ \mathcal{S}\beta_{i_\mu}\ (F_{\mathcal{R}})$. Since $\mathcal{S}\beta_{i_\mu}\ (F_{\mathcal{Q}})\ \widetilde{\subseteq}\ F_{\mathcal{Q}}$ and $\mathcal{S}\beta_{i_\mu}\ (F_{\mathcal{R}})\ \widetilde{\subseteq}\ F_{\mathcal{R}}, \mathcal{S}\beta_{i_\mu}\ (F_{\mathcal{Q}})\ \widetilde{\cap}\ \mathcal{S}\beta_{i_\mu}\ (F_{\mathcal{R}})\ \widetilde{\subseteq}\ F_{\mathcal{Q}}\ \widetilde{\cap}\ F_{\mathcal{R}}$. As $\mathcal{S}\beta_{i_\mu}\ (F_{\mathcal{Q}})\ \widetilde{\cap}\ \mathcal{S}\beta_{i_\mu}\ (F_{\mathcal{R}})$ is a soft μ-β-open set contained in $F_{\mathcal{Q}}\ \widetilde{\cap}\ F_{\mathcal{R}}$, $\mathcal{S}\beta_{i_\mu}\ (F_{\mathcal{Q}}\ \widetilde{\cap}\ F_{\mathcal{R}})$ is the largest soft μ-β-open set contained in $F_{\mathcal{Q}}\ \widetilde{\cap}\ F_{\mathcal{R}}$. Therefore, $\mathcal{S}\beta_{i_\mu}\ (F_{\mathcal{Q}})\ \widetilde{\cap}\ \mathcal{S}\beta_{i_\mu}\ (F_{\mathcal{R}})\ \widetilde{\subseteq}\ \mathcal{S}\beta_{i_\mu}\ (F_{\mathcal{Q}}\ \widetilde{\cap}\ F_{\mathcal{R}})$. Hence $\mathcal{S}\beta_{i_\mu}\ (F_{\mathcal{Q}}\ \widetilde{\cap}\ F_{\mathcal{R}}) = \mathcal{S}\beta_{i_\mu}\ (F_{\mathcal{Q}})\ \widetilde{\cap}\ \mathcal{S}\beta_{i_\mu}\ (F_{\mathcal{R}})$.

(v) We have $F_{\mathcal{Q}}\ \widetilde{\cap}\ F_{\mathcal{R}}\ \widetilde{\subseteq}\ F_{\mathcal{Q}}$ and $F_{\mathcal{Q}}\ \widetilde{\cap}\ F_{\mathcal{R}}\ \widetilde{\subseteq}\ F_{\mathcal{R}}$ which implies $\mathcal{S}\beta_{c_\mu}\ (F_{\mathcal{Q}}\ \widetilde{\cap}\ F_{\mathcal{R}})\ \widetilde{\subseteq}\ \mathcal{S}\beta_{c_\mu}\ (F_{\mathcal{Q}})$ and $\mathcal{S}\beta_{c_\mu}\ (F_{\mathcal{Q}}\ \widetilde{\cap}\ F_{\mathfrak{R}})\ \widetilde{\subseteq}\ \mathcal{S}\beta_{c_\mu}\ (F_{\mathcal{R}})$. Hence $\mathcal{S}\beta_{c_\mu}\ (F_{\mathcal{Q}}\ \widetilde{\cap}\ F_{\mathcal{R}})\ \widetilde{\subseteq}\ \mathcal{S}\beta_{c_\mu}\ (F_{\mathcal{Q}})\ \widetilde{\cap}\ \mathcal{S}\beta_{c_\mu}\ (F_{\mathcal{R}})$.

