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Stochastic Perishable Inventory System Analysis With Two Types Of Customers In The Supply Chain

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Abstract

In the supply chain model being examined in this study, there are two different client expectations. A warehouse, a single distribution centre, a single retailer, and the handling of a single perishable product make up the system. Retailer node (lower echelon) assumes a $A(s, S)$ type inventory system with Poisson demand and exponentially dispersed lead times, and distribution centre assumes a $(0, kQ)$ policy (middle echelon). Demands that happen during the times when there is no stock are taken to be lost sales. The merchandise is sent from the distribution centre to the stores in packs of $Q(= S - s)$ items. It is believed that only the retailer nodes with rate γ experience item perishability. The upper echelon (warehouse) replaces the distribution center's stock with exponentially distributed lead times due to its ample supply. Demands arrive at the retailer node in two categories: (i) normal customers; and (ii) priority customers, with arrival rates of α and β. It is possible to obtain the metrics of system performance as well as the steady state probability distribution of system states. The proposed paradigm is demonstrated with numerical examples.

Keywords: Supply chain management, Perishable Inventory System, Markov process, Perishable inventory Optimization. [1]

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1 Introduction to Inventory Control System

For a healthy business, the inventory system should be updated daily or on a regular basis. These updates also make the system portable. What is in your factory, office, or retail space serves as the basis of your inventory system. This gives notice of goods for sale or allows you to create products to readily meet customer demands. The stock of goods, commodities, or other economic resources that are kept on hand or set aside for future use in manufacturing or satisfying demand can be widely referred to as inventory. When products are needed, the inventory system alerts the user to buy or supply them. To deal with circumstances where demand, lead time, or both fluctuate, an inventory control system was designed.

The study of supply chain management (SCM) started in the late 1980s and has acquired a growing amount of interest from both companies and researchers over the following 3 decades. The term supply chain management has several different definitions. The Stanford Global Supply Chain Management Forum's (1999) director, Hau Lee, provides the following clear and concise definition on the forum's website:

The management of resources, information, and financial flows in a network made up of suppliers, manufacturers, distributors, and customers is the focus of supply chain management. From this description, it is clear that SCM affects service and financial enterprises in addition to manufacturing businesses. We can even assert that practically all businesses will experience SCM issues, and that SCM decisions will have an impact on how the entire organisation runs. More and more businesses are recognising the value of SCM and making efforts to enhance the efficiency of their supply chains.

A supply chain can be described as an integrated process where a number of different corporate entities (suppliers, distributors, and retailers) collaborate to (1) obtain raw materials, (2) convert them into useful products, (3) produce and convey the finished goods to retailers. The movement of commodities through this network requires an effective transportation and communication system. The forward movement of products and materials and the backward flow of data and money are the two traditional characteristics of the supply chain.

One of the key elements of supply chain management is inventory control. Stock control models are usually always stochastic optimization problems with either expected costs, projected profits, or hazards as the objectives. In reality, a merchant might seek the best option that minimises expected costs or maximises expected profits with the least amount of chance of deviating from the goal.

Perishable commodities are those that have a fixed or predetermined lifespan beyond which they are deemed useless and cannot be used to satisfy demand. Because in real life products like milk, blood, drugs, food, vegetables, and some chemicals actually have defined life spans after which they would perish, planning and controlling perishable inventory systems is crucial. The existence of these kind products after their lifetime will not only consume space of the store, but also harm the lifetime (damage) of the nearby objects. At specific times, the things that have degraded (exponentially degraded) may be taken out of the stock.

As a result of recent technology improvements and innovations in the sector of telecommunications and transportation, the supply chain management (SCM) industry is growing. This SCM performs a very significant function in each and every field. Retailers of goods such as food, medicine, and household appliances have opened locations all across the country in order to better service their clients.

Benita [1998] performed an exhaustive review. Yet, the performance, design, and analysis of the supply chain as a whole are receiving more and more attention. This type of investigation was started in 1977 by the Strategic Planning and Modeling (SPaM) team of HP (Hawlett Packard). From a practical standpoint, a number of changes in the manufacturing environment?including rising manufacturing costs, declining manufacturing base resources, shortened product life cycles, levelling of the playing field in manufacturing planning, inventory-driven costs (IDC) associated with distribution, and the globalisation of market economics leads to the development of the supply chain concept. The supply chain concept emerged from two-stage multi-echelon inventory models in manufacturing research, and it is significant to note that much of the research in this field is based on the seminal work of Clark and Scarf [1960].

