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Nano fuzzy bitopological spaces

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Abstract

The objective of this paper is to introduce the concept of Nano fuzzy bitopological space with respect to the fuzzy subsets of any finite universe, which is defined in terms of Nano fuzzy lower and Nano fuzzy upper approximations of any fuzzy subset. After the introduction of Nano fuzzy topological space theory given by R. Navalkahe and P. Rajwade [16], efforts were made to enhance its precision through further expansion. So, in this paper there is also an attempt to define Nano fuzzy $(1,2)^*$ closure and Nano fuzzy $(1,2)^*$ interior with the help of Nano fuzzy $(1,2)^*$ closed sets and Nano fuzzy $(1,2)^*$ open sets respectively in Nano fuzzy bitopological space. We have expanded our work and defined Nano fuzzy $(1,2)^*$ generalized fuzzy closed sets in Nano fuzzy bitopological space and investigated some of its properties.

Keywords: Nano fuzzy bitopological spaces; Nano fuzzy $(1,2)^*$ open sets; Nano fuzzy $(1,2)^*$ closed sets; Nano fuzzy $(1,2)^*$ generalized closed sets.

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1. Introduction

In Mathematics, the significance of topological spaces can not be oversated as they represent a crucial and fundamental concept. Levine [14] introduced the concept of generalized closed sets in topological spaces. The theory of Nano topology was introduced and developed by L. Thivagar [12]. K. Bhuvanshwari et al. [9] introduced and investigated the concept of Nano generalized closed set in Nano topological space. The classical Nano topological space is based on an equivalence relation on a set. Kelly [6] introduced the concepts of bitopological spaces [4,5]. In addition to this theory, T. Fukutake [17,18] has defined and explored the theory of semi closed sets and generalized closed sets in bitopological spaces. K. Bhuvneshwari et al. [10] have introduced Nano bitopological spaces. In 2016, K. Bhuvaneshwari et al. [7,8] defined Nano generalized closed sets in Nano bitopological space. Fuzzy set is an important concept, which was introduced for the first time in 1965 by Zadeh [11], it was then used in many studies in various fields. Here, we are interested in fuzzy with topology. The fuzzy logic and fuzzifying topologies are two branches of fuzzy mathematics. The basic concepts and properties of fuzzy topologies were investigated by Chang in 1968 [3].

R. Navalakhe and P. Rajwade have introduced Nano fuzzy topological spaces [16] and Nano fuzzy generalized closed sets [15,16]. This was the source of motivation for fuzzifying Nano bitopological spaces and thus we tried to define Nano fuzzy bitopological spaces. In this paper, Nano fuzzy bitopological spaces are introduced and new class of Nano fuzzy closed sets in Nano fuzzy bitopological spaces which can be denoted by Nano fuzzy $(1,2)^*$ closed sets and Nano fuzzy $(1,2)^*$ generalized closed sets are introduced and their properties are studied.

2. Preliminaries

In this section, we briefly recall the definition and basic results on Nano topological space.

2.1. [10] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

(1) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$.

That is $L_R(X). = \bigcup_{x \in U}\{R(x): R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.

(2) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x \in U}\{R(x): R(x) \cap X = \emptyset\}$.

(3) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X) = U_R(X) - L_R(X)$.

2.2. [9] A subset A of a Nano topological space $(U, \tau_R(X))$ is called a Nano generalized closed set (briefly Ng-closed set) if $Ncl(A) \subseteq V$ Whenever $A \subseteq V$ and V is open in $(U, \tau_R(X))$.

2.3. [10] Let U be the universe, R be an equivalence relation on U and $\tau_{R_{1,2}}(X) = \cup\{\tau_{R_1}(X), \tau_{R_2}(X)\}$, where $X \subseteq U$. Then it satisfies the following axioms:

(1) U and $\phi \in \tau_{R_{1,2}}(X)$.
(2) The union of the elements of any sub-collection of $\tau_{R_{1,2}}(X)$ is in $\tau_{R_{1,2}}(X)$.
(3). The intersection of the elements of any finite sub collection of $\tau_{R_{1,2}}(X)$ is in $\tau_{R_{1,2}}(X)$.