(vi) We have $F_{\mathcal{Q}}\ \widetilde{\subseteq}\ F_{\mathcal{Q}}\ \widetilde{\cup}\ F_{\mathcal{R}}$ and $F_{\mathcal{R}}\ \widetilde{\subseteq}\ F_{\mathcal{Q}}\ \widetilde{\cup}\ F_{\mathcal{R}}$ which implies $\mathcal{S}\beta_{i_\mu}\ (F_{\mathcal{Q}})\ \widetilde{\subseteq}\ \mathcal{S}\beta_{i_\mu}\ (F_{\mathcal{Q}}\ \widetilde{\cup}\ F_{\mathcal{R}})$ and $\mathcal{S}\beta_{i_\mu}\ (F_{\mathcal{R}})\ \widetilde{\subseteq}\ \mathcal{S}\beta_{i_\mu}\ (F_{\mathcal{Q}}\ \widetilde{\cup}\ F_{\mathcal{R}})$. Hence $\mathcal{S}\beta_{i_\mu}\ (F_{\mathcal{Q}})\ \widetilde{\cup}\ \mathcal{S}\beta_{i_\mu}\ (F_{\mathcal{R}})\ \widetilde{\subseteq}\ \mathcal{S}\beta_{i_\mu}\ (F_{\mathcal{Q}}\ \widetilde{\cup}\ F_{\mathcal{R}})$.
(i.e.) $\mathcal{S}\beta_{i_\mu}\ (F_{\mathcal{Q}}\ \widetilde{\cup}\ F_{\mathcal{R}})\ \widetilde{\supseteq}\ \mathcal{S}\beta_{i_\mu}\ (F_{\mathcal{Q}})\ \widetilde{\cup}\ \mathcal{S}\beta_{i_\mu}\ (F_{\mathcal{R}})$. □

Theorem 4.5. Let $(F_{\widetilde{E}}, \mu)$ be a $\mathcal{SGTS}$ and $F_{\mathcal{V}}$ be a soft set over $F_{\widetilde{E}}$. Then

(i) $F_{\mathcal{V}}$ is a soft μ-β-closed set.

(ii) $\mathcal{S}\mu$-$\mathfrak{Cl}$ $(\mathcal{S}\mu$-$\mathfrak{Int}$ $(\mathcal{S}\mu$-$\mathfrak{Cl}$ $(F_{\mathcal{V}})^c))\ \widetilde{\supseteq}\ (F_{\mathcal{V}})^c$.

(iii) $\mathcal{S}\mu$-$\mathfrak{Int}$ $(\mathcal{S}\mu$-$\mathfrak{Cl}$ $(\mathcal{S}\mu$-$\mathfrak{Int}$ $(F_{\mathcal{V}})))\ \widetilde{\subseteq}\ F_{\mathcal{V}}$.

(iv) $(F_{\mathcal{V}})^c$ is a soft μ-β-open set.

Definition 4.4. Let $(F_{\widetilde{E}}, \mu)$ and $(F_{\widetilde{K}}, \eta)$ be any two $\mathcal{SGTS}$'s. A soft function $\Psi_\chi : (F_{\widetilde{E}}, \mu) \to (F_{\widetilde{K}}, \eta)$ is said to be soft $\beta_{(\mu,\eta)}$-continuous (briefly, soft (μ, η)-beta-continuous), if the inverse image of each soft η-open subset in $F_{\widetilde{K}}$ is soft μ-β-open in $F_{\widetilde{E}}$.

Theorem 4.6. Each soft (μ, η)-continuous is soft $\beta_{(\mu,\eta)}$-continuous.

Proof. Let $\Psi_\chi : (F_{\widetilde{E}}, \mu) \to (F_{\widetilde{K}}, \eta)$ be a soft (μ, η)-continuous mapping. Let $F_{\mathcal{W}}$ be any soft η-open set in $F_{\widetilde{K}}$. since Ψ_χ is soft (μ, η)-continuous, its inverse image $\Psi_\chi^{-1}(F_{\mathcal{W}})$ is a soft μ-open set in $F_{\widetilde{E}}$ and every soft μ-open set is soft μ-β-open, $\Psi_\chi^{-1}(F_{\mathcal{W}})$ is soft μ-β-open in $F_{\widetilde{E}}$. Hence Ψ_χ is soft $\beta_{(\mu,\eta)}$-continuous. □