Federgruen [1993] compiled a thorough analysis of this development. Significant advancements in two-echelon models can be found in Ming and Jewkes [2000], Axaster [1993], and Nahimas [1982]. Krishnan.K. and Elango.C. 2005 studied a continuous review (s, S) strategy with favourable lead times in a two-echelon supply chain. In supply chains with partial backorders, stochastic inventory control systems were thought to exist by Bakthavachalam et al. [2012]. Inventory policy for perishable inventory system with (s, Q) policy in supply chain is introduced by Kumaresan et al. [2014]. In the supply chain, an integrated inventory control system is being built by Kartheeswari and Bakthavachalam [2017]. Multi-Echelon Inventory Optimization for Fresh Goods in Supply Chains is being considered by Zhang et al. [2021].

The other sections of the study are structured as follows: section 2 describes the model formulation, section 3, and section 4 performs steady state analyses for our novel model. The system's operational features are covered in Section 4, and the cost analysis for the operation is covered in Section 5. A few numerical examples are provided in section 6, and section 7 concludes the paper.

2 The Model Description

The supply chain inventory control system taken into account in this paper is defined as follows: A perishable product is sent from the warehouse to the distribution centre, which follows the $(0, kQ)$ policy and from the distribution centre to the retailer, who follows the (s, S) policy. A Poisson distribution with rate $\alpha > 0$ and Priority demand $(1 + 1)$ with rate $\beta > 0$ describe the normal demand at the retailer node, respectively. online shopping plays a very essential part in the industry nowadays, they are provided priority and some discounts(1+1 Offer) by distinguishing themselves as the most important customer of their firm in order to attract customers. An exponential lead time with a parameter of $\mu > 0$ is used to administer the supply to the store in packets of $Q(= S - s)$ items. Demands that happen during the times when there is no stock are taken to be lost sales. The maximum inventory levels nQ and S in our model are constant, whereas the retailer's reorder level s varies so that $S - s = Q$. As stated in the definition above, both nodes' on-hand inventory amounts are determined by random processes. It is believed that only the retailer nodes with rate γ experience item perishability.

3 Analysis

Let $IR(t)$ and $ID(t)$ denote, respectively, the retailer node and distribution center's (DC) on-hand inventory levels at time t^+. Given the assumptions on the input and output process, $I(t) = (IR(t), ID(t)) : t \geq 0$ is a Markov process with state space
$E = \{(i, j)/i = S, S-1 \ldots, s, s-1, \ldots, 3, 2, 1, 0.$ and $j = nQ, (n-1)Q, \ldots Q\}$.

Theorem 3.1. *The vector process* $I(t) : t \geq 0$ *where* $I(t) = (I_0(t), I_1(t))$ *for* $t \geq 0$ *is a continuous time Markov Chain with state space*

$$E = \{(j, q)/j = 0, 1, 2..., S; q = Q, 2Q, ..., nQ\}$$

Proof. The stochastic process $\{I(t) : t \geq 0\}$ has a discrete state space with order relation $' \leq'$ that $(j, q) \leq (k, r)$ if and only if $j \leq k$ and $q \leq r$.To prove that $I(t) : t \geq 0$ is a Markov chain ,first we do a transformation for state space E to E' such that $(j, q) \longrightarrow j + q \in E'$,where $E' = \{Q, Q + 1, ..., Q + S, ..., nQ + S\}$.

Now we may realize that $\{I(t) : t \geq 0\}$ is a stochastic process with discrete state space E'.

The joint distribution of random variables $\{I(t_1), I(t_2), ..., I(t_n)\}$ and $\{I(t_1 + \tau), I(t_2 + \tau), ..., I(t_n + \tau)\}$ with $\tau > 0$ (an arbitrary real number) are equal.

In particular the conditional probability

$$P_r\{I_n = k|I_{n-1} = j, I_{n-2} = i, ...I_0 = 1\} = P_r\{I_n = k|I_{n-1} = j\}$$

due to the single step transition of states in E.
Hence $\{I(t) : t \geq 0\}$ is a continuous time Markov Chain.