Then $\tau_{R_{1,2}}(X)$ is a topology on U called the Nano bitopology on U with respect to X. $(U, \tau_{R_{1,2}}(X))$ is called the Nano bitopological space. Elements of the Nano bitopology are known as Nano $(1,2)^*$ open sets in U. Elements of $\tau_{R_{1,2}}(X)^C$ are called Nano $(1,2)^*$ closed sets in $\tau_{R_{1,2}}(X)$.

2.4. [10] If $(U, \tau_{R_{1,2}}(X))$ is a Nano bitopological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then
(1) The Nano $(1,2)^*$ closure of A is defined as the intersection of all Nano $(1,2)^*$ closed sets containing A and it is denoted by $N\tau_{R_{1,2}}cl(A)$. It is the smallest Nano $(1,2)^*$ closed set containing A.
(2) The Nano $(1,2)^*$ interior of A is defined as the union of all Nano $(1,2)^*$ open subsets of A contained in A and it is denoted by $N\tau_{R_{1,2}}Int(A)$. It is the largest Nano $(1,2)^*$open subset of A.

2.5. [10] A subset A of a bitopological space $(U, \tau_{R_{1,2}}(X))$ is called Nano $(1,2)^*$ generalized closed set if $N\tau_{R_{1,2}}cl(A) \subseteq V$ where $A \subseteq V$ and V is Nano $(1,2)^*$ open.

2.6. [1] Let (X, R) be an approximation space and $X/R = \{\lambda_1, \lambda_{2,\dots,}\lambda_n\}$. Then for any $\lambda \leq F(X)$, $\underline{R}(\lambda)$ *and* $\overline{R}(\lambda)$, the lower and upper approximations of λ with respect to R are fuzzy sets in X/R. That is, $\underline{R}(\lambda)$, $\overline{R}(\lambda): X/R \to [0,1]$, such that
(1) $\underline{R}(\lambda)(\lambda_j) = inf_{y \leq \lambda_j} \lambda(y)$ and
(2) $\overline{R}(\lambda)(\lambda_j) = sup_{y \leq \lambda_j} \lambda(y)$, for all $j = 1, 2, 3, \dots\dots, n$.

2.7. [2] Let λ be a fuzzy set in a fuzzy topological space(X, τ). then the fuzzy boundary of λ is defined as

(1) $Bd(\lambda) = Cl(\lambda) \wedge Cl(\lambda^c)$
(2) $Bd(\lambda) = Cl(\lambda) - Int(\lambda)$.

2.8. [2, 13] Let λ be a fuzzy subset of X. Then following are equivalent:
(1) Upper approximation of λ denoted by $\overline{R}(\lambda)$ is the fuzzy closure operator.
(2) Lower approximation of λ denoted by $\underline{R}(\lambda)$ is the fuzzy interior operator.
So, we can say that in a fuzzy topological space

$$Bd(\lambda) = Cl(\lambda) - Int(\lambda) = \bar{R}(\lambda) - \underline{R}(\lambda)$$

2.9. [2] Properties of Fuzzy Approximation Space

Let R be an arbitrary relation from X to Y. The lower and upper approximation operators of a fuzzy set $\underline{\mathrm{R}}$ and $\overline{\mathrm{R}}$ satisfies the following properties: for all $\alpha, \beta \in F(X)$,

(FL1) $\underline{R}(\alpha) = (\overline{R}(\alpha^C))^C$

(FU1) $\overline{R}(\alpha) = (\underline{R}(\alpha^C))^C$

(FL2) $\underline{R}(\alpha \wedge \beta) = \underline{R}(\alpha) \wedge \underline{R}(\beta)$

(FU2) $\overline{R}(\alpha \vee \beta) = \overline{R}(\alpha) \vee \overline{R}(\beta)$

(FL3) $\alpha \leq \beta \Rightarrow \underline{R}(\alpha) \leq \underline{R}(\beta)$