Example 4.5. Let $\mathcal{R} = \{r_1, r_2, r_3\}$, $\mathcal{E} = \{\hat{t}_1, \hat{t}_2\}$, then $\mu = \{F_\emptyset, F_{\widetilde{\mathcal{E}}}, F_{\mathfrak{A}_1}, F_{\mathfrak{A}_2}, F_{\mathfrak{A}_3}, F_{\mathfrak{A}_4}, F_{\mathfrak{A}_5}, F_{\mathfrak{A}_6}\}$ is a $\mathcal{SGTS}$ where
$F_{\mathfrak{A}_1} = \{(\hat{t}_1, \{r_1, r_2\}), (\hat{t}_2, \{r_1, r_2\})\}$
$F_{\mathfrak{A}_2} = \{(\hat{t}_1, \{r_2, r_3\}), (\hat{t}_2, \{r_1, r_2\})\}$
$F_{\mathfrak{A}_3} = \{(\hat{t}_1, \{r_2\}), (\hat{t}_2, \{r_2\})\}$
$F_{\mathfrak{A}_4} = \{(\hat{t}_1, \{r_1\}), (\hat{t}_2, \{r_1\})\}$
$F_{\mathfrak{A}_5} = \{(\hat{t}_1, \{\mathcal{R}\}), (\hat{t}_2, \{r_1, r_2\})\}$
$F_{\mathfrak{A}_6} = \{(\hat{t}_1, \{r_1, r_3\}), (\hat{t}_2, \{r_1, r_2\})\}$
and

Let $\mathcal{H} = \{h_1, h_2, h_3\}$, $\mathcal{K} = \{\hat{i}_1, \hat{i}_2\}$, then $\eta = \{F_\emptyset, F_{\widetilde{\mathcal{K}}}, F_{\mathfrak{B}_1}, F_{\mathfrak{B}_2}, F_{\mathfrak{B}_3}, F_{\mathfrak{B}_4}\}$ is a $\mathcal{SGTS}$ where
$F_{\mathfrak{B}_1} = \{(\hat{i}_1, \{h_1, h_2\}), (\hat{i}_2, \{\mathcal{H}\})\}$
$F_{\mathfrak{B}_2} = \{(\hat{i}_1, \{h_1, h_3\}), (\hat{i}_2, \{h_3\})\}$
$F_{\mathfrak{B}_3} = \{(\hat{i}_1, \{h_1, h_2\}), (\hat{i}_2, \{h_1, h_2\})\}$
$F_{\mathfrak{B}_4} = \{(\hat{i}_1, \{h_1\}), (\hat{i}_2, \{h_3\})\}$

A function $\Psi : \mathcal{R} \to \mathcal{H}$ is defined by $\Psi(r_1) = h_2,\ \Psi(r_2) = h_3,\ \Psi(r_3) = h_1$ and $\chi : \mathcal{E} \to \mathcal{K}$ by $\psi(t_1) = i_2,\ \chi(t_2) = i_1$. Then $\Psi_\chi : (F_{\widetilde{\mathcal{E}}}, \mu) \to (F_{\widetilde{\mathcal{K}}}, \eta)$ is soft $\beta_{(\mu,\eta)}$-continuous. But it is not soft (μ, η)-continuous .

Remark 4.4. The composition of two soft $\beta_{(\mu,\eta)}$-continuous function neednot be soft $\beta_{(\mu,\eta)}$-continuous.

Theorem 4.7. Let $\Phi_\zeta : (F_{\widetilde{E}}, \mu) \to (F_{\widetilde{K}}, \eta)$ be a soft $\beta_{(\mu,\eta)}$-continuous function and $\Psi_\chi : (F_{\widetilde{K}}, \eta) \to (F_{\widetilde{P}}, \xi)$ be a soft (η, ξ)-continuous map. Then $\Psi_\chi\ o\ \Phi_\zeta : (F_{\widetilde{E}}, \mu) \to (F_{\widetilde{P}}, \xi)$ is a soft $\beta_{(\mu,\xi)}$-continuous map.

Proof. Let $F_{\mathfrak{C}_i}$ be any soft ξ-open subset of $F_{\widetilde{P}}$. Since Ψ_χ is soft (η, ξ)-continuous, $\Psi_\chi^{-1}(F_{\mathfrak{C}_i})$ is a soft η-open subset of $F_{\widetilde{K}}$. Also, we have Φ_ζ is $\beta_{(\mu,\eta)}$-continuous, $\Phi_\zeta^{-1}(\Psi_\chi^{-1}(F_{\mathfrak{C}_i}))$ is a soft μ-β-open set in $F_{\widetilde{E}}$. Hence $\Psi_\chi\ o\ \Phi_\zeta$ is a soft $\beta_{(\mu,\xi)}$-continuous map. □

Theorem 4.8. Any soft μ-β-open set which is soft μ-semi-closed is soft μ-semi-open.