□

The following inputs can be used to get this process' infinitesimal generator, R.

1. The Markov Process transitions from (i, j) to $(i - 1, j)$ with an intensity of transition of $\alpha > 0$ in response to the arrival of a normal demand or perish at retailer node.
2. When a Priority demand or perish arrives at the retailer node, the Markov Process transitions from (i, j) to $(i - 2, j)$ with an intensity of transition of $\beta > 0$.
3. Inventory replenishment from a destruction centre to a retailer changes from (i, j) to $(i + Q, j - Q)$ and from (i, Q) to $(i + Q, nQ)$ with a transition rate of $\mu(> 0)$.

The infinitesimal generator R is defined as follows:

$$R = \begin{array}{r} nQ \\ (n-1)Q \\ (n-2)Q \\ \cdots \\ Q \end{array} \begin{bmatrix} A & B & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & A & B & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & A & B & 0 & 0 & \cdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ A & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & B \end{bmatrix} \quad (1)$$

As follows is a way to express the entries of the matrix R.

$$[R]_{p \times q} = \begin{cases} A \text{ if p } = \text{ q, q } = \text{ nQ , (n-1)Q,(n-2)Q...2Q} \\ A \text{ if p } = \text{ Q} \\ B \text{ if p } = \text{ q+Q, q = (n-1)Q,(n-2)Q...2Q,Q} \\ 0 \text{ otherwise} \end{cases} \quad (2)$$

$$[A]_{m \times n} = \begin{cases} \alpha + i\gamma & \text{n=m+1} & \text{m=S,S-1,....,s+1,s,s-1,...3,2,1.} \\ \beta + i\gamma & \text{n=m+2} & \text{m=S,S-1,....,s+1,s,s-1,...3,2.} \\ -(\alpha + \beta + i\gamma) & \text{n=m} & \text{m=S,S-1,....,s+1} \\ -(\alpha + \beta + \mu + i\gamma) & \text{n=m} & m = \text{s,s-1,...3,2.} \\ -(\alpha + \mu + i\gamma) & \text{n=m} & \text{m=1.} \\ -\mu & \text{n=m} & \text{m=0.} \\ 0 & \text{otherwise} & \end{cases} \quad (3)$$

$$[\mathrm{B}]_{\mathrm{m\,x\,n}} = \begin{cases} \mu & \text{if} \qquad \mathrm{m} = \mathrm{n} + \mathrm{Q}\,,\, j = \mathrm{s,s\text{-}1,...1,0.} \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

3.1 Steady state analysis

The state space E of the Markov process, $\{I(t); t \geq 0\}$, is finite and irreducible, as shown by the structure of the infinitesimal matrix R. Allowing for the limiting probability Distribution of the inventory level process to be

$\Pi_i^j = \lim_{t\to\infty} \Pr\{(IR(t), ID(t) = (i,j)\}$, where$\Pi_i^j$ is the steady state probability that the system be in state (i,j), (Cinlar Hadley and Whitin [1963]).

Let $\Pi = \left(\Pi^{kQ}, \Pi^{(k-1)Q}, \Pi^{(k-2)Q}, \ldots, \Pi^{Q}.\right)$ be the probability vector indicating the steady state probability distribution for the system under investigation. By resolving the matrix equation $\Pi R = 0$, it is possible to get Π_j^q for each (i,j). Hence, the steady state balance equations are

$$\Pi^{kQ} A + \sum_{\mathrm{i}=0}^{\mathrm{k\text{-}1}} \Pi^{iQ} B = 0 \tag{5}$$

$$\Pi^{Q}\, B + \Pi^{kQ}\, \mathrm{A} = 0 \tag{6}$$

furthermore, the normalising equation

$$\sum_{(i,j)\in E} \Pi_j^q = 1. \tag{7}$$

4 System performance measures

4.1 Mean Inventory Levels

The steady-state mean inventory level at the retailer node is represented by the overline symbol $\overline{IR}$ and is calculated as follows:

$$\overline{IR} = {\sum_{j=Q}^{nQ}}^{*} \left(\sum_{i=0}^{S} i.\Pi_i^j \right) \tag{8}$$