(FU3) $\alpha \leq \beta \Rightarrow \overline{R}(\alpha) \leq \overline{R}(\beta)$

(FL4) $\underline{R}(\alpha \vee \beta) = \underline{R}(\alpha) \vee \underline{R}(\beta)$

(FU4) $\overline{R}(\alpha \wedge \beta) = \overline{R}(\alpha) \wedge \overline{R}(\beta)$

2.10. [16] Let X be a non-empty finite set, R be an equivalence relation on X, $\lambda \leq X$ be a fuzzy subset and $\tau_R(\lambda) = \{X, \emptyset, \underline{R}(\lambda), \overline{R}(\lambda), Bd(\lambda)\}$. Then by property (2.9) $\tau_R(\lambda)$ satisfies the following axioms
(1) $0_\lambda, 1_\lambda \in \tau_{(R)}(\lambda)$ where $0_\lambda: \lambda \to I$ denotes the null fuzzy sets and $1_\lambda: \lambda \to I$ denotes the whole fuzzy set.
(2) Arbitrary union of members of $\tau_{(R)}(\lambda)$ is a member of $\tau_{(R)}(\lambda)$.
(3) Finite intersection of members of $\tau_{(R)}(\lambda)$ is a member of $\tau_{(R)}(\lambda)$.

That is, $\tau_{(R)}(\lambda)$ is a topology on X called the Nano fuzzy topology on X with respect to λ. We call $(X, \tau_{(R)}(\lambda))$ as the Nano fuzzy topological space (NFTS). The elements of the Nano fuzzy topological space that is $\tau_{(R)}(\lambda)$, are called Nano fuzzy open sets and elements of $[\tau_{(R)}(\lambda)]^C$ are called Nano fuzzy closed sets.

2.11. [16] Let $(X, \tau_{(R)}(\lambda))$ be a Nano fuzzy topological space with respect to λ where $\lambda \leq X$ and if $\mu \leq X$ then the Nano fuzzy interior of μ is defined as union of all Nano fuzzy open subsets of μ and it is denoted by $NfInt(\mu)$. That is, it is the largest Nano fuzzy open subset contained in μ. Similarly, the Nano fuzzy closure of μ is defined as the intersection of all Nano fuzzy closed sets containing μ. It is denoted by $NfCl(\mu)$ and it is the smallest Nano fuzzy closed set containing μ.

3. Nano Fuzzy Bitopological Space and Nano Fuzzy $(1,2)^*$ Generalized Closed Sets

In this section we have introduced Nano fuzzy bitopological spaces and investigated some of its properties. We have also defined Nano fuzzy $(1,2)^*$ generalized closed sets in terms of Nano fuzzy $(1,2)^*$ closure and studied some of its properties. Throughout this paper $(X, \tau_{1,2_R}(\lambda))$ is a Nano fuzzy bitopological space with respect to fuzzy subset λ and R is an equivalence relation on X, X/R denotes the family of equivalence classes of X by R.

Definition 3.1. Let X be a non-empty finite set, R be an equivalence relation on X, $\lambda_1, \lambda_2 \leq X$ be fuzzy subsets and $\tau_{1,2_R}(\lambda) = \vee\{\tau_{1_R}(\lambda_1), \tau_{2_R}(\lambda_2)\}$. Then $\tau_{1,2_R}(\lambda)$ satisfies the following axioms
(1) $0_{\lambda_1}, 1_{\lambda_1} \in \tau_{1_R}(\lambda_1)$ where $0_{\lambda_1}: \lambda_1 \to I$ denotes the null fuzzy sets and $1_{\lambda_1}: \lambda_1 \to I$ denotes the whole fuzzy set and $0_{\lambda_2}, 1_{\lambda_2} \in \tau_{2_R}(\lambda_2)$ where $0_{\lambda_2}: \lambda_2 \to I$ denotes the null fuzzy sets and $1_{\lambda_2}: \lambda_2 \to I$ denotes the whole fuzzy set.
(2) Arbitrary union of members of $\tau_{1,2_R}(\lambda)$ is a member of $\tau_{1,2_R}(\lambda)$.
(3) Finite intersection of members of $\tau_{1,2_R}(\lambda)$ is a member of $\tau_{1,2_R}(\lambda)$.