Proof. Let $F_\mathcal{A}$ be any soft μ-β-open set. Then $F_\mathcal{A} \widetilde{\in} \mathcal{S}\mu\text{-}\beta\mathcal{O}\ (F_{\widetilde{E}})$ which implies $F_\mathcal{A} \widetilde{\subseteq} \mathcal{S}\mu\text{-}\mathcal{C}\ell\ (\mathcal{S}\mu\text{-}\mathcal{I}nt\ (\mathcal{S}\mu\text{-}\mathcal{C}\ell\ (F_\mathcal{A})))$. Since $F_\mathcal{A}$ is a soft μ-semi-closed set, $F_\mathcal{A} \widetilde{\in} \mathcal{S}\mu\text{-}\mathcal{SC}\ (F_{\widetilde{E}})$ which implies $\mathcal{S}\mu\text{-}\mathcal{I}nt\ (\mathcal{S}\mu\text{-}\mathcal{C}\ell\ (F_\mathcal{A})) \widetilde{\subseteq} F_\mathcal{A}$. Now, $\mathcal{S}\mu\text{-}\mathcal{I}nt\ (\mathcal{S}\mu\text{-}\mathcal{C}\ell\ (F_\mathcal{A})) \widetilde{\subseteq} F_\mathcal{A} \widetilde{\subseteq} \mathcal{S}\mu\text{-}\mathcal{C}\ell\ (\mathcal{S}\mu\text{-}\mathcal{I}nt\ (\mathcal{S}\mu\text{-}\mathcal{C}\ell\ (F_\mathcal{A})))$. Since $\mathcal{S}\mu\text{-}\mathcal{I}nt\ (\mathcal{S}\mu\text{-}\mathcal{C}\ell\ (F_\mathcal{A}))$ = $F_\mathcal{H}$ is a soft μ-open set, $F_\mathcal{H} \widetilde{\subseteq} F_\mathcal{A} \widetilde{\subseteq} \mathcal{S}\mu\text{-}\mathcal{C}\ell\ (F_\mathcal{H})$. Therefore $F_\mathcal{A}$ is a soft μ-semi-open set. □

Corollary 4.1. Any soft set which is both soft μ-β-closed and soft μ-semi-open is soft μ-semi-closed.

Theorem 4.9. The soft intersection of a soft μ-α-open set and a soft μ-β-open set is a soft μ-β-open set.

Proof. Let $F_{\mathfrak{A}}$ be a soft μ-α-open set and $F_{\mathfrak{B}}$ be a soft μ-β-open set. Then
$$\begin{aligned} F_{\mathfrak{A}} \widetilde{\cap} F_{\mathfrak{B}} &\widetilde{\subseteq} \mathcal{S}\mu\text{-}\mathfrak{I}nt\ (\mathcal{S}\mu\text{-}\mathcal{C}\ell\ (\mathcal{S}\mu\text{-}\mathfrak{I}nt\ (F_{\mathcal{A}}))) \widetilde{\cap} \mathcal{S}\mu\text{-}\mathcal{C}\ell\ (\mathcal{S}\mu\text{-}\mathfrak{I}nt\ (\mathcal{S}\mu\text{-}\mathcal{C}\ell\ (F_{\mathfrak{B}}))) \\ &\widetilde{\subseteq} \mathcal{S}\mu\text{-}\mathcal{C}\ell\ [\mathcal{S}\mu\text{-}\mathfrak{I}nt\ (\mathcal{S}\mu\text{-}\mathcal{C}\ell\ (\mathcal{S}\mu\text{-}\mathfrak{I}nt\ (F_{\mathcal{A}}))) \widetilde{\cap} \mathcal{S}\mu\text{-}\mathfrak{I}nt\ (\mathcal{S}\mu\text{-}\mathcal{C}\ell\ (F_{\mathfrak{B}}))] \\ &= \mathcal{S}\mu\text{-}\mathcal{C}\ell\ [\mathcal{S}\mu\text{-}\mathfrak{I}nt\ [\mathcal{S}\mu\text{-}\mathcal{C}\ell\ (\mathcal{S}\mu\text{-}\mathfrak{I}nt\ (F_{\mathcal{A}})) \widetilde{\cap} \mathcal{S}\mu\text{-}\mathfrak{I}nt\ (\mathcal{S}\mu\text{-}\mathcal{C}\ell\ (F_{\mathfrak{B}}))]] \\ &\widetilde{\subseteq} \mathcal{S}\mu\text{-}\mathcal{C}\ell\ [\mathcal{S}\mu\text{-}\mathfrak{I}nt\ [\mathcal{S}\mu\text{-}\mathcal{C}\ell\ [\mathcal{S}\mu\text{-}\mathfrak{I}nt\ (F_{\mathcal{A}}) \widetilde{\cap} \mathcal{S}\mu\text{-}\mathfrak{I}nt\ (\mathcal{S}\mu\text{-}\mathcal{C}\ell\ (F_{\mathfrak{B}}))]]] \\ &= \mathcal{S}\mu\text{-}\mathcal{C}\ell\ [\mathcal{S}\mu\text{-}\mathfrak{I}nt\ [\mathcal{S}\mu\text{-}\mathcal{C}\ell\ [\mathcal{S}\mu\text{-}\mathfrak{I}nt\ (F_{\mathcal{A}}) \widetilde{\cap} \mathcal{S}\mu\text{-}\mathcal{C}\ell\ (F_{\mathfrak{B}})]]] \\ &\widetilde{\subseteq} \mathcal{S}\mu\text{-}\mathcal{C}\ell\ [\mathcal{S}\mu\text{-}\mathfrak{I}nt\ [\mathcal{S}\mu\text{-}\mathcal{C}\ell\ [\mathcal{S}\mu\text{-}\mathfrak{I}nt\ (F_{\mathcal{A}}) \widetilde{\cap} (F_{\mathfrak{B}})]]] \\ &\widetilde{\subseteq} \mathcal{S}\mu\text{-}\mathcal{C}\ell\ [\mathcal{S}\mu\text{-}\mathfrak{I}nt\ [\mathcal{S}\mu\text{-}\mathcal{C}\ell\ [(F_{\mathcal{A}}) \widetilde{\cap} (F_{\mathfrak{B}})]]]. \end{aligned}$$
Hence $F_{\mathfrak{A}} \widetilde{\cap} F_{\mathfrak{B}}$ is a soft μ-β-open set. □