The Distribution Center's average inventory level is indicated by the symbol $\overline{ID}$ and is provided by

$$\overline{ID} = \sum_{i=0}^{S} \left(\sum_{j=Q}^{nQ\,*} j.\Pi_i^j \right). \tag{9}$$

Algorithm 1

1. Recursively determine the matrix $D_n, 0 \leq n \leq N$ by using
$$D_0 = A_0$$
$$D_n = A_n + B_n(-D_{n-1}^{-1})\, C, \quad n = 1, 2, ...K$$
2. the system's solution
$$\Pi^{<N>} D_N = 0$$
3. Calculate the vector iteratively $\Pi^{<n>}, \quad n = N-1, ..., 0$ using
$$\Pi^{<n>} \;=\; \Pi^{<n+1>} B_{n+1}(-D_n^{-1}), n = n-1, ..., 0$$
4. Renormalizing the vector Π, using $\Pi e = 1$.

4.2 Mean Reorder Rates

The mean reorder rate at the retailer node is represented by R_r and is calculated as follows:

$$R_r = (\alpha + \beta + \gamma) \sum_{j=Q}^{nQ} {}^{*}\Pi_{s+1}^{j} + (\beta + \gamma) \sum_{j=Q}^{nQ} {}^{*}\Pi_{s+2}^{j} \tag{10}$$

The Distribution Center's mean reorder rate is shown by the symbol R_d and is calculated as follows:

$$R_d = \mu \sum_{j=0}^{s} \Pi_j^Q \tag{11}$$

4.3 Mean Reorder Rates

The mean shortage rate at the retailer node is shown by T and is calculated as follows:

$$T = (\alpha + \beta) \sum_{j=Q}^{nQ} {}^{*}\Pi_0^j + \beta \sum_{j=Q}^{nQ} {}^{*}\Pi_1^j \tag{12}$$

5 Cost Analysis

The cost structure for the suggested model is imposed in this part, and it is evaluated according to the criterion of minimising the steady state (long run) total predicted cost per unit of time. Let k_r and k_d be setup costs, and h_r and h_d be holding costs per unit item per unit time at the retailer and distribution centre, respectively. The cost of the scarcity, which only occurs at retailer nodes, is shown by the symbol S_r. The estimated long-term cost rate $C(s, Q)$ is provided by

$$C(s,Q) = h_r\overline{IR} + h_d\overline{IR} + k_rR_r + k_dR_d + s_rT \tag{13}$$

6 Numerical Example and Sensitivity Analysis

This section discusses the challenge of reducing the long-term total predicted cost per unit of time given the following cost structure. The following numerical example might be used to show the results from the steady state scenarios. In the example that follows, it is presummated that $S = 12, n = 3, \alpha = 5, \beta = 1.75, \mu = 1.25, h_r = 0.025, h_d = 0.075, k_r = 4.5, k_d = 5.5, T = 0.25$ For various reorder levels, the total anticipated cost rate is provided by

s	Cost	Q
2	248.5732765	28
3	261.6411365	27
4	252.9833491	26
5	262.6257595	25
6	252.5650419	24
7	259.6892737	23
8	248.8257261	22
9	253.7464519	21
10	257.2118433	20
11	244.9712722	19
12	246.2623777	18
13	246.0741916	17
14	244.3801994*	16

Table 1: Expected Cost Rate Analysis (C(s, Q)) in a Perishable Inventory System with S=30 and M=300

In Table 1, we present the total expected cost rate for different combinations of reorder level (s) and order quantity (Q) when S=30 and M=300. The minimum cost is achieved when s=14 and Q=16, with a cost of 240.3801994. This suggests that the optimum reorder level is 14, and the optimum order quantity is 16.

Figure 1: Expected Cost Rate Analysis (C(s, Q)) in a Perishable Inventory System with S=30 and M=300

Figure 1 visually represents the data from Table 1 as a line graph. The x-axis represents the reorder level (s), and the y-axis represents the total expected cost rate. Each line on the graph corresponds to a different order quantity (Q). It is evident from the graph that there is a clear minimum point, indicating the combination of s=14 and Q=16, which results in the lowest total expected cost rate.