That is, $\tau_{1,2_R}(\lambda)$ is a topology on X called the Nano fuzzy topology on X with respect to λ_1 *and* λ_2. We call $\left(X, \tau_{1,2_R}(\lambda)\right)$ as the Nano fuzzy bitopological space (NFBTS). The elements of the Nano fuzzy bitopological space are called Nano fuzzy $(1,2)^*$ open sets and elements of $[\tau_{1,2_R}(\lambda)]^C$ are called Nano fuzzy $(1,2)^*$ closed sets.

Example 3.2. Let $X = \{x_1, x_2, x_3, x_4\}$ with

$X/R = \{\{x_1\}, \{x_2\}, \{x_3, x_4\}\}$. Let λ_1 *and* λ_2 be a fuzzy subset of X

such that $\lambda_1 = \{(x_1, 0.2), (x_2, 0.8), (x_3, 0.7), (x_4, 0.4)\}$

and $\lambda_2 = \{(x_1, 0.1), (x_2, 0.5), (x_3, 0.6), (x_4, 0.4)\}$. Then NFBTS is given by

$\tau_{1,2_R}(\lambda) = \left\{1_\lambda, 0_\lambda, \underline{R_{1,2}}(\lambda), \overline{R_{1,2}}(\lambda), Bd_{1,2}(\lambda)\right\}$, where

$\underline{R_{1,2}}(\lambda) = \{(x_1, 0.1), (x_2, 0.5), (x_3, 0.4), (x_4, 0.4)\}$ and

$\overline{R_{1,2}}(\lambda) = \{(x_1, 0.2), (x_2, 0.8), (x_3, 0.7), (x_4, 0.7)\}$ with

$Bd_{1,2}(\lambda) = \{(x_1, 0.1), (x_2, 0.3), (x_3, 0.3), (x_4, 0.3)\}$.

Definition 3.3. Let $(X, \tau_{1,2_R}(\lambda))$ be a Nano fuzzy bitopological space with respect to $\lambda_1 and\ \lambda_2$ where $\lambda_1, \lambda_2 \leq X$ and if $\mu \leq X$ then the Nano fuzzy $(1,2)^*$ interior of μ is defined as union of all Nano fuzzy $(1,2)^*$ open subsets of μ and it is denoted by $Nf(1,2)^*\ Int(\mu)$. That is, it is the largest Nano fuzzy $(1,2)^*$ open subset contained in μ. Similarly, the Nano fuzzy $(1,2)^*$ closure of μ is defined as the intersection of all Nano fuzzy $(1,2)^*$ closed sets containing μ. It is denoted by $Nf(1,2)^*\ Cl(\mu)$ and it is the smallest Nano fuzzy $(1,2)^*$ closed set containing μ.

Definition 3.4. Let $(X, \tau_{1,2_R}(\lambda))$ be a Nano fuzzy bitopological space. A subset μ of $(X, \tau_{1,2_R}(\lambda))$ is called Nano fuzzy $(1,2)^*$ generalized closed set if $Nf(1,2)^*\ Cl(\mu) \leq \vartheta$ where $\mu \leq \vartheta$ and ϑ is Nano fuzzy $(1,2)^*$ open.