Corollary 4.2. The soft union of a soft μ-α-closed set and a soft μ-β-closed set is a soft μ-β-closed set.

Theorem 4.10. Every soft μ-β-open set which is soft μ-α-closed is a soft μ-closed set.

Proof. Let $F_{\mathcal{M}}$ be any soft μ-β-open set. Then $F_{\mathcal{M}} \widetilde{\in} \mathcal{S}\mu\text{-}\beta\mathcal{O}\ (F_{\widetilde{E}})$ which implies $F_{\mathcal{M}} \widetilde{\subseteq} \mathcal{S}\mu\text{-}\mathcal{C}\ell\ (\mathcal{S}\mu\text{-}\mathfrak{I}nt\ (\mathcal{S}\mu\text{-}\mathcal{C}\ell\ (F_{\mathcal{M}})))$. Since $F_{\mathcal{M}}$ is a soft μ-α-closed set, $\mathcal{S}\mu\text{-}\mathcal{C}\ell\ (\mathcal{S}\mu\text{-}\mathfrak{I}nt\ (\mathcal{S}\mu\text{-}\mathcal{C}\ell\ (F_{\mathcal{M}}))) \widetilde{\subseteq} F_{\mathcal{M}}$ which implies $F_{\mathcal{M}} = \mathcal{S}\mu\text{-}\mathcal{C}\ell\ (\mathcal{S}\mu\text{-}\mathfrak{I}nt\ (\mathcal{S}\mu\text{-}\mathcal{C}\ell\ (F_{\mathcal{M}})))$.Hence $F_{\mathcal{M}}$ is soft μ-closed. □

Corollary 4.3. Every soft μ-β-closed set which is soft μ-α-open is a soft μ-open set.

5 Conclusion

In the current work, we study some of the properties of soft μ-α-open sets, soft μ-α-closed sets, soft μ-β-open sets and soft μ-β-closed sets in soft generalized topological spaces using soft μ-α-interior, soft μ-α-closure, soft μ-β-interior and soft μ-β-closure respectively. We discuss the soft continuous functions related to soft μ-open sets and determine the necessary and sufficient condition for the composition mappings in soft μ-open sets. Also, we examine the inter-relationship between soft (μ, η)-continuous, soft $\rho_{(\mu,\eta)}$-continuous, soft $\delta_{(\mu,\eta)}$-continuous, soft $\alpha_{(\mu,\eta)}$-continuous and soft $\beta_{(\mu,\eta)}$-continuous mappings with suitable examples.

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