This graphical representation reinforces the findings from Table 1, highlighting the importance of selecting the right combination of reorder level and order quantity to minimize the total expected cost in the stochastic perishable inventory system with two types of customers in the supply chain. The optimal values of s=14 and Q=16 should be considered for cost-efficient inventory management in this system.

s	Cost	Q
2	280.9923	48
3	277.6517	47
4	273.0369	46
5	267.6404	45
6	261.8845	44
7	256.0216	43
8	281.6608	42
9	275.125	41
10	268.6639	40
11	262.2744	39
12	255.9477*	38
13	277.4151	37
14	270.435	36
15	263.4905	35
16	256.575	34
17	274.4263	33
18	266.8035	32
19	259.1963	31
20	274.0963	30
21	265.761	29
22	257.4307	28
23	269.3426	27
24	260.235	26

Table 2: Expected Cost Rate Analysis (C(s, Q)) in a Perishable Inventory System with S=50 and M=300

The results presented in Table 2 demonstrate the relationship between the total expected cost rate, reorder level (s), and order quantity (Q) in the context of the stochastic perishable inventory system with two types of customers. Notably, with fixed values of S=50 and M=300, the minimum cost of 255.9477 is attained when employing an optimal reorder level of 12 and an optimal order quantity of 38. This signifies that in the studied supply chain, the most cost-efficient strategy involves maintaining a reorder level of 12 units and placing orders of 38 units.

Figure 2: Expected Cost Rate Analysis (C(s, Q)) in a Perishable Inventory System with S=50 and M=300

Figure 2 visually represents the data from Table 2 through a line graph, reinforcing the observation that the total expected cost rate is minimized at the specified reorder level and order quantity. The graph provides a clear visualization of the cost-saving benefits associated with this optimal inventory management strategy.

s	Cost	Q	s	Cost	Q	s	Cost	Q
2	498.3686	98	18	483.9871	82	34	447.7754	66
3	497.8622	97	19	478.5274	81	35	441.7263	65
4	494.7201	96	20	473.0933	80	36	435.6802	64
5	489.9647	95	21	467.6803	79	37	429.6367	63
6	484.4736	94	22	462.2851	78	38	470.0873	62
7	478.7646	93	23	456.905	77	39	463.2988	61
8	473.0843	92	24	451.5376	76	40	456.5122	60
9	467.5287	91	25	446.1812	75	41	449.7275	59
10	462.1217	90	26	440.8343	74	42	442.9445	58
11	456.8565	89	27	435.4956	73	43	436.1631	57
12	451.7155	88	28	430.1641	72	44	429.3829*	56
13	446.6787	87	29	478.0778	71	45	463.8472	55
14	441.7277	86	30	472.0084	70	46	456.3177	54
15	436.8465	85	31	465.9439	69	47	448.7853	53
16	432.022	84	32	459.8838	68	48	441.243	52
17	489.4777	83	33	453.8278	67	49	433.6725	51

Table 3: Expected Cost Rate Analysis (C(s, Q)) in a Perishable Inventory System with S=100 and M=500

Figure 3: Expected Cost Rate Analysis (C(s, Q)) in a Perishable Inventory System with S=100 and M=500

Table 3 presents the total expected cost rate in the stochastic perishable inventory system with two customer types. With S=100 and M=500, the optimal reorder level (s=44) and order quantity (Q=56) result in the lowest cost of 429.3829. Figure 3 visualizes this data in a line graph.

h_r/h_d	0.1	0.2	0.3	0.4	0.5
0.1	41.94327	67.14327	92.34327	117.5433	142.7433
0.2	52.08175	77.28175	102.4817	127.6817	152.8817
0.3	62.22022	87.42022	112.6202	137.8202	163.0202
0.4	72.3587	97.5587	122.7587	147.9587	173.1587
0.5	82.49717	107.6972	132.8972	158.0972	183.2972

Table 4: Comparison of h_r & h_d

The results, as presented in Table 4 and illustrated in Figure 4, indicate that as both h_r & h_d increase, there is a noticeable increase in the total cost. This suggests a direct relationship between higher reliability and demand levels and increased operational costs in the perishable inventory system.

Figure 4: Comparison of h_r & h_d

k_r/k_d	0.1	0.2	0.3	0.4	0.5
0.1	435.2638	435.2676	435.2714	435.2753	435.2791
0.2	435.257	435.2608	435.2647	435.2685	435.2723
0.3	435.2502	435.254	435.2579	435.2617	435.2655
0.4	435.2434	435.2472	435.2511	435.2549	435.2588
0.5	435.2366	435.2404	435.2443	435.2481	435.252

Table 5: Comparison of k_r & k_d

It was observed that as both k_r & k_d increased, the total cost exhibited a consistent decrease trend, as illustrated in Figure 5, which corresponds to the data presented in Table 5.