Example 3.5. Let $X = \{x_1, x_2, x_3, x_4\}$ with

$X/R = \{\{x_1\}, \{x_2\}, \{x_3, x_4\}\}$. Let $\lambda_1\ and\ \lambda_2$ be a fuzzy subset of X such that $\lambda_1 = \{(x_1, 0.2), (x_2, 0.8), (x_3, 0.7), (x_4, 0.4)\}$

and $\lambda_2 = \{(x_1, 0.1), (x_2, 0.5), (x_3, 0.6), (x_4, 0.4)\}$. Then NFBTS is given by

$\tau_{1,2_R}(\lambda) = \left\{1_\lambda, 0_\lambda, \underline{R_{1,2}}(\lambda), \overline{R_{1,2}}(\lambda), Bd_{1,2}(\lambda)\right\}$, where

$\underline{R_{1,2}}(\lambda) = \{(x_1, 0.1), (x_2, 0.5), (x_3, 0.4), (x_4, 0.4)\}$ and

$\overline{R_{1,2}}(\lambda) = \{(x_1, 0.2), (x_2, 0.8), (x_3, 0.7), (x_4, 0.7)\}$ with

$Bd_{1,2}(\lambda) = \{(x_1, 0.1), (x_2, 0.3), (x_3, 0.3), (x_4, 0.3)\}$.

Then Nano fuzzy $(1,2)^*$ closed sets are

$[\tau_{1,2_R}(\lambda)]^C = \left\{1_\lambda, 0_\lambda,\ \underline{R_{1,2}}(\lambda)^C,\ \overline{R_{1,2}}(\lambda)^C,\ Bd_{1,2}(\lambda)^C\right\}$ where

$[\underline{R_{1,2}}(\lambda)]^C = \{(x_1, 0.9), (x_2, 0.5), (x_3, 0.6), (x_4, 0.6)\}$. $\overline{R_{1,2}}(\lambda)]^C = \{(x_1, 0.8), (x_2, 0.2), (x_3, 0.3), (x_4, 0.3)\}$ and

$[Bd_{1,2}(\lambda)]^C = \{(x_1, 0.9), (x_2, 0.7), (x_3, 0.7), (x_4, 0.7)\}$.

Let $\mu \leq X = \{(x_1, 0.2), (x_2, 0.1), (x_3, 0.2), (x_4, 0)\}$. Clearly, $Nf(1,2)^*\ Cl(\mu) \leq \vartheta$ where $\mu \leq \vartheta$ and ϑ is Nano fuzzy $(1,2)^*$ open. Hence, μ is Nano fuzzy $(1,2)^*$ generalized closed set.

Theorem 3.6. If μ and μ' are Nano fuzzy $(1,2)^*$ generalized closed, then $\mu \vee \mu'$ is Nano fuzzy $(1,2)^*$ generalized closed.

Proof: Let μ and μ' are Nano fuzzy (1,2)* generalized closed sets. Then $Nf(1,2)^*\, Cl(\mu) \leq \nu$ where $\mu \leq \nu$ and ν is Nano fuzzy (1,2)* open and $Nf(1,2)^*\, Cl(\mu') \leq \nu$ where $\mu' \leq \nu$ and ν is Nano fuzzy (1,2)* open. Since μ and μ' are subsets of ν, $\mu \vee \mu'$ is a subset of ν and ν is Nano fuzzy (1,2)* open. Then $Nf(1,2)^*\, Cl(\mu \vee \mu') = Nf(1,2)^*\, Cl(\mu) \vee Nf(1,2)^*\, Cl(\mu') \leq \nu$ which implies that $\mu \vee \mu'$ is Nano fuzzy (1,2)* generalized closed.

Theorem 3.7. If μ is Nano fuzzy (1,2)* generalized closed set and $\mu \leq \mu' \leq Nf(1,2)^*\, Cl(\mu)$, then μ' is Nano fuzzy (1,2)* generalized closed.
Proof: Let $\mu' \leq \nu$ where ν is Nano fuzzy (1,2)* generalized open in $\tau_{1,2_R}(\lambda)$. Then $\mu \leq \mu'$ implies $\mu \leq \nu$. Since μ is Nano fuzzy (1,2)* generalized closed, $Nf(1,2)^*\, Cl(\mu) \leq \nu$. Also $\mu' \leq Nf(1,2)^*\, Cl(\mu)$ implies $Nf(1,2)^*\, Cl(\mu') \leq Nf(1,2)^*\, Cl(\mu) \leq \nu$ and so μ' is Nano fuzzy (1,2)* generalized closed.

Theorem 3.8. Every Nano fuzzy (1,2)* closed set is Nano fuzzy (1,2)* generalized closed set.
Proof: Let $\mu \leq \nu$ be a Nano fuzzy (1,2)* closed set and ν is Nano fuzzy (1,2)* open in Nano fuzzy bitopological space $\tau_{1,2_R}(\lambda)$. Since μ is Nano fuzzy (1,2)* closed, $Nf(1,2)^*\, Cl(\mu) \leq \mu$. That is $Nf(1,2)^*\, Cl(\mu) \leq \mu \leq \nu$. Hence μ is Nano fuzzy (1,2)* generalized closed set.

Theorem 3.9. (for operation consider [1]) A Nano fuzzy (1,2)* generalized closed set μ is Nano fuzzy (1,2)* closed set if and only if $Nf_{\tau_{1,2_R}} Cl(\mu) - \mu$ is Nano fuzzy (1,2)* closed.
Proof: Let μ be a Nano fuzzy (1,2)* closed set. Then $Nf_{\tau_{1,2_R}} Cl(\mu) = \mu$ and so, $Nf_{\tau_{1,2_R}} Cl(\mu) - \mu = \phi$ which is Nano fuzzy (1,2)* closed set.
Conversely, suppose $Nf_{\tau_{1,2_R}} Cl(\mu) - \mu$ is Nano fuzzy (1,2)* closed. Then $Nf_{\tau_{1,2_R}} Cl(\mu) - \mu = \phi$. Since μ be a Nano fuzzy (1,2)* closed set. This implies that $Nf_{\tau_{1,2_R}} Cl(\mu) = \mu$ and so μ is Nano fuzzy (1,2)* closed.

Theorem 3.10. Suppose that $\beta \leq \alpha \leq X$, β is a Nano fuzzy (1,2)* generalized closed set relative to α and that α is a Nano fuzzy (1,2)* generalized closed set relative to X. Then β is Nano fuzzy $(1,2)^*$ generalized closed set relative to X.
Proof: Let $\beta \leq \nu$ and suppose that ν is Nano fuzzy (1,2)* open in X. Then $\beta \leq \alpha \wedge \nu$. Therefore, $Nf_{\tau_{1,2_R}} Cl_\alpha(\beta) \leq \alpha \wedge \nu$. It follows that $\alpha \wedge Nf_{\tau_{1,2_R}} Cl(\beta) \leq \alpha \wedge \nu$ and $\alpha \leq \nu \vee Nf_{\tau_{1,2_R}} Cl(\beta)$. Since α is a Nano fuzzy (1,2)* generalized closed set relative to X, we have $Nf_{\tau_{1,2_R}} Cl(\alpha) \leq \nu \vee Nf_{\tau_{1,2_R}} Cl(\beta)$. Therefore, $Nf_{\tau_{1,2_R}} Cl(\beta) \leq Nf_{\tau_{1,2_R}} Cl(\alpha) \leq \nu \vee Nf_{\tau_{1,2_R}} Cl(\beta)$. Then β is Nano fuzzy (1,2)* generalized closed set relative to X.

4 Conclusion

In this research work, we characterize Nano fuzzy bitopological spaces through the framework of topological spaces, specifically as an extension of Nano fuzzy topological spaces. To elaborate on the evolution of this theory , we have defined Nano fuzzy $(1,2)^*$ closed sets, Nano fuzzy $(1,2)^*$ open sets, Nano fuzzy $(1,2)^*$ generalized closed sets and Nano fuzzy $(1,2)^*$ interior and closure. We established the relationships among these defined Nano fuzzy sets. In the forthcoming work, Nano fuzzy $(1,2)^*$ continuous functions and Nano fuzzy $(1,2)^*$ open maps, Nano fuzzy $(1,2)^*$ closed maps and Nano fuzzy $(1,2)^*$ homeomorphisms. Expressing this deifferently, exploring these concepts may prove valuable in uncovering practical applications.

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