Figure 5: Comparison of k_r & k_d

α/β	0.1	0.2	0.3	0.4	0.5
0.1	428.989	429.1205	429.252	429.3835	429.5151
0.2	429.1199	429.2514	429.3829	429.5144	429.646
0.3	429.2508	429.3823	429.5138	429.6453	429.7769
0.4	429.3817	429.5132	429.6447	429.7762	429.9077
0.5	429.5126	429.6441	429.7756	429.9071	430.0386

Table 6: Comparison of α & β

Table 6. clearly demonstrates that as both α & β increase, the total cost also increases. This relationship is further illustrated in Figure 6, where the data from Table 6. is graphically represented, reaffirming the positive correlation between α, β, and total cost.

Figure 6: Comparison of α & β

α/μ	0.1	0.2	0.3	0.4	0.5
0.1	428.76	429.004	429.252	429.5043	429.7608
0.2	428.8903	429.1345	429.3829	429.6355	429.8924
0.3	429.0205	429.2651	429.5138	429.7667	430.024
0.4	429.1508	429.3957	429.6447	429.898	430.1556
0.5	429.2811	429.5263	429.7756	430.0292	430.2872

Table 7: Comparison of α & μ

The cost value increases as the values of α & μ increase, according to Table 7.

Table 7 and Figure 7 show that the total expected cost rate increases as the arrival rate (α) and deterioration rate (μ) increase. This is because a higher α and μ mean that the retailer needs to hold a higher inventory level to meet demand and to prevent perishable items from deteriorating, which increases the holding costs.

Figure 7: Comparison of α & μ

7 Conclusion

In this research paper, our primary focus is on analyzing a complex supply chain scenario that involves two distinct client categories. Within this supply chain, we implement a continuous review inventory control system to efficiently manage inventory levels. Our investigation aims to shed light on various aspects of this system, ranging from its overall performance metrics to the steady-state probability distribution of different system states.

To provide a comprehensive understanding of the subject matter, we begin by outlining the key elements of our study. We delve into the intricacies of this supply chain to extract valuable insights and metrics that can assist in optimizing inventory management for businesses facing similar challenges.

One crucial aspect of our study is to quantify the system's performance. We will employ a variety of performance indicators and metrics to evaluate how effectively the supply chain functions under different conditions. These measurements will provide valuable insights into the efficiency, reliability, and responsiveness of the inventory control system.

Additionally, we will delve into the steady-state probability distribution of system states. This analysis will help us gain a deeper understanding of the system's behavior over time. By examining the probability distribution, we can identify potential bottlenecks, vulnerabilities, or areas for improvement within the supply chain.

As part of our research, we will also provide practical demonstrations of the suggested model. These numerical examples will illustrate the real-world applicability of our approach and offer valuable insights for practitioners and decision-makers in the field of supply chain management.

However, our research does not stop here. We have plans for further expansion and refinement of our model. In the future, we intend to incorporate perishable goods into the equation, taking into account their unique characteristics and handling requirements. This expansion will enable us to address the complexities associated with managing goods that have limited shelf lives.

Furthermore, we will consider partial backlogging as a crucial factor in our extended model. This addition will allow us to tackle situations where clients may not receive their orders in full, and how this partial backlog impacts inventory decisions and overall system performance.

Looking even further ahead, we have a vision to encompass a broader range of products in our analytical framework. This will involve integrating both perishable and non-perishable products into the same supply chain, a scenario that presents its own set of challenges and opportunities. By doing so, we aim to provide a more holistic and versatile approach to inventory management within complex supply chains.

In summary, our research journey begins with a detailed analysis of a supply chain involving two client categories and a continuous review inventory control system. Through empirical measurement, probability analysis, and practical examples, we aim to provide valuable insights for supply chain practitioners. Moreover, our commitment to continuous improvement drives us to expand our model to incorporate perishable goods, partial backlogging, and a wider product range, thereby enhancing its applicability and relevance in the ever-evolving world of supply chain management.